CS311: Discrete Math for Computer Science, Spring 2015

Test 1, with Solutions

Open notes. No books, no calculators.

1. Simplify the given formula. Justify your answers.

(a) $n > 4 \wedge n^2 < 30$. *Answer:* $n = 5$.

(b) $n > 4 \vee (n^2)! = 0$. *Answer:* $n > 4$.

2. Determine whether the given formula is true. If it is, prove it. If not, find a counterexample.

(a) $\forall n(1 \leq n \leq 4 \rightarrow 4 \cdot 2^{2-n} > 1)$.

Answer: false; counterexample: $n = 4$. Indeed, $4 \cdot 2^{2-4} = 1$.

(b) $\forall n(1 \leq n \leq 4 \rightarrow n! \leq 2^{n+1})$.

Answer: true. Proof by exhaustion:

n	$n!$	2^{n+1}	$n! \leq 2^{n+1}$
1	1	4	T
2	2	8	T
3	6	16	T
4	24	32	T

3. Translate into logical notation:

(a) There exists a positive integer that is less than 5.

Answer: $\exists n(0 < n < 5)$.

(b) The square of every negative real number is positive.

Answer: $\forall x(x < 0 \rightarrow x^2 > 0)$.

4. Determine whether the given formula is true when the value of each of its free variables is 1. If it is then give a witness.

(a) $\exists xz(x < y \wedge z < y)$.

Answer: True; witness: $x = z = 0$.

(b) $\exists x(x < y \wedge y < z)$.

Answer: False. Indeed, when the value of each of the variables y, z is 1, the conjunctive term $y < z$ is false.

5. Rewrite the given expression in Sigma-notation.

(a) $\sqrt{3} + \sqrt[3]{4} + \cdots + \sqrt[30]{31}$.   *Answer:* $\displaystyle\sum_{i=2}^{30} \sqrt[i]{i+1}$.

(b) $\frac{1}{2^n} + \frac{1}{2^{n+1}} + \cdots + \frac{1}{2^{2n}}$.   *Answer:* $\displaystyle\sum_{i=n}^{2n} \frac{1}{2^i}$.