Translation Synchronization via Truncated Least Squares

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Abstract

- 1D Translation Synchronization problem: recover global coordinates $\{x_i\}$ from noisy relative measurements $t_{ij} = x_i - x_j$+noise.
- We introduce a robust algorithm *TranSync* for this problem by applying truncated least squares to gradually prune out noisy measurements.
- We provide analysis of our algorithm under both deterministic and randomized settings.
- In experiments, our algorithm achieved superior robustness and stability against state-of-the-art convex formulations on both synthetic and real datasets.

Formulations

- l_1 minimization:

$$\arg\min_x \sum_{i,j\in\mathcal{E}} |t_{ij} - (x_i - x_j)|$$

- Least Squares:

$$\arg\min_x \sum_{i,j\in\mathcal{E}} |t_{ij} - (x_i - x_j)|^2$$

- Truncated Least Squares (ours):

$$\arg\min_x \sum_{i,j\in\mathcal{E}} w_{ij}^{(k)} |t_{ij} - (x_i - x_j)|^2$$

Our Algorithm

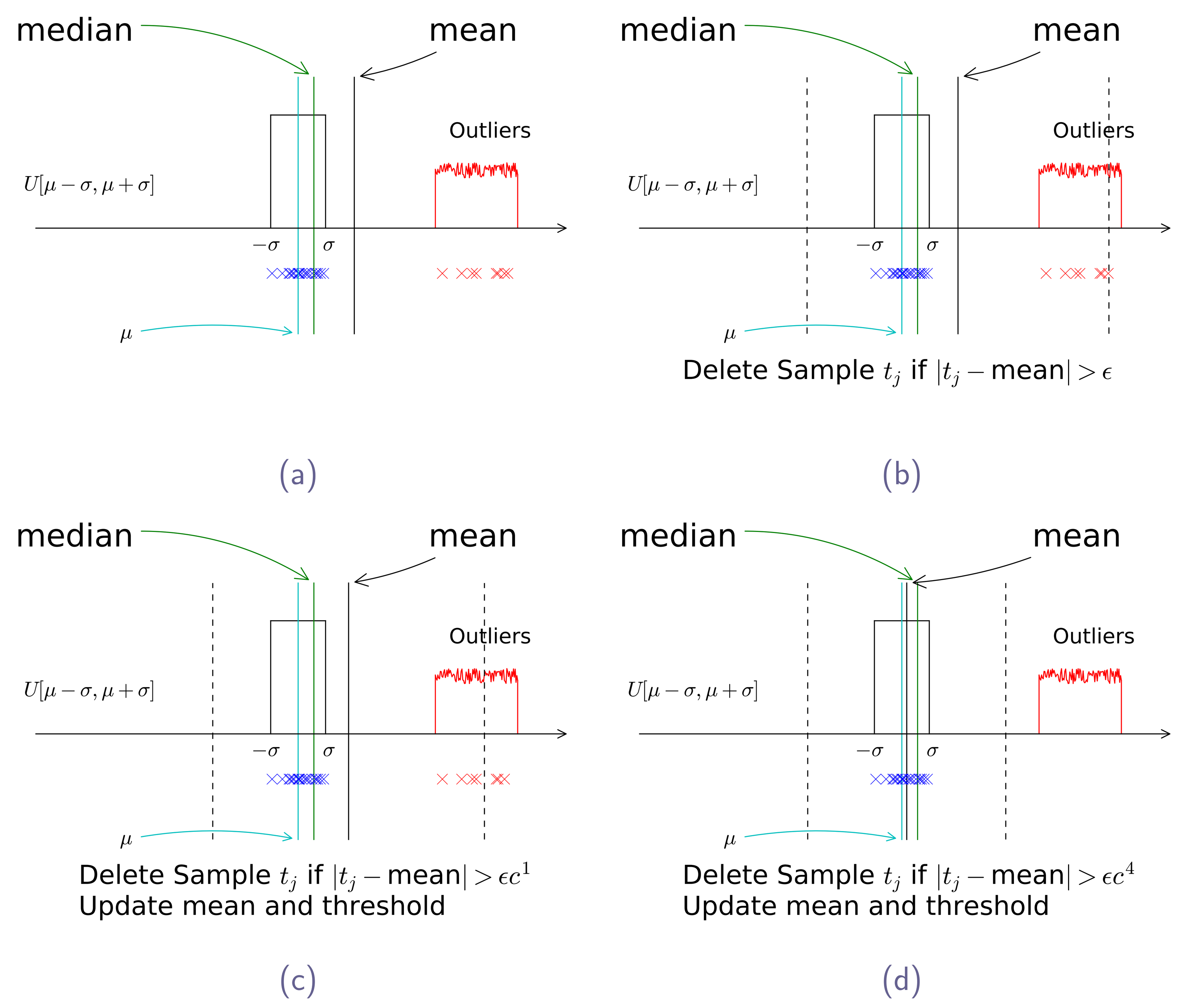
1. $x^{(-1)} \leftarrow 0$. $\delta_{-1} \leftarrow \infty$. $c < 1$.

