

Introduction to Mathematical Logic, Handout 4

Satisfiability and Entailment

A set Γ of formulas is *satisfiable* if there exists an interpretation that satisfies all formulas in Γ , and *unsatisfiable* otherwise.

Problem 4.1 (a) For any set Γ of formulas, if every two-element subset of Γ is satisfiable then Γ is satisfiable. True or false? (b) Let Γ be a set of literals. Show that Γ is satisfiable iff there is no atom A for which both A and $\neg A$ belong to Γ .

A set Γ of formulas *entails* a formula F (symbolically, $\Gamma \models F$), if every interpretation that satisfies all formulas in Γ satisfies F also.

Problem 4.2 For any formulas F_1, \dots, F_n, G , the following conditions are equivalent:

- $F_1, \dots, F_n \models G$,
- $(F_1 \wedge \dots \wedge F_n) \rightarrow G$ is a tautology,
- the set $\{F_1, \dots, F_n, \neg G\}$ is unsatisfiable.