

CS311: Discrete Math for Computer Science, Spring 2015

Additional Exercises

1. Prove that the assertion

$$m = F_i \wedge n = F_{i+1} \wedge i \leq 10$$

where F_i is the i -th Fibonacci number, is a loop invariant for the loop

```
while  $i < 10$  do  
   $i \leftarrow i + 1$ ;  
   $k \leftarrow m + n$ ;  
   $m \leftarrow n$ ;  
   $n \leftarrow k$   
enddo
```

2. Determine which of the assertions

$$n \geq 10, \quad n \mid 10, \quad 10 \mid n$$

are loop invariants for the loop

```
while  $n < 10$  do  
   $n \leftarrow n \times 2$   
enddo
```

3. The sequence A_0, A_1, \dots is defined by the formulas

$$A_n = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even,} \\ \frac{n+1}{2}, & \text{otherwise.} \end{cases}$$

- (i) Prove that all members of this sequence are integers. (ii) Prove that for every n , $A_{n+2} = A_n + 1$.

4. The sequence B_0, B_1, \dots is defined by the formulas

$$\begin{aligned} B_0 &= 0, \\ B_1 &= 1, \\ B_{n+2} &= 4B_{n+1} - B_n. \end{aligned}$$

- (i) Find an explicit formula for B_n . (ii) Determine how the formula will change if we replace the first two equations by

$$\begin{aligned} B_0 &= 1, \\ B_1 &= 2. \end{aligned}$$

5. Draw a graph with the adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$