Refraction as Minimization Solution

Consider the situation in which a layer with transmission speed $c_1$ is beneath a layer with transmission speed $c_2$. The distance from $(0,0)$ to $(x,1)$ is $\sqrt{1+x^2}$, thus the time for the beam of light to travel the distance is $\frac{\sqrt{1+x^2}}{c_1}$. The distance from $(1,x)$ to $(1,1)$ is $\sqrt{1+(1-x)^2}$, thus the time for the beam of light to travel the distance is $\frac{\sqrt{1+(1-x)^2}}{c_2}$ and the total transit time is $\frac{\sqrt{1+x^2}}{c_1} + \frac{\sqrt{1+(1-x)^2}}{c_2} = \frac{\sqrt{1+x^2} + r\sqrt{1+(1-x)^2}}{c_1}$, where $r$ is just the ratio $c_1 / c_2$. In units of inverse $c$, the transit time is $\frac{\sqrt{1+x^2}}{c} + \frac{r\sqrt{1+(1-x)^2}}{c}$, and this will suffice for our purposes here since the units are relevant.

1. Construct a MATLAB function `transtime` that has inputs $x$ and $r$ and returns the time the ray requires to transit from $(0,0)$ to $(1,2)$.

   ```matlab
   function y = transtime (x, r)
   y = sqrt(1+x^2)+r*sqrt(1+(1-x)^2);
   ```

2. Let $r$ be an array of 101 linearly spaced points from 1 to 10. For each element of the array, use `fminbnd` to compute a minimum of `transtime` to get the optimal positions $x$. This results in an array $x$ as a function of the refraction index $r$. Plot $x$ versus $r$ (i.e., the dependent variable is $x$).

   ```matlab
   r = linspace (1, 10, 101);
   for k = 1:101
       x(k) = fminbnd ('transtime', 0, 1, optimset, r(k));
   end
   plot (r, x)
   ```

3. Consider the angle of incidence $\theta_1$ and angle of refraction $\theta_2$. Recognize that $\sin \theta_1 = \frac{x}{\sqrt{1+x^2}}$ and $\sin \theta_2 = \frac{1-x}{\sqrt{1+(1-x)^2}}$. Compute arrays `sintheta1` and `sintheta2` from $x$. Create a plot of `sintheta1./sintheta2` versus $r$. You should be able to conclude Snell’s Law from this.

   ```matlab
   sintheta1 = x./sqrt(1+x.^2);
   sintheta2 = (1-x)./sqrt(1+(1-x).^2);
   plot (r, sintheta1./sintheta2)
   ```