for $k = 0, 1, 2, k_{\max}$ **do**

2. Obtain the truncated graph $\mathcal{G}^{(k)}$ using $x^{(k-1)}$ and δ_{k-1}.
3. **Break if** $\mathcal{G}^{(k)}$ is disconnected
4. Solve Truncated Least Squares w.r.t. $\mathcal{G}^{(k)}$ to obtain $x^{(k)}$.
5. $\delta_k = \min\left(\max_{(i,j)\in\mathcal{E}} |t_{ij} - (x_i^{(0)} - x_j^{(0)})|, c\delta_{k-1}\right)$.

end for

Output: $x^{(k)}$.



Deterministic Exact Recovery Condition

- **Biased Noise Model (Unbounded Outliers):**

$$t_{ij} = \begin{cases} x_i^{gt} - x_j^{gt} + U[-\sigma,\sigma] & \text{with probability } p \\ \text{Any real number} & \text{with probability } 1-p \end{cases} \quad (1)$$

- For some constants p, q only depend on graph structure, during optimization we have

$$\|x^{(k)} - x^{gt}\|_\infty \le q\sigma + 2p\epsilon c^{k-1}$$

and eventually we'll reach an $\hat{x}$

$$\|\hat{x} - x^{gt}\|_\infty \le \frac{2p + cq}{c - 4p}\sigma$$

where the RHS is independent of ϵ.

Randomized Case

Biased Noise Model:

$$t_{ij} = \begin{cases} x_i^{gt} - x_j^{gt} + U[-\sigma,\sigma] & \text{with probability } p \\ x_i^{gt} - x_j^{gt} + U[-a,b] & \text{with probability } 1-p \end{cases} \quad (2)$$

