



Fluid Updates: Beyond Strong vs. Weak Updates

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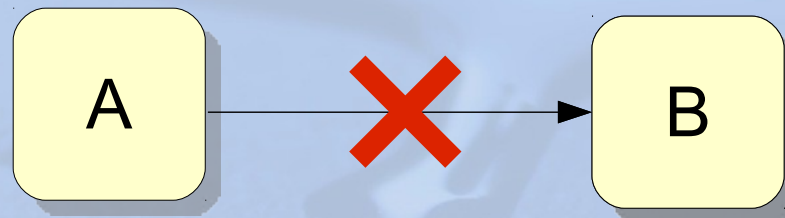


Strong and Weak Updates

- In *static analysis*, there is a distinction between two kinds of updates to *abstract* memory locations:

Strong vs *weak*

- Consider points-to edge from abstract location A to B:



- A *strong update* removes existing points-to edges from location A and adds new edge to C.

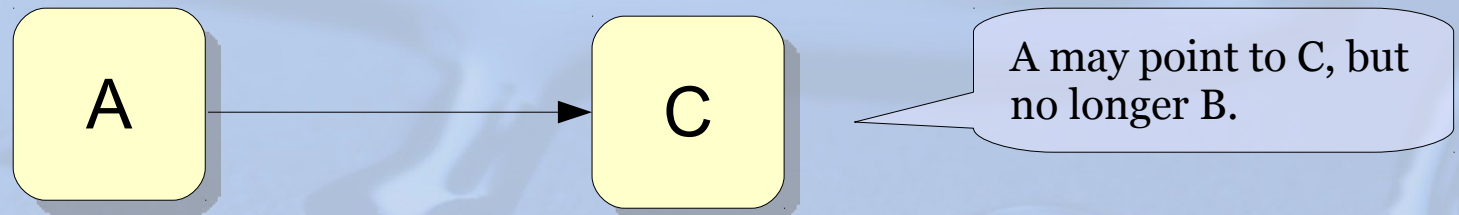


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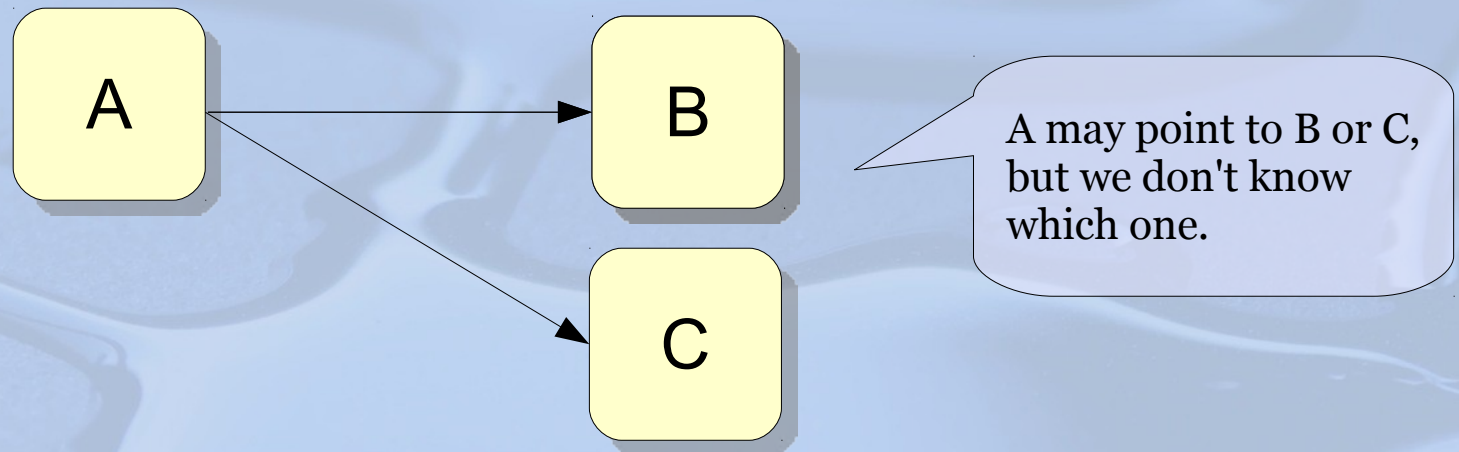


Strong and Weak Updates

- In *static analysis*, there is a distinction between two kinds of updates to *abstract* memory locations:

Strong vs *weak*

- Consider points-to edge from abstract location A to B:



- A *weak update* adds a new edge without removing existing edges.

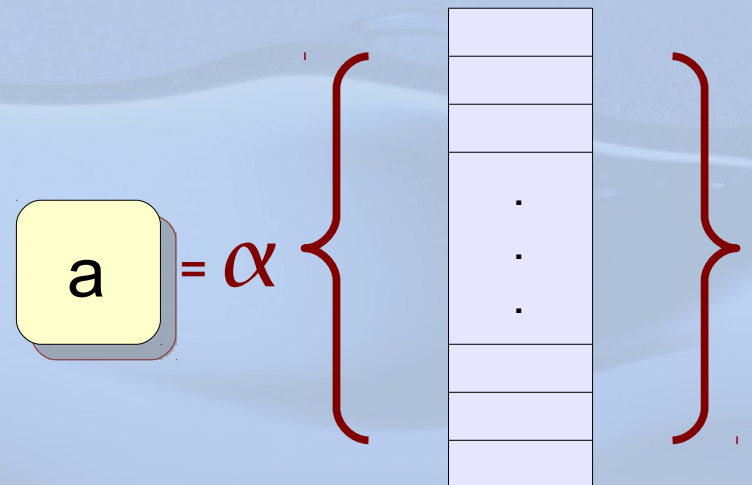


When are Strong Updates Applicable?

- ♦ In general, it is preferable to apply strong updates because it is more precise.
- ♦ However, we can only apply strong updates if the abstract location corresponds to *one concrete location*.
 - ♦ This makes it difficult to apply strong updates to elements of unbounded data structures, such as arrays.

Statically unknown size n.

```
int* a = malloc(n*sizeof(int))
```





Strong Updates to Arrays

- Many approaches overcome this difficulty by creating *partitions* of the array.

$a[k] = 7;$

This abstract location contains only the k'th element of the array.

$a : < k$

$a := k$ **7**

$a : > k$





Strong Updates to Arrays

- Many approaches overcome this difficulty by creating *partitions* of the array.

```
for (int k=0;  
     k < size; k++)  
    a[k] = 7;
```

a:<k 7

a:=k 7

a:>k

We need to avoid creating an unbounded number of partitions.



Strong Updates to Arrays

- Many approaches overcome this difficulty by creating *partitions* of the array.

```
for(int k=0;  
    k < size; k++)  
    a[k] = 7;
```

a:<=k 7

a:>k

Finalizes the number
of partitions.



Drawbacks

- ♦ This approach has some drawbacks:
 - ♦ Can create *large number* of explicit partitions
 - ➡ Limits scalability
 - ♦ *Heuristics* to decide when to focus/blur
 - ➡ Can be difficult to automate
 - ♦ „*Shape*” of partitions are *fixed a priori*
 - ➡ Typically contiguous ranges
- ♦ The rest of this talk is about a *heap abstraction* and *update mechanism* that does not have these drawbacks.



Symbolic Heap Abstraction

- ♦ Key Idea #1:
 - ♦ Arrays are represented by abstract locations that are qualified by *index variables*.

