Declarative Programming:
Lecture Notes and Exercises

Part 9

An implication $F \leftarrow G$ is simple if $F$ and $G$ don’t contain implications. For instance, formulas of the form

$$A_0 \leftarrow A_1 \land \cdots \land A_m \land \neg A_{m+1} \land \cdots \land \neg A_n,$$

where each $A_i$ is an atom, are simple implications. If a formula $F$ doesn’t contain implications then we can rewrite it as the simple implication $F \leftarrow \top$.

We will now generalize the concept of a stable model, defined in Part 7 of these lecture notes, to sets of simple implications. This more general definition, like the one given earlier, is based on the process of constructing the reduct of the given set of formulas.

A critical part of a simple implication is a subformula that begins with negation but is not part of a larger subformula that begins with negation. For instance, the critical parts of implication (1) are the negative literals $\neg A_{m+1}, \ldots, \neg A_n$. The critical parts of the implication

$$p \lor \neg q \leftarrow \neg (p \land \neg r)$$

are $\neg q$ and $\neg (p \land \neg r)$.

Let $\Gamma$ be a set of simple implications, and let $S$ be a set of atoms. The reduct $\Gamma^S$ of $\Gamma$ relative to $S$ is the set obtained from $\Gamma$ by replacing each critical part $\neg H$ in each of its formulas by $\top$ if $S$ satisfies $\neg H$, and by $\bot$ otherwise. If $S$ is a minimal model of the reduct $\Gamma^S$ then we say that $S$ is a stable model of $\Gamma$.

For instance, let $\Gamma$ be the pair of formulas

$$p \lor q,$$

$$s \leftarrow \neg q.$$

We will show that $\{p, s\}$ is a stable model of $\Gamma$. To this end, consider the reduct of $\Gamma$ relative to $\{p, s\}$:

$$p \lor q,$$

$$s \leftarrow \top.$$

The set $\{p, s\}$ that we started with is indeed one of the two minimal models of the reduct.
9.1. Find all other stable models of $\Gamma$. Run CLINGO to check that your answer is correct.

In each of the following problems, find all stable models of the given set of formulas and run CLINGO to check that your answer is correct.

9.2.

\[ p \lor q, \]
\[ \bot \leftarrow \neg q. \]

9.3.

\[ p \lor q, \]
\[ \neg q. \]

9.4.

\[ p \lor \neg q. \]

9.5.

\[ p \lor \neg p. \]

9.4.

\[ p \leftarrow \neg \neg p. \]