Declarative Programming:
Lecture Notes and Exercises

Part 8

In Part 7 of these lecture notes we defined the concept of a stable model for sets of formulas that consist of atoms and implications of the form

\[ A_0 \leftarrow A_1 \land \cdots \land A_m \land \neg A_{m+1} \land \cdots \land \neg A_n, \]

where each \( A_i \) is an atom. If we are given a CLINGO program such that every instance of each of its rules is a formula of this type, and all terms occurring in these instances are precomputed, then the definition of a stable model can serve as a theoretical justification of the solutions generated by CLINGO for that program.

Take, for instance, the program

\[
\begin{align*}
p(a), & \quad (1) \\
q(b), & \quad (2) \\
q(X) & \leftarrow p(X), \quad (3) \\
s(X) & \leftarrow q(X) \land \neg p(X). \quad (4)
\end{align*}
\]

The instances of the last two rules are the rules

\[
\begin{align*}
q(r) & \leftarrow p(r), \quad (5) \\
s(r) & \leftarrow q(r) \land \neg p(r) \quad (6)
\end{align*}
\]

for all precomputed terms \( r \). The solutions that an ASP system such as CLINGO is supposed to generate for the given program are exactly the stable models of formulas (1), (2), (5), (6).

Denote this set of formulas by \( \Gamma \). We will now check that the set of atoms

\[
\{p(a), q(a), q(b), s(b)\} \quad (7)
\]

is a stable model of \( \Gamma \). The reduct of \( \Gamma \) relative to (7) consists of formulas (1), (2), (5),

\[ s(a) \leftarrow q(a) \land \bot \quad (8) \]

and

\[ s(r) \leftarrow q(r) \land \top \quad (r \neq a). \quad (9) \]

Formula (8) is a tautology, and formulas (9) are equivalent to

\[ s(r) \leftarrow q(r) \quad (r \neq a). \quad (10) \]

All formulas (1), (2), (5), (10) are definite, and the minimal model of this set of formulas is (7).

Set \( \Gamma \) has no stable models other than (7). Accordingly, given the rules
\texttt{p(a).}
\texttt{q(b).}
\texttt{q(X) :- p(X).}
\texttt{s(X) :- q(X), not p(X).}

CLINGO generates a single answer:

\texttt{p(a) q(b) q(a) s(b)}

\textbf{8.1.} Check that the set of atoms

\{\texttt{p(a), p(b), q(a), q(b)}\}

is not a stable model of $\Gamma$.

\textbf{8.2.} Consider the program

\begin{align*}
p(1), \\
p(2), \\
q(2), \\
q(3), \\
s(X) & \leftarrow p(X) \land \neg q(X).
\end{align*}

The set of instances of its rules has a unique stable model. Guess what it is, and prove that your guess is correct.

\textbf{8.3.} Consider the program

\begin{align*}
p(a), \\
q(1, X) & \leftarrow p(X) \land \neg q(2, X), \\
q(2, X) & \leftarrow p(X) \land \neg q(1, X).
\end{align*}

The set of instances of its rules has two stable models. Guess what they are, and prove that your guess is correct.