

CS311: Discrete Math for Computer Science, Spring 2015

Homework Assignment 2, with Solutions

Determine whether the given formula is true or false. Justify your answers.

(a) $\forall n \exists x (n < x < n + 1)$.

Answer: true; take $x = n + \frac{1}{2}$.

(b) $\forall n \exists x (n < x^2 < n + 1)$.

Answer: false; $n = -1$ is a counterexample. Indeed, there is no real number x such that $-1 < x^2 < 0$, because x^2 is nonnegative.

(c) $\forall x \exists y (y^3 + 1 = x)$.

Answer: true; take $y = \sqrt[3]{x - 1}$.

(d) $\exists x \forall y (x + 4 < y^4)$.

Answer: true; take $x = -5$. Indeed, for all y , $-1 < y^4$, because y^4 is nonnegative.

(e) $\exists x y \forall z (xz = y)$.

Answer: true; take $x = y = 0$. It is clear that for all z , $0 \cdot z = 0$.