Sound, Complete, and Scalable Path-Sensitive Analysis

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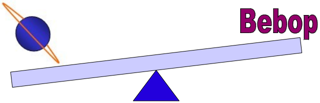
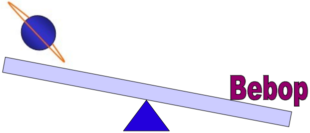
- Therefore, there are many proposed techniques for path- and context-sensitive program analysis.
  - Model checking tools: Bebop, BLAST, SLAM, ...
  - Lighter-weight static analysis tools: Saturn, ESP, ...

Tradeoff?

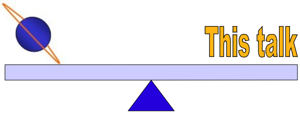


The Scalability Scale



Sound & Complete Scale

This Talk



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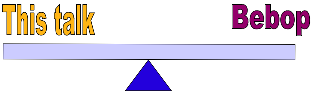

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Technique for path- and context-sensitive analysis that guarantees:

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Key Insight:

- We can distinguish a special class of variables called unobservable variables

- These variables can be eliminated from formulas used to express path-sensitive conditions without any loss of precision
  - Smaller formulas ⇒ Better scalability

An Example

```
bool queryUser(bool featureEnabled) {
   if(!featureEnabled) return false;
   char userInput = getUserInput();
   if(userInput == 'y') return true;
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When does queryUser return true?

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Given an arbitrary argument α, what is the constraint $\Pi_{\alpha,\text{true}}$ under which queryUser returns true?

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The existential quantifier expresses:

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- Note: The existential has slightly non-standard semantics.

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- $\exists$-bound variables cause problems with termination.

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int* p = malloc(sizeof(int));
if(!p) return;
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if(arr[i]==0) return;
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- Return Variables (Π)
  - Represent unknowns we want to solve for

Generalized Recursive Constraints

$$E = \begin{bmatrix} [\vec{\Pi}_{f_1,\alpha,C_i}] & = & \exists \vec{\beta_1}.\ [\vec{\phi}_{1i}(\vec{\alpha}_1, \vec{\beta}_1, \vec{\Pi}[\vec{b}_1/\vec{\alpha}])] \\ \vdots & & \vdots \\ [\vec{\Pi}_{f_k,\alpha,C_i}] & = & \exists \vec{\beta_k}.\ [\vec{\phi}_{ki}(\vec{\alpha}_k, \vec{\beta}_k, \vec{\Pi}[\vec{b}_k/\vec{\alpha}])] \end{bmatrix}$$

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Bad News

Unfortunately, we do not know of a way to obtain an exact solution to these constraints.

Good News

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- For must queries, we only need a **validity** preserving solution.

Strongest Necessary and Weakest Sufficient Conditions

- For any formula ϕ, the **strongest necessary condition** $\lceil\phi\rceil$ of ϕ containing *only observable variables* preserves satisfiability.

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\begin{array}{ll}
(1) & \phi \Rightarrow \lceil\phi\rceil \\
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- Similarly, for any formula ϕ the **weakest sufficient condition** $\lfloor \phi \rfloor$ over *only observable variables* preserves validity of ϕ.

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(1) & \lfloor \phi \rfloor \Rightarrow \phi \\
(2) & \forall \phi'.((\phi' \Rightarrow \phi) \Rightarrow (\phi' \Rightarrow \lfloor \phi \rfloor))
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Strongest Necessary and Weakest Sufficient Conditions (2)

If ϕ is the constraint under which a program property P holds, we have the following guarantees:

$$\text{SAT}(\lceil \phi \rceil) \Leftrightarrow \text{P } \textbf{MAY} \text{ hold}$$

$$\text{VALID}(\lfloor \phi \rfloor) \Leftrightarrow \text{P } \textbf{MUST} \text{ hold}$$

Example Revisited

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Original constraint:

$$\Pi_{\alpha,\mathsf{true}} = \exists\beta.\ ((\alpha = \mathsf{true}) \wedge (\beta = \text{'y'} \vee (\beta \neq \text{'n'} \wedge \Pi_{\alpha,\mathsf{true}}[\mathsf{true}/\alpha])))$$

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Strongest Necessary Condition: $\lceil \Pi_{\alpha,\text{true}} \rceil = (\alpha = \text{true})$

Weakest Sufficient Condition: $\lfloor \Pi_{\alpha,\text{true}} \rfloor = \text{false}$

Generalized Recursive Constraints Revisited

