Subdivision Surfaces
Subdivision Curves

Idea: Repeatedly refine control polygon:

\[ P^1 \rightarrow P^2 \rightarrow P^3 \]

Curve will be limit of infinite process
Chaikin’s Algorithm

“Corner-cutting” scheme
1. Start with piecewise linear curve
2. Insert new vertices at midpoint (splitting)
3. Average vertex with “next” neighbor (averaging)
4. Repeat splitting and averaging
Averaging Mask

Rather than average with nearest neighbor, apply weighted averaging mask during averaging step:

\[ r = (\ldots, r_{-1}, r_0, r_1, \ldots) \]

Chaikin’s algorithm:

\[ r = (1/2, 1/2) \]
Averaging Example

Split → Average →
Averaging Example

Cubic B-spline subdivision mask: $1/4(1\ 2\ 1)$

Split: $a = 1/2(A+B)$, $b = 1/2(B+C)$

Average: $c = 1/4(a + 2C + b)$
Extending to Surfaces

Subdivision curves extend to surfaces

Used in all major 3D modeling programs

Preserves lower polymeshs while allowing for high-quality models
NURBS

• Non-uniform rational basis splines
• Patches generated from curves
• Model curves and surfaces
• Intuitive control points
• Efficient evaluation

https://www.youtube.com/watch?v=m9U_XmnHQMUr
Subdivision Surfaces

Iteratively refine a **control polyhedron** (or **control mesh**) to produce the limit surface using splitting and averaging steps.

Allow for more regional control (good for artists).
Subdivisions for Modeling

https://www.youtube.com/watch?v=zDIn3ESrPEY
Approximating Schemes

Limit surfaces approximate initial meshes
Generated control points not on surface

Examples:
- Doo-Sabin
- Catmull-Clark
- Loop
Doo-Sabin Scheme

Edge points formed from midpoint of each edge
Face point formed as centroid of polygon
New vertex averages vertex, face point, and two edge points
Blue vertices and yellow edges show topological relationship to subdivision.
Vertex Schemes

Vertices create more vertices:

A vertex surrounded by $n$ faces is split into $n$ sub-vertices (one per face)

Note: **valence** is number of edges incident to a vertex

**extraordinary vertices** do not have standard valence of topology (generally unavoidable)
Face Schemes

Faces create more faces
Can also insert vertices along edges and at face centroids
Catmull-Clark Scheme

Face point (F) average of original vertices (O)
Edge points (E) at midpoint along original edges
Weight mask based on valence n:
\[ O(n-3)/3 + F_{\text{avg}}/n + 2E_{\text{avg}}/n \]
Catmull-Clark in Practice

Works best on quads (4:1 subdivision)
Turns all polys into quads
Common subdivision method in modern commercial tools
Finding the Limit

Possible to evaluate limit of Catmull-Clark surfaces without explicit subdivision

• Patches have same limit surface regardless of valence after subdivision
• Can be evaluated analytically as an eigenbasis

Loop Scheme

Subdivides triangles into smaller triangles (4:1 subdivision)
Each face is split into four subfaces based on weight mask
Loop Scheme in Practice

Defined for triangle meshes (not a general algorithm)
Always has extraordinary vertices (valence not 6)
$C^1$ at extraordinary points, $C^2$ elsewhere
Interpolation Schemes

Original mesh’s control points and generated control points interpolated along limit’s surface

Examples:
  • Butterfly scheme
Butterfly Subdivision

Averaging mask:

$t = 0$ gives original polyhedron
Small values of $t$ smooths the surface
$t = 1/8$ has provable $G^1$ continuity
Preserving Creases

Sometimes we want features like creases to be preserved:

How to do this?
Trim Curves

Modify subdivision mask:

Results in $G^0$ continuity (no tangent plane continuity)
ZBrush Example

Allows for both subdivision and “dynamic” meshes
DynaMesh provides clay analogy in 3D
  • Retopologizes mesh to match volume, resolution, and polygon distribution
In practice, both are used at different stages of art pipeline

https://pixologic.com/zclassroom/lesson/subdivision-vs-dynamesh
Additional Reading