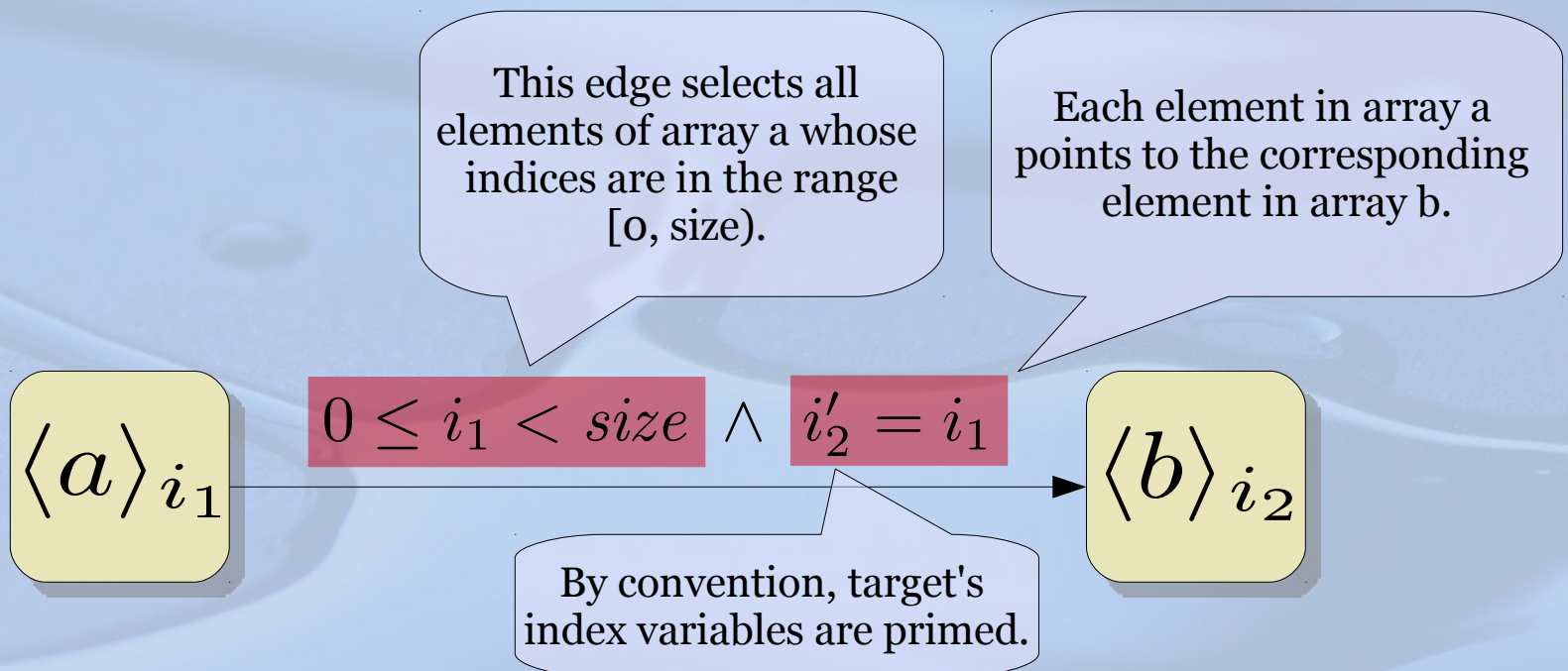
$$\langle a \rangle_i$$

- ♦ These index variables range over *possible indices* of the array.



Symbolic Heap Abstraction

- ♦ **Key Idea #2:**
 - ♦ *Constraints on index variables* select which concrete elements in the source location point to which concrete elements in the target location.



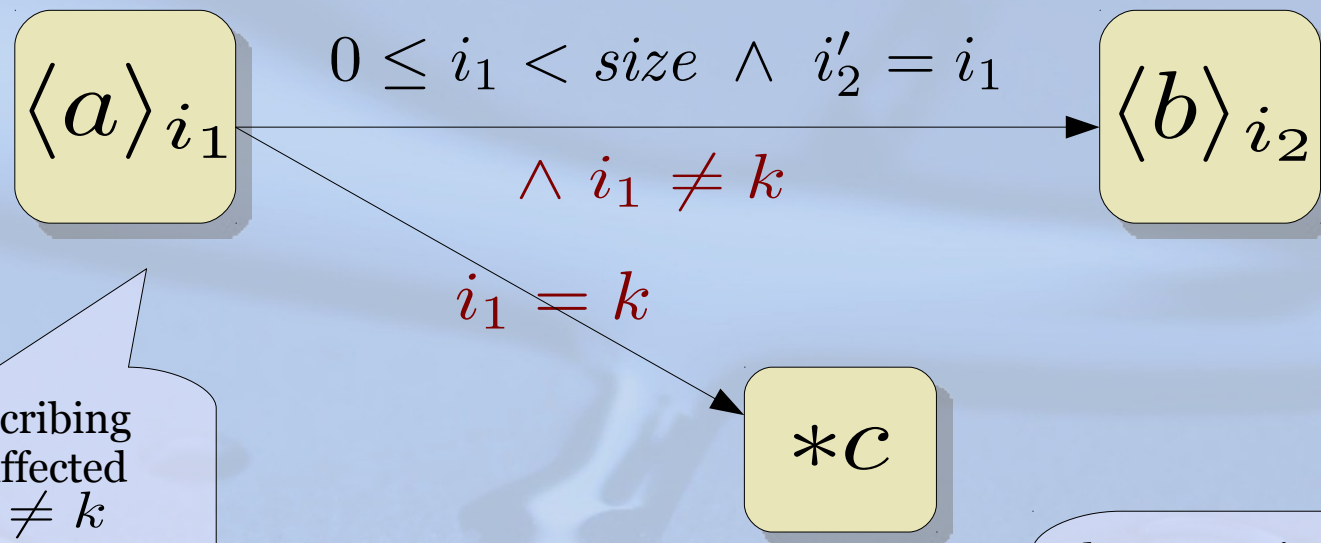


Fluid Updates

- ♦ To perform a *fluid update* on the symbolic heap:
 - ♦ Compute a constraint Φ specifying which elements in an abstract location A are modified by the update.
 - ♦ Now, the negation of Φ , $\neg\Phi$, specifies the concrete elements in A *not affected* by the update.
 - ♦ Hence, a fluid update conjoins $\neg\Phi$ with all *existing edges* from A , while adding *new edge* under Φ .



Fluid Update Example



Constraint describing
elements not affected
by update: $i_1 \neq k$

The constraint describing
updated elements is:
 $i_1 = k$

Consider a statement: $\mathbf{a[k] = c;}$



What if we aren't sure which elements are updated?

- In the previous example, we could precisely describe the *exact* set of elements that were written to.
- In general, we cannot always precisely specify which elements are updated.

```
for(i=0; i<size; i++)  
{  
    if(rand())  
        a[i] = NULL;  
}
```

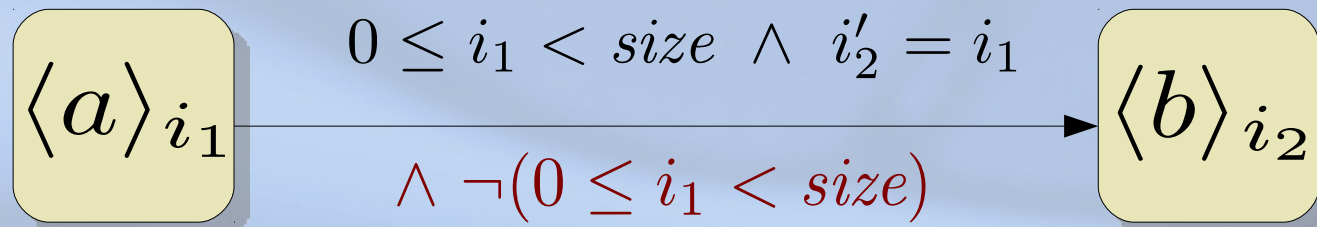
All elements in range [0, size)
may be set to NULL,
but no element
must be set to NULL.