$$E = \left[\begin{array}{lcl} [\vec{\Pi}_{f_1,\alpha,C_i}] & = & \exists \vec{\beta}_1.\ [\vec{\phi}_{1i}(\vec{\alpha}_1, \vec{\beta}_1, \vec{\Pi}[\vec{b}_1/\vec{\alpha}])] \\ \vdots & & \vdots \\ [\vec{\Pi}_{f_k,\alpha,C_i}] & = & \exists \vec{\beta}_k.\ [\vec{\phi}_{ki}(\vec{\alpha}_k, \vec{\beta}_k, \vec{\Pi}[\vec{b}_k/\vec{\alpha}])] \end{array}\right]$$

Goal: Compute observable strongest necessary and weakest sufficient conditions for the solution of E.

Outline of the Algorithm

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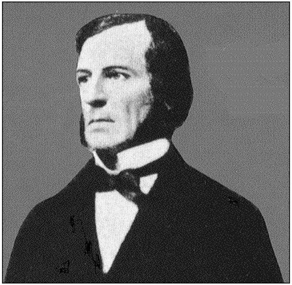
- Step 3: Solve the recursive constraints via fixed-point computation (syntactic substitution)

Step 1: Eliminate Unobservable Variables

$$\text{SNC}(\phi, \beta) = \phi[\text{true}/\beta] \vee \phi[\text{false}/\beta]$$

$$\text{WSC}(\phi, \beta) = \phi[\text{true}/\beta] \wedge \phi[\text{false}/\beta]$$



Result of Step 1

$$E_{\mathsf{NC}} = \begin{bmatrix} \lceil \Pi_{f_1,\alpha,C_1} \rceil & = & \phi'_{11}(\vec{\alpha_1}, \lceil \vec{\Pi} \rceil [\vec{b}_1/\vec{\alpha}]) \\ & \vdots & \\ \lceil \Pi_{f_k,\alpha,C_n} \rceil & = & \phi'_{kn}(\vec{\alpha}_k, \lceil \vec{\Pi} \rceil [\vec{b}_k/\vec{\alpha}]) \end{bmatrix}$$

$$E_{\mathsf{SC}} = \begin{bmatrix} \lfloor \Pi_{f_1,\alpha,C_1} \rfloor & = & \phi'_{11}(\vec{\alpha_1}, \lfloor \vec{\Pi} \rfloor [\vec{b}_1/\vec{\alpha}]) \\ & \vdots & \\ \lfloor \Pi_{f_k,\alpha,C_n} \rfloor & = & \phi'_{kn}(\vec{\alpha}_k, \lfloor \vec{\Pi} \rfloor [\vec{b}_k/\vec{\alpha}]) \end{bmatrix}$$

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  - Constraints contain negated Π literals.
    But $\neg\lceil\phi\rceil \not\Leftrightarrow \lceil\neg\phi\rceil$ and $\neg\lfloor\phi\rfloor \not\Leftrightarrow \lfloor\neg\phi\rfloor$

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  - Implicit constraints: Existence and uniqueness

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  - Either replace $\neg\Pi_{f,\alpha,c_i}$ with $\bigvee_{j\neq i}\Pi_{f,\alpha,c_j}$.
  - Or use the property $\lceil\neg\phi\rceil \Leftrightarrow \neg\lfloor\phi\rfloor$ and $\lfloor\neg\phi\rfloor \Leftrightarrow \neg\lceil\phi\rceil$

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- The latter requires simultaneous fixpoint computation of strongest necessary and weakest sufficient conditions
- But important for a practical implementation

Step 2: Preservation under Syntactic Substitution II

- To eliminate implicit existence and uniqueness constraints:
  - Convert to DNF and drop contradictions
    (for necessary conditions)
  - Convert to CNF and drop tautologies
    (for sufficient conditions)

Step 2: Preservation under Syntactic Substitution II

- To eliminate implicit existence and uniqueness constraints:
  - Convert to DNF and drop contradictions
    (for necessary conditions)
  - Convert to CNF and drop tautologies
    (for sufficient conditions)
- The resulting constraints preserve strongest necessary and weakest sufficient conditions under syntactic substitution.

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- Furthermore, since the computed strongest necesssary and weakest sufficient conditions do not contain any unobservable variables, the resulting constraints are small in practice, allowing the technique to scale to large programs.

Experiments I

- We compute the full interprocedural constraint -in terms of SNC's and WSC's- for every pointer dereference in OpenSSH, Samba and the Linux kernel ($>$6 MLOC).

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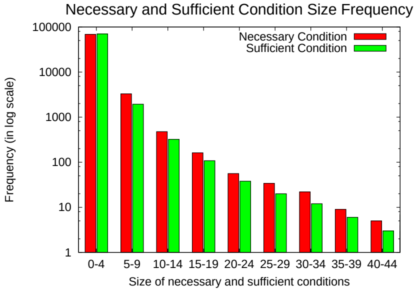

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	Interprocedurally Path-sensitive			Intraprocedurally Path-sensitive		
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Total Reports	3	48	171	21	379	1495
Bugs	1	17	134	1	17	134
False Positives	2	25	37	20	356	1344
Undecided	0	6	17	0	6	17
Report to Bug Ratio	3	2.8	1.3	21	22.3	11.2

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- Observed close to an order of magnitude reduction of false positives without resorting to (potentially unsound) ad-hoc heuristics.

Future Work

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    - Underlying framework collapses all unbounded data structures into one summary node
    - Imprecise for verifying memory safety.
- Shape analysis is our current work-in-progress.

Related Work

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