Theorem

There exists a constant c so that if $p > c/\sqrt{\log(n)}$, then w.h.p,

$$\|x^{(k)} - x^{gt}\|_\infty \le (1 - p/2)^k (b - a),$$

$$\forall\, k = 0, \cdots, [-\log(\frac{b+a}{2\sigma})/log(1 - p/2)].$$

Experiments

Synthetic Graphs in $\{$Dense, Sparse$\}$ $\times$ $\{$Regular, Irregular$\}$

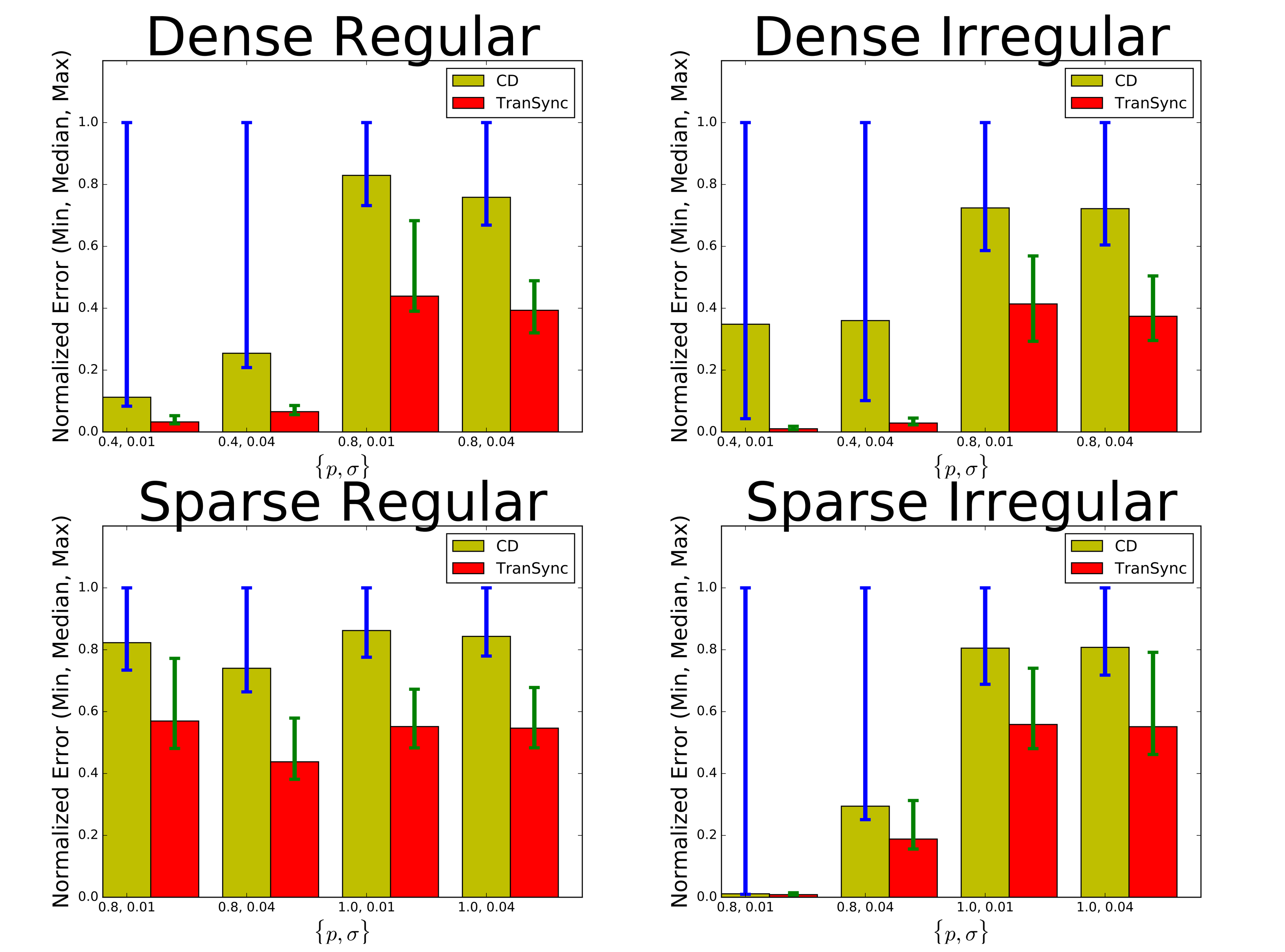


Figure: Experimental results comparing TranSync and Coordinate Descent (CD) under different settings. All statistics (min, median, max) are computed among 100 independent experiments with the same setting. Bars under the same setting are normalized for better display.

Joint alignment of point clouds.