So, an *overapproximation*
of the elements updated in
the loop is:

$$0 \leq i_1 < size$$



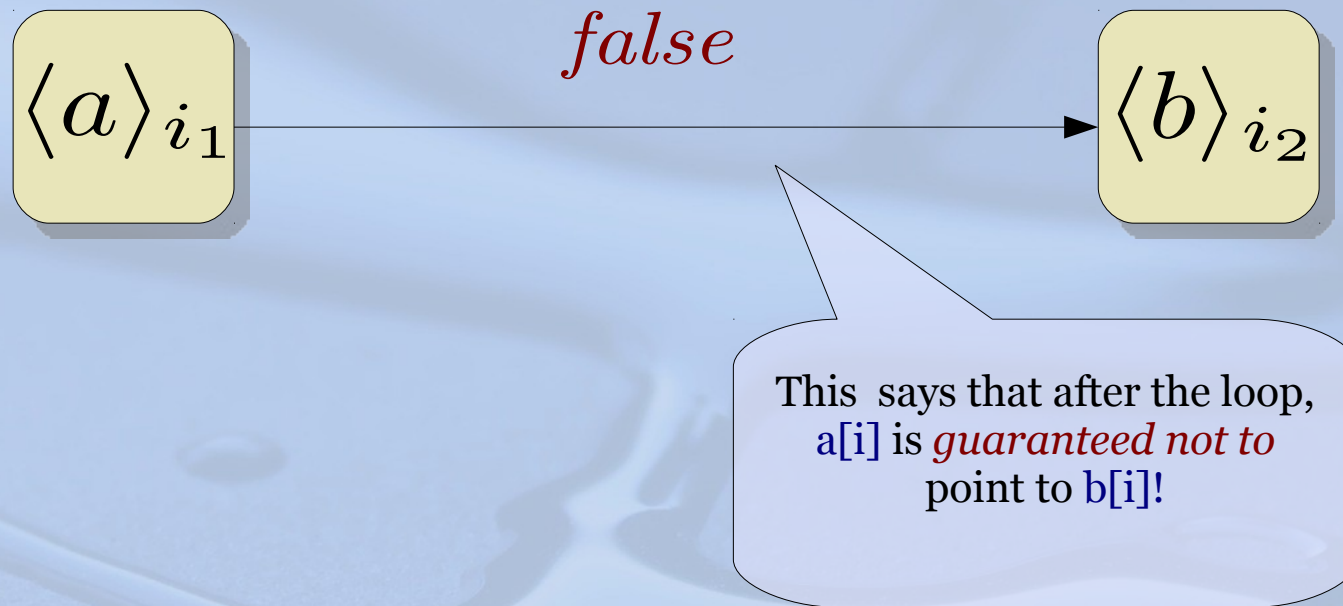
Fluid Update with Overapproximations



Suppose we *erroneously* use the **overapproximation** for the update



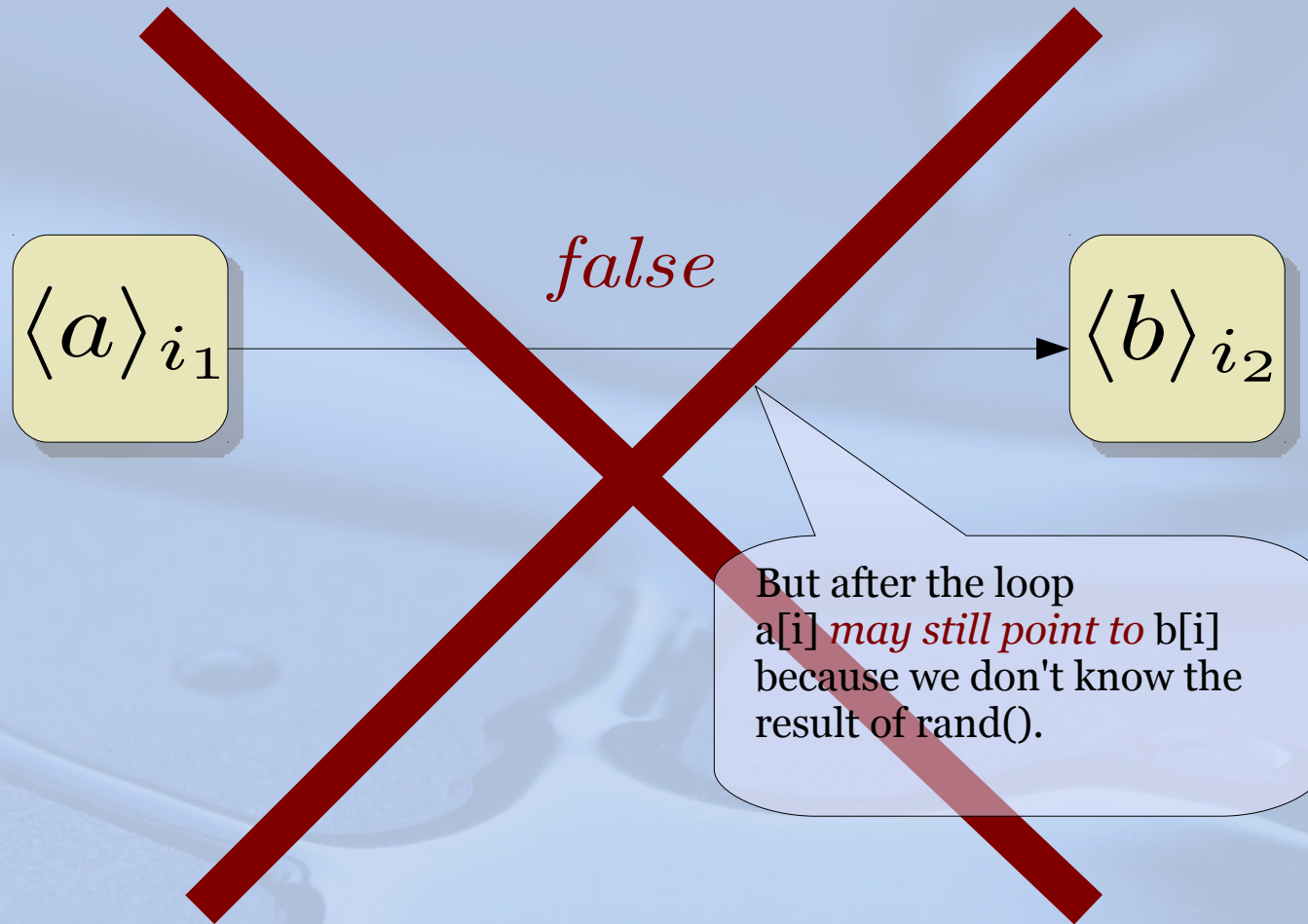
Fluid Update with Overapproximations



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Fluid Update with Overapproximations



Suppose we *erroneously* use the **overapproximation** for the update



What went wrong?

- ♦ The *negation of an overapproximation* is an *underapproximation*.
- ♦ Hence, if the fluid update uses an overapproximation, we *underapproximate* the set of elements *not affected* by the update.



UNSOUND!





Bracketing Constraints

- ♦ **Solution:** Constraints in the symbolic heaps are pairs of constraints, called *bracketing constraints*:

$$\langle \phi_{may}, \phi_{must} \rangle$$

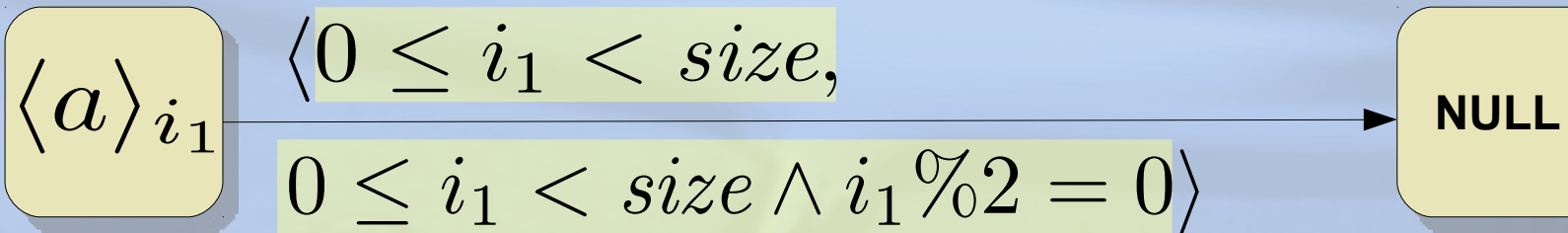
This specifies which elements in the source location **may** point to which elements in the target.

This specifies which elements in the source location **must** point to which elements in the target.



