CS311: Discrete Math for Computer Science, Spring 2015

Homework Assignment 8, with Solutions

1. Find the partition of the set $\{1, 2, \ldots, 6\}$ corresponding to the equivalence relation $|m - 3| = |n - 3|$.

Answer: $\{\{1, 5\}, \{2, 4\}, \{3\}, \{6\}\}$.

2. Consider the equivalence relation between non-empty subsets A, B of $\{1, 2, \ldots, 100\}$ defined by the condition: the greatest element of A is the same as the greatest element of B. Let P be the partition corresponding to this equivalence relation. (a) Find the cardinality of P. (b) Find an element of P. (c) Find one more element of P.

Solution. Each equivalence class of this relation consist of the non-empty subsets of $\{1, 2, \ldots, 100\}$ that have the same greatest element. (a) There are 100 equivalence classes, because there are 100 choices for the greatest element. So the cardinality of P is 100. (b) $\{\{1\}\}$ is an element of P. (c) $\{\{1, 2\}, \{2\}\}$ is another element of P.

3. Find a partition of $\mathbf{N}$ that consists of one infinite set and infinitely many finite sets.

Solution: One possible answer is $\{\{0, 2, 4, 6, \cdots\}, \{1\}, \{3\}, \{5\}, \cdots\}$.

4. For each of the following relations between positive integers m, n, determine whether it is a partial order, and whether it is a total order:

(a) $m|n$.

Solution: This relation is reflexive (every number evenly divides itself), anti-symmetric (if m divides n and n divides m then $m = n$), and transitive (if k divides m and m divides n then k divides n). Consequently this is a partial order. But it is not total: for example, 2 doesn't divide 3 and 3 doesn't divide 2.

(b) $m|n^2$.

Solution: This relation is not anti-symmetric; for instance, 2 divides 4^2 and 4 divides 2^2. So it is not a partial order and hence not a total order.

(c) $m^2|n$.

Solution: This relation is not reflexive; for instance, 2^2 doesn't divide 2. So it is not a partial order and hence not a total order.

(d) the first digit of m in decimal notation is less than or equal to the first digit of n.

Solution: This relation R is not anti-symmetric; for instance, $15\,R\,16$ and $16\,R\,15$. So it is not a partial order and hence not a total order.