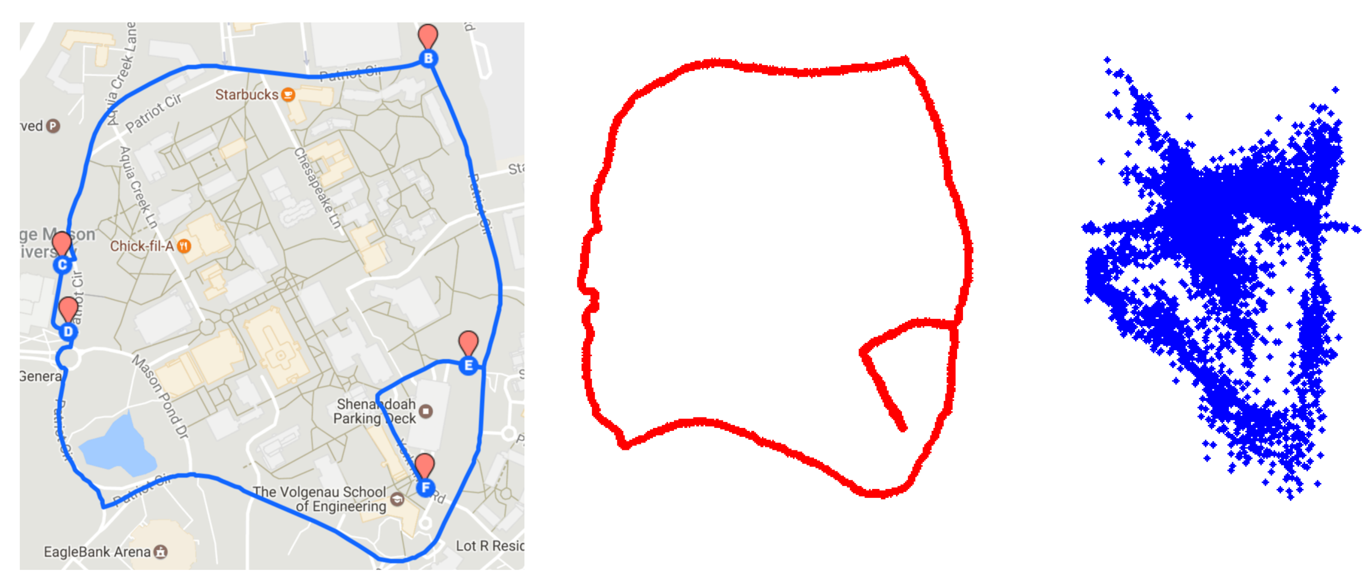


Figure: The application of TranSync in joint alignment of 6K Lidar scans around a city block. (a) Snapshot of the underlying scanning trajectory. (b) Reconstruction using TranSync (c) Reconstruction using Coordinate Descent.

Ranking from relative comparisons.

Movie	Global ranking (score)					
	MRQE	Hodge-Diff.	Hodge-Ratio	Hodge-Binary	TS-Init	TS-Final
Shakespeare in Love	1(85)	1(0.247)	2(0.078)	1 (0.138)	1(0.135)	1(0.219)
Witness	2(77)	2(0.217)	1(0.088)	3(0.107)	3(0.076)	2(0.095)
October Sky	3(76)	3(0.213)	3(0.078)	2(0.111)	2(0.092)	3(0.0714)
The Waterboy	4(66)	6(-0.464)	6(-0.162)	6(-0.252)	5(-0.134)	4(-0.112)
Interview with the Vampire	5(65)	4(-0.031)	4(-0.012)	4(-0.120)	4 (-0.098)	5(-0.140)
Dune	6(44)	5(-0.183)	5(-0.069)	5(-0.092)	6(-0.216)	6(-0.281)

Table: Global ranking of selected six movies via different methods: MRQE, HodgeRank[1] with 1) arithmetic mean score difference, 2) geometric mean score ratio and 3) and binary comparisons, and the initial and final predictions of TranSync. TranSync results in the most consistent result with MRQE.

Reference

Xiaoye Jiang, Lek-Heng Lim, Yuan Yao, and Yinyu Ye.
Statistical ranking and combinatorial hodge theory.
Math. Program., 127(1):203–244, 2011.