Bracketing Constraint Example

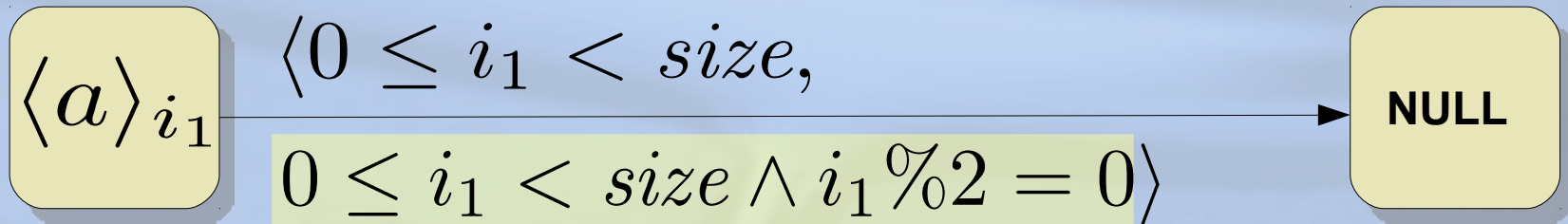
Any element in the range
[0, size) **may** point to null.



All **even** elements in the range
[0, size) **must** point to null.



Bracketing Constraint Example



But, **odd** elements in the range
[0, size) **may or may not**
point to null.



Fluid Update Using Bracketing Constraints

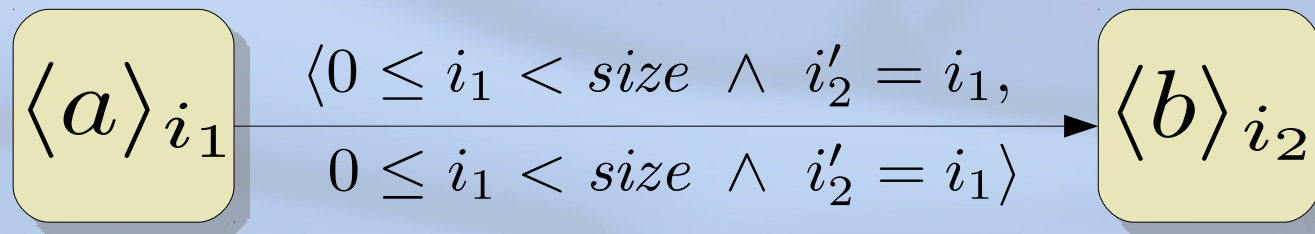
- ♦ If we use bracketing constraints, the fluid update operation described earlier is sound.
 - ♦ This is because negating a bracketing constraint yields correct over- and underapproximations.
- ♦ In particular, the negation of a bracketing constraint $\langle \phi_{may}, \phi_{must} \rangle$ is given by:

$$\neg \langle \phi_{may}, \phi_{must} \rangle \iff \langle \neg \phi_{must}, \neg \phi_{may} \rangle$$



Fluid Update Using Bracketing Constraints

- Consider again the following initial symbolic heap:



If a bracketing constraint $\langle \phi_{may}, \phi_{must} \rangle$ is **precise**, i.e., $\phi_{may} \Leftrightarrow \phi_{must}$ then we write a single constraint instead of a pair.



Fluid Update Using Bracketing Constraints

- Consider again the following initial symbolic heap:



From now on, think of a single constraint as a bracketing constraint where *may* and *must* conditions are the same.



Fluid Update Using Bracketing Constraints

- Consider again the following initial symbolic heap:



The negation of this constraint is:

$\langle \neg false, \neg(0 \leq i_1 < size) \rangle$

```
for(i=0; i<size; i++)  
{  
    if(rand())  
        a[i] = NULL;  
}
```

A bracketing constraint describing the update condition is:

$\langle 0 \leq i_1 < size, false \rangle$



Fluid Update Using Bracketing Constraints

- Consider again the following initial symbolic heap:



The negation of this constraint is:

$\langle true, i_1 < 0 \vee i_1 \geq size \rangle$

```
for(i=0; i<size; i++)  
{  
    if(rand())  
        a[i] = NULL;  
}
```

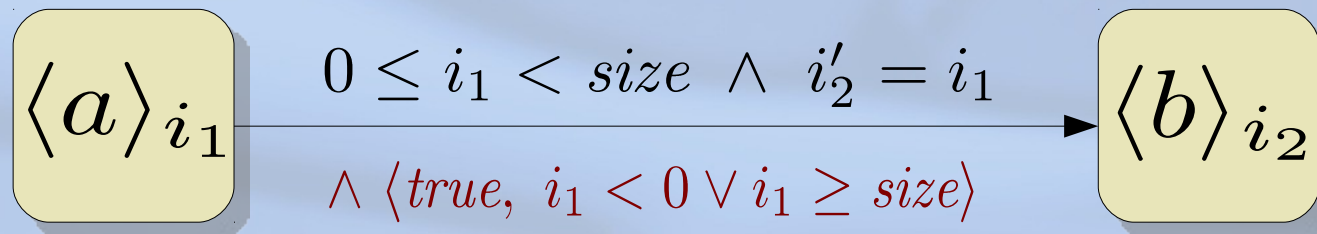
A bracketing constraint describing the update condition is:

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Example Revisited

- Consider again the following initial symbolic heap:



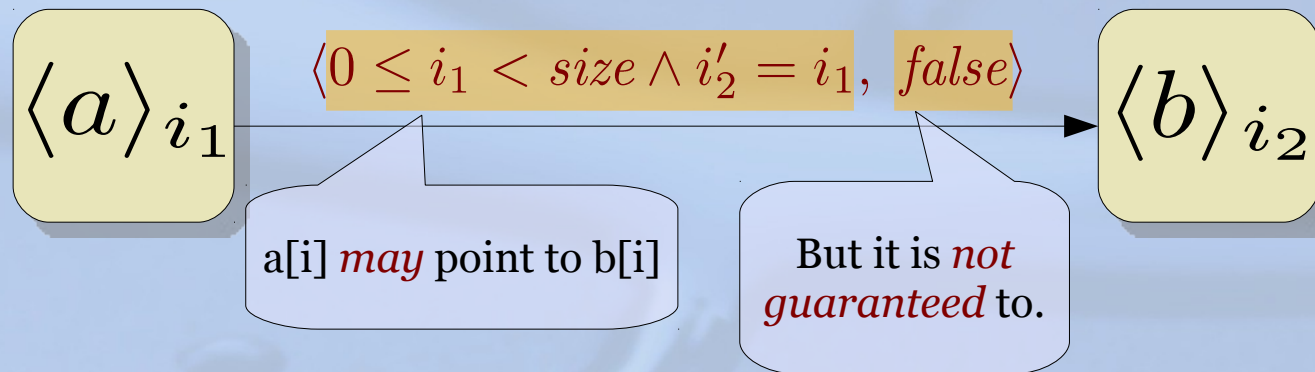
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}
```

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Example Revisited

- Consider again the following initial symbolic heap:



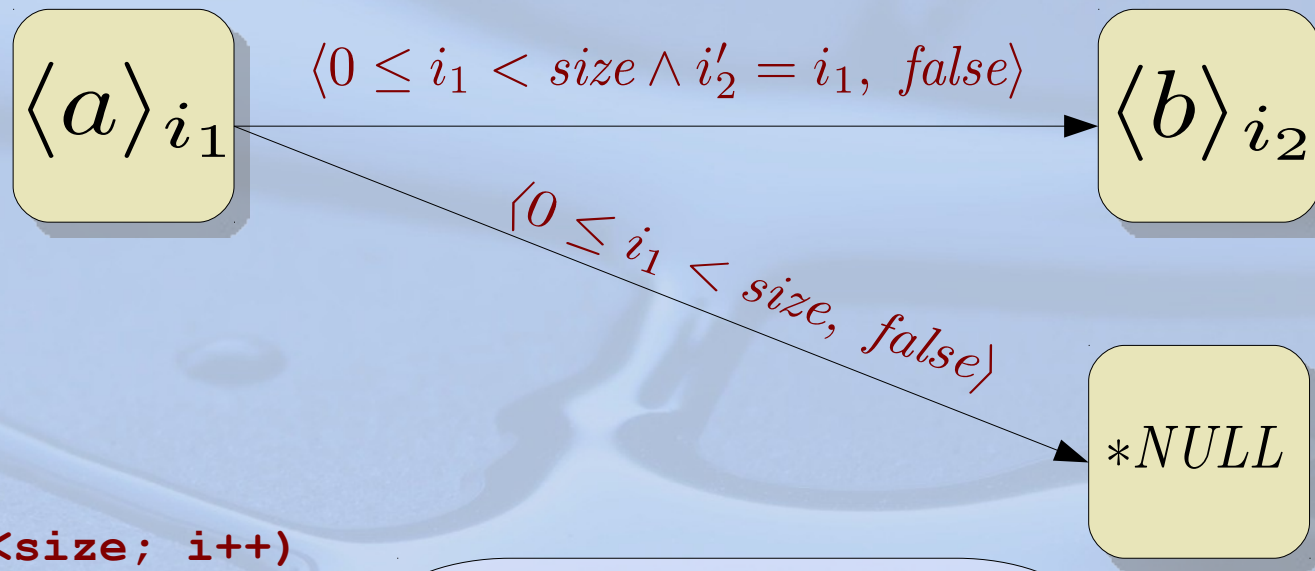
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Example Revisited

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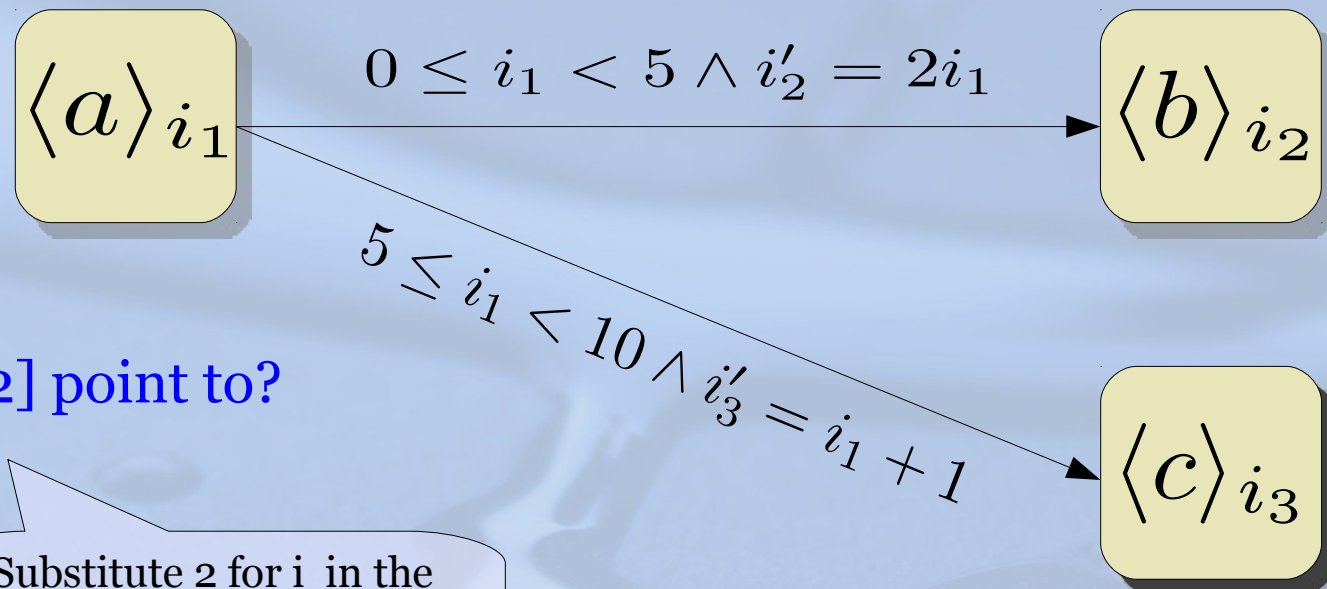
Interpreting the Symbolic Heap

- ♦ So far, we described what a symbolic heap looks like and defined a fluid update operation on this heap.
- ♦ But how do we interpret/understand the facts that are encoded by the symbolic heap?



Interpreting the Symbolic Heap

- Consider the following:



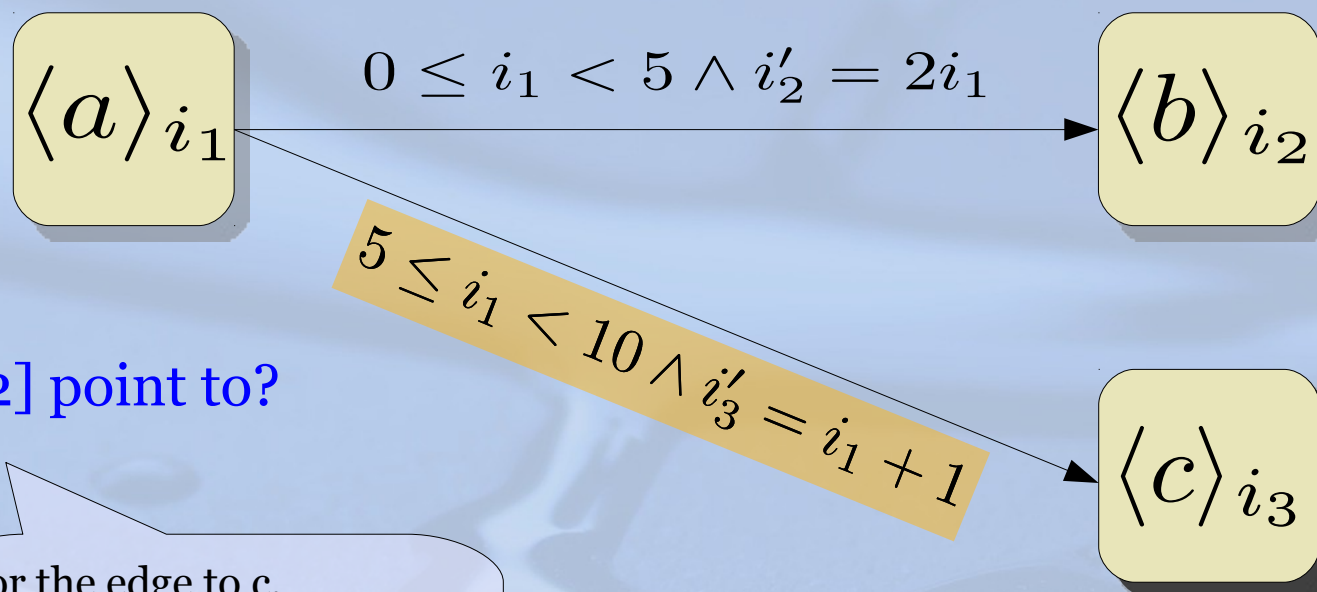
Where does $a[2]$ point to?

Substitute 2 for i_1 in the edge constraints.



Interpreting the Symbolic Heap

- Consider the following:



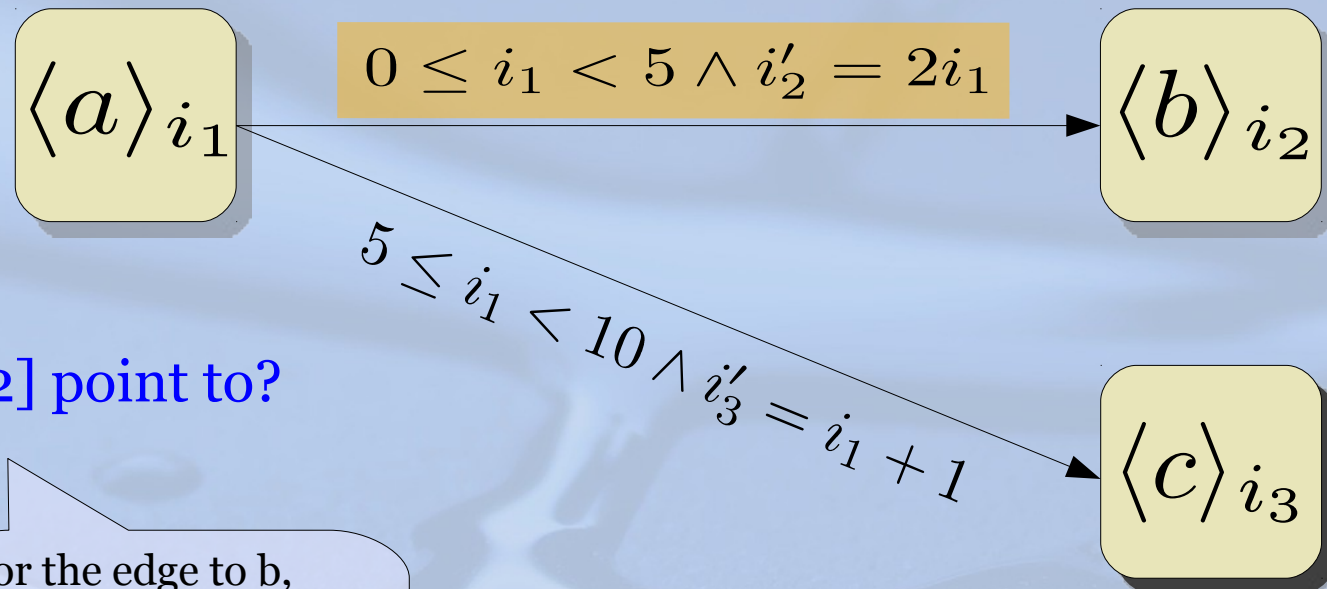
Where does $a[2]$ point to?

For the edge to c ,
the constraint $5 \leq i_1$
is unsatisfiable, hence, $a[2]$
cannot point to $c[3]$.



Interpreting the Symbolic Heap

- Consider the following:



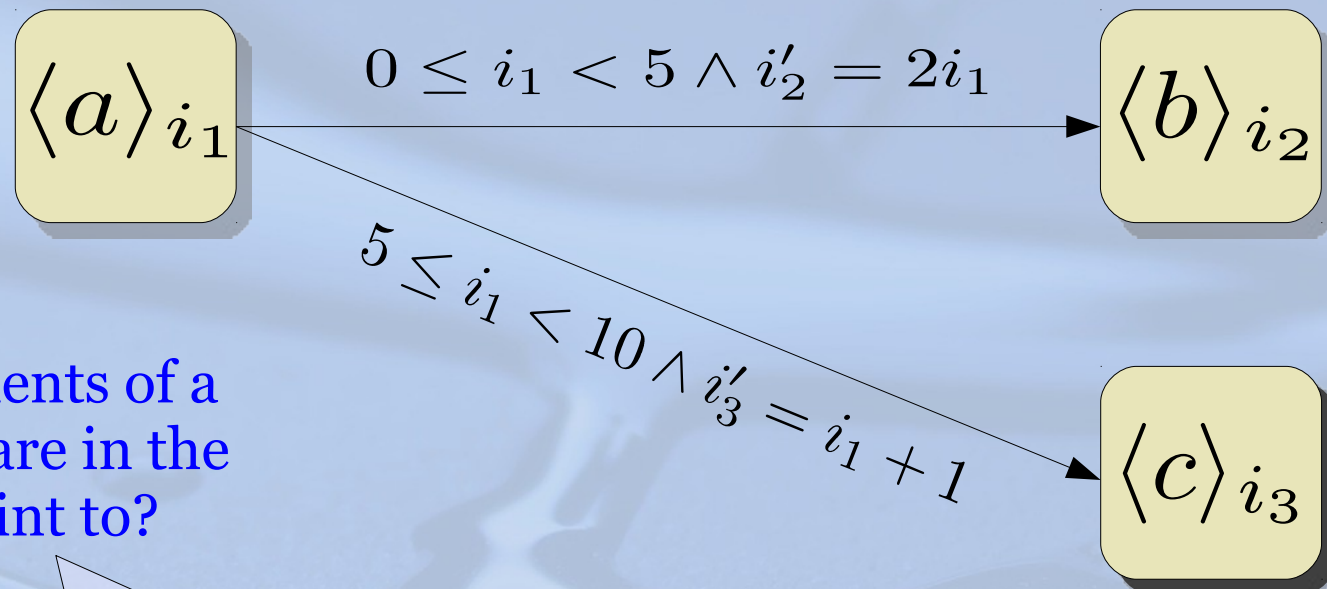
Where does $a[2]$ point to?

For the edge to b ,
substituting 2 yields
 $i'_2 = 4$; so $a[2]$ points
to $b[4]$.



Interpreting the Symbolic Heap

- Consider the following:



Where do elements of a whose indices are in the range $[0, 3]$ point to?

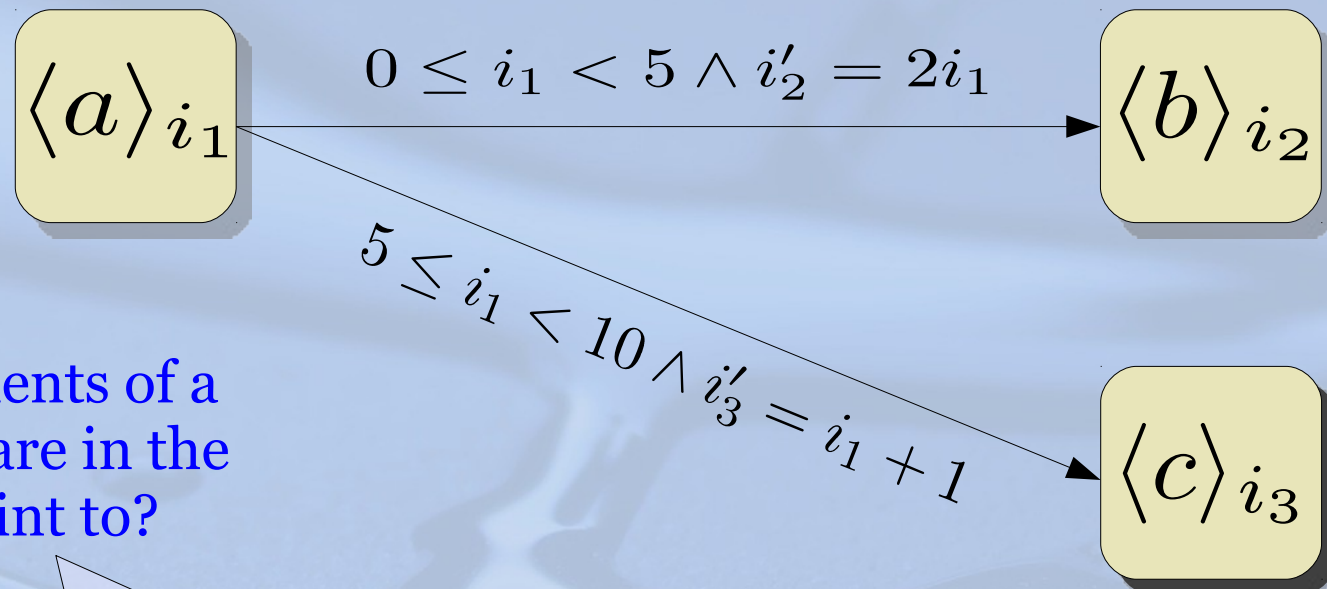
In general, we need to know the points-to targets of those elements whose indices *satisfy some constraint*.

