Written Assignment 3
Due October 7, 2015 at 12:30pm

You may discuss this assignment with other students and work on the problems together. However, your write-up should be your own individual work and you must acknowledge the students you work with. Written assignments must be turned in at the start of lecture on the indicated due date.

1. (12 points) Give a context-free grammar (CFG) for each of the following languages over the alphabet \( \Sigma = \{a, b\} \)

   (a) (3 points) All strings in the language \( L : \{a^n b^m a^{2n} \mid n, m \geq 0\} \)

   (b) (3 points) All non-empty strings that start and end with the same symbol

   (c) (3 points) All strings with more a’s than b’s

   (d) (3 points) All palindromes (a palindrome is a string that reads the same forwards and backwards)

2. (8 points, 2 each) For each of the languages of (1), give an equivalent regular expression or explain why the language is not regular.

3. (45 points) In the old days, scientific calculators and many programs took their input in what is known as reverse polish notation. In this notation, the operator is written after the operands. For example, the expression \(3 4 +\) means \((3 + 4)\) and the expression \(3 4 + 7 *\) means \((3 + 4) * 7\). Consider the following grammar for expressions in RPN:

\[
S \rightarrow \text{int} \mid S_1 S_2 + \mid S_1 S_2 *
\]

   (a) (5 points) Is this grammar ambiguous? If so, give two derivations for the same string, if not, explain why not.

   (b) (5 points) What is the advantage of taking input in RPN over in-fix notation?

   (c) (15 points) Give large-step operational semantics for the language of RPN expressions. Your semantics should encode the “intuitive” and expected semantics.

   (d) (5 points) Derive the meaning of \(3 4 + 2 *\) using your operational semantics. Hint: You need to draw a derivation tree.

   (e) (10 points) Give small-step operational semantics for the language of RPN expressions. Your semantics should the same semantics as part (c)

   (f) (5 points) Derive the meaning of \(3 4 + 2 *\) using your small-step operational semantics from (e).