Here, $0 \leq i_1 \leq 3$



Interpreting the Symbolic Heap

- Consider the following:



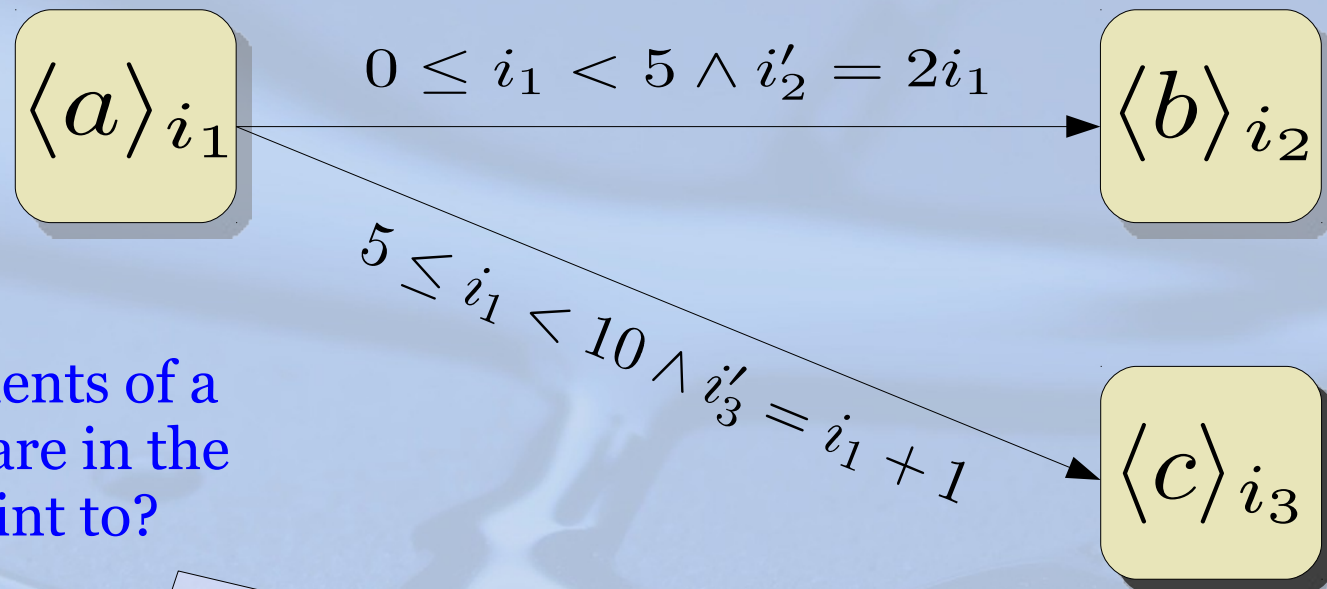
Where do elements of a
whose indices are in the
range $[0, 3]$ point to?

So, we need a generalized
form of substitution, i.e.,
*existential quantifier
elimination*.



Interpreting the Symbolic Heap

- Consider the following:



Where do elements of a whose indices are in the range $[0, 3]$ point to?

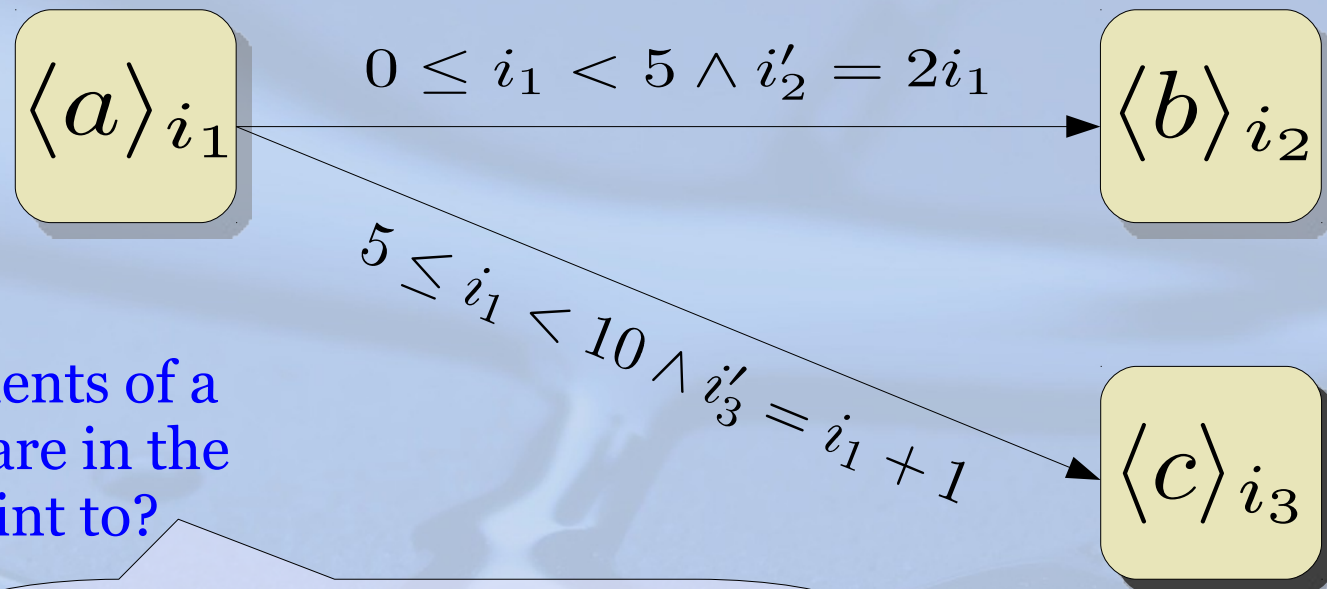
We can determine the points-to targets of elements whose indices are in $[0, 3]$, by *eliminating the quantifier* from the following formula (for the edge to b):

$$\exists i_1. 0 \leq i_1 \leq 3 \wedge (0 \leq i_1 < 5 \wedge i'_2 = 2i_1)$$



Interpreting the Symbolic Heap

- Consider the following:



Where do elements of a whose indices are in the range $[0, 3]$ point to?

This yields: $i'_2 \% 2 = 0 \wedge 0 \leq i'_2 \leq 6$

Elements of a in range $[0, 3]$ point to **even** elements of b in range $[0, 6]$.



Interpreting the Symbolic Heap

- ♦ To summarize, *traversing edges* (i.e., going from one abstract location to its points-to target) requires *existential quantifier elimination*.
- ♦ Similar to image computation and computation of strongest post-conditions.



Implementation

- ♦ The combination of symbolic heap abstraction and fluid updates forms the core of the *Compass* program verification system for C programs.
 - ♦ Compass is path- and context-sensitive
 - ♦ *Summary-based*
 - ♦ Used for checking *memory safety properties* (buffer overruns, null errors, uninitialized reads, casting errors, ...) as well as arbitrary *user-provided assertions*.
 - ♦ Has successfully been scaled to real programs in the range of a few 10,000 lines of code.



Case Study

- We first evaluate this technique on a set of challenging array benchmarks.

Program	Time	Memory	#Sat queries	Solve time
init	0.01s	< 1 MB	172	0s
init_nonconst	0.02s	< 1 MB	184	0.01s
init_partial	0.01s	< 1MB	166	0.01s
init_partial_buggy	0.02s	< 1 MB	168	0s
init_even	0.04s	< 1 MB	146	0.04s
init_even_buggy	0.04s	< 1 MB	166	0.03s
2D_array_init	0.04s	< 1 MB	311	0.04s
copy	0.01s	< 1 MB	209	0.01s
copy_partial	0.01s	< 1 MB	220	0.01s
copy_odd	0.04s	< 1 MB	243	0.02s
copy_odd_buggy	0.05s	< 1 MB	246	0.05s
reverse	0.03s	< 1 MB	273	0.01s
reverse_buggy	0.04s	< 1 MB	281	0.02s
swap	0.12s	2 MB	590	0.11s
swap_buggy	0.11s	2 MB	557	0.06s
double_swap	0.16s	2 MB	601	0.1s
strcpy	0.07s	< 1 MB	355	0.04s
strlen	0.02s	< 1 MB	165	0.01s
strlen_buggy	0.01s	< 1 MB	89	0.01s
memcpy	0.04s	< 1 MB	225	0.04s
find	0.02s	< 1 MB	119	0.02s
find_first_nonnull	0.02s	< 1 MB	183	0.02s
append	0.02s	< 1 MB	183	0.01s
merge_interleave	0.09s	< 1 MB	296	0.07s
merge_interleave_buggy	0.11s	< 1 MB	305	0.09s
alloc_fixed_size	0.02s	< 1 MB	176	0.02s
alloc_fixed_size_buggy	0.02s	< 1 MB	172	0.02s
alloc_nonfixed_size	0.03s	< 1 MB	214	0.02

Also very memory efficient.

Very fast despite being very precise.

Enter Code Callgraph Cfg Summary Statistics Step through Unit Regressions

Project used: current Name: Case Study/Swap

Files Units Callg < >

Unit Time

check_swap
check_swap@3
check_swap@7
swap
swap@3

test.c

```
void swap(int* a, int* b, int size)
{
    int i;
    for(i=0; i<size; i++) {
        int t = a[i];
        a[i] = b[i];
        b[i] = t;
    }
}

void check_swap(int size, int* a, int* b)
{
    int i;
    int* a_copy = malloc(sizeof(int)*size);
    int* b_copy = malloc(sizeof(int)*size);
    for(i=0; i<size; i++) {
        a_copy[i] = a[i];
        b_copy[i] = b[i];
    }
    swap(a, b, size);
    for(i=0; i<size; i++) {
        static_assert(a[i] == b_copy[i]);
        static_assert(b[i] == a_copy[i]);
    }

    free(a_copy);
    free(b_copy);
}
```

New File

Status

Num CPUs 1

Walltime 0

CPU time 0

0%

5/ 0

☐ Check Buffers

☐ Check Null

☐ Check Uninit

Run

Cancel

Add as regression

File Line Message

Enter Code Callgraph Cfg Summary Statistics Step through Unit Regressions

Project used: current Name: Case Study/Swap

Files Units Callg < >

Unit	Time
check_swap	0.03
check_swap@3	0.02
check_swap@7	0.02
swap	0.01
swap@3	0.03

test.c

```
void swap(int* a, int* b, int size)
{
    int i;
    for(i=0; i<size; i++) {
        int t = a[i];
        a[i] = b[i];
        b[i] = t;
    }
}

void check_swap(int size, int* a, int* b)
{
    int i;
    int* a_copy = malloc(sizeof(int)*size);
    int* b_copy = malloc(sizeof(int)*size);
    for(i=0; i<size; i++) {
        a_copy[i] = a[i];
        b_copy[i] = b[i];
    }
    swap(a, b, size);
    for(i=0; i<size; i++) {
        static_assert(a[i] == b_copy[i]);
        static_assert(b[i] == a_copy[i]);
    }

    free(a_copy);
    free(b_copy);
}
```

New File

Status

Num CPUs 1

Walltime 0

CPU time 0.11

100%

5/ 5

29 lines

☐ Check Buffers

☐ Check Null

☐ Check Uninit

Run

Cancel

Add as regression

File Line Message

Enter Code Callgraph Cfg Summary Statistics Step through Unit Regressions

Project used: current Name: Case Study/Swap

Files Units Callg < >

Unit	Time
check_swap	0.03
check_swap@3	0.02
check_swap@7	0.02
★ swap	0.01
swap@3	0.03

test.c

```
void swap(int* a, int* b, int size)
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    int i;
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        a[i] = b[i];
        b[i] = t;
    }
}

void check_swap(int size, int* a, int* b)
{
    int i;
    int* a_copy = malloc(sizeof(int)*size);
    int* b_copy = malloc(sizeof(int)*size);
    for(i=0; i<size; i++) {
        a_copy[i] = a[i];
        b_copy[i] = b[i];
    }
    swap(a, b, size);
    for(i=0; i<size; i++) {
        static_assert(a[i] == b_copy[i]);
        static_assert(b[i] == a_copy[i]);
    }

    free(a_copy);
    free(b_copy);
}
```

New File

Status

Num CPUs 1

Walltime 0

CPU time 0.11

100%

5/ 5

29 lines

☐ Check Buffers

☐ Check Null

☐ Check Uninit

Run

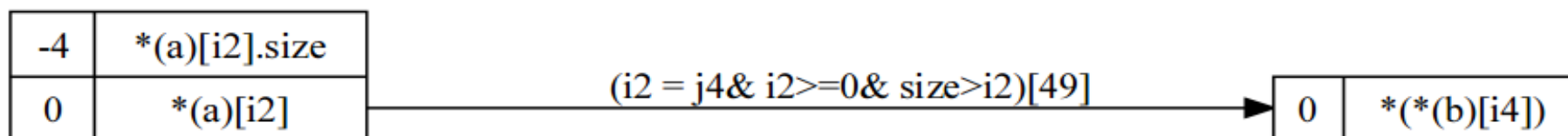
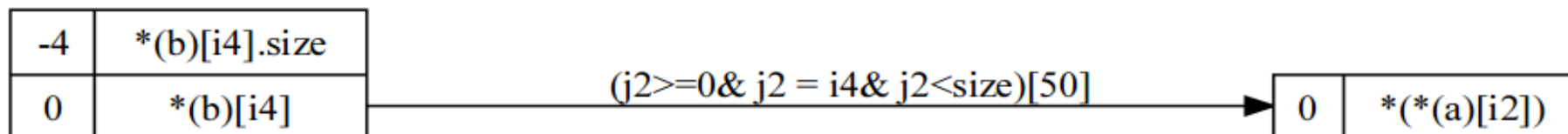
Cancel

Add as regression

File Line Message

Enter Code Callgraph Cfg Summary Statistics Step through Unit Regressions

Summary Unit: /home/tdillig/compass/current/test.c/swap<[nonret] (int*, int*, int)>



Return Condition: true
Loop Counters:
Error trace summary:

Enter Code Callgraph Cfg Summary Statistics Step through Unit Regressions

Project used: current Name: Case Study/Swap

Files Units Callg < >

Name

+ compass
+ sail
test.c

test.c

New File

```
void swap(int* a, int* b, int size)
{
    int i;
    for(i=0; i<size; i++) {
        int t = a[i];
        a[i] = b[i];
        b[i] = t;
    }
}

void check_swap(int size, int* a, int* b)
{
    int i;
    int* a_copy = malloc(sizeof(int)*size);
    int* b_copy = malloc(sizeof(int)*size);
    for(i=0; i<size; i++) {
        a_copy[i] = a[i];
        b_copy[i] = b[i];
    }
    swap(a, b, size);
    for(i=0; i<size; i++) {
        static_assert(a[i] == b_copy[i]);
        static_assert(b[i] == a_copy[i]);
    }

    free(a_copy);
    free(b_copy);
}
```

Status

Num CPUs 1

Walltime 0

CPU time 0

0%

5/ 0

☐ Check Buffers

☐ Check Null

☐ Check Uninit

Run

Cancel

Add as regression

File

Line

Message

Enter Code Callgraph Cfg Summary Statistics Step through Unit Regressions

Project used: current Name: Case Study/Swap

Files Units Callg < >

Name

+ compass
+ sail
test.c

test.c

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        b[i] = t;
    }
}

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    int i;
    int* a_copy = malloc(sizeof(int)*size);
    int* b_copy = malloc(sizeof(int)*size);
    for(i=0; i<size; i++) {
        a_copy[i] = a[i];
        b_copy[i] = b[i];
    }
    swap(a, b, size);
    for(i=0; i<size; i++) {
        static_assert(a[i] == b_copy[i]);
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    }

    free(a_copy);
    free(b_copy);
}
```

New File

Status

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Walltime 0

CPU time 0

0%

5/ 0

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Run

Cancel

Add as regression

File

Line

Message

Enter Code Callgraph Cfg Summary Statistics Step through Unit Regressions

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Files Units Callg < >

Name

+ compass
+ sail
test.c

test.c

New File

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        b[i] = a[i];
        a[i] = t;
    }
}

void check_swap(int size, int* a, int* b)
{
    int i;
    int* a_copy = malloc(sizeof(int)*size);
    int* b_copy = malloc(sizeof(int)*size);
    for(i=0; i<size; i++) {
        a_copy[i] = a[i];
        b_copy[i] = b[i];
    }
    swap(a, b, size);
    for(i=0; i<size; i++) {
        static_assert(a[i] == b_copy[i]);
        static_assert(b[i] == a_copy[i]);
    }

    free(a_copy);
    free(b_copy);
}
```

Status

Num CPUs 1

Walltime 0

CPU time 0.1

100%

5/ 5

29 lines

☐ Check Buffers

☐ Check Null

☐ Check Uninit

Run

Cancel

Add as regression

File	Line	Message
test.c	23	Static assert failed



Unix Coreutils

- A collection of widely used command-line utilities.
- Heavily use arrays, pointers, and string buffers.
- Precise heap analysis necessary for successful verification.

Program	Lines	Total Time	Memory	#Sat queries	Solve Time
hostname	304	0.13s	5 MB	1533	0.12s
chroot	371	0.13s	3 MB	1821	0.10s
rmdir	483	1.05s	12 MB	3461	1.02s
su	1047	1.86s	32 MB	6088	1.69s
mv	1151	0.70s	21 MB	7427	0.68s
Total	3356	3.87s	73 MB	20330	3.61

Compass verified absence of **buffer overruns** and **null dereferences** with *no false positives* and *no annotations*.



Thank You

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