A Spring/Damper Suspension ODE Problem
Due Friday, December 6 by 12 noon

(From Recktenwald Problem 26, pp732-3)

The following is a simplified model of the suspension system of one wheel of an automobile.

The input to the system is the time-varying displacement $y_0(t)$ corresponding to changes in the terrain. The shock absorber is characterized by its spring rate $k_2$ and damping coefficient $c_2$. Damping in the tire is neglected. (There is no $c_1$ term.)

Applying Newton’s law of motion and force balances to the wheel and vehicle chassis yields the following system of equations:

$$m_1y_1''(t) + c_2(y_1'(t) - y_2'(t)) + k_2(y_1(t) - y_2(t)) + k_1y_1(t) = k_1y_0(t),$$

$$m_2y_2''(t) - c_2(y_1'(t) - y_2'(t)) - k_2(y_1(t) - y_2(t)) = 0.$$  

(a) Convert these two second-order equations into an equivalent system of first-order equations. (How many first-order equations are required?). Write a Matlab function $yp = spring(t, y, m, k, c)$ that takes as input the time $t$, a column array $y$, and the constants $m$, $k$, and $c$ (as arrays). Imbed the forcing function $y_0(t) = 0.05\sin(3\pi t)$.

We construct the new array $y = \begin{bmatrix} y_1^{\text{old}} \\ y_2^{\text{old}} \\ y_1' \\ y_2' \end{bmatrix}$ so

$$y' = \begin{bmatrix} y_3 \\ y_4 \\ (k_1y_0(t) - c_2(y_1'(t) - y_2'(t)) - k_2(y_1(t) - y_2(t)) - k_1y_1(t))/m_1 \\ (c_2(y_1'(t) - y_2'(t)) + k_2(y_1(t) - y_2(t)))/m_2 \end{bmatrix}$$

and this
is implemented in the Matlab function:

```matlab
function yp = spring(t, y, m, k, c)
yp = zeros(4,1);
yp(1) = y(3);
yp(2) = y(4);
yp(3) = (.05*sin(3*pi*t)-c(2)*(y(3)-y(4))-k(2)*(y(1)-y(2))-k(1)*y(1))/m(1);
yp(4) = (c(2)*(y(3)-y(4))+k(2)*(y(1)-y(2)))/m(2);
```

(b) Use Matlab function ode45 integration routine to solve this system on the time interval \([0, 5]\) for \(m_1 = 110\) kg, \(k_1 = 136\) N/m, \(m_2 = 1900\) kg, \(k_2 = 16\) N/m, and \(c_2 = 176\) Ns/m. Assume the system is at rest at \(t = 0\) (i.e., \(y_1(0) = 0, y_2(0) = 0, y_1'(0) = 0, \) and \(y_2'(0) = 0\)). Produce a plot that shows both \(y_1\) and \(y_2\) versus \(t\).

```matlab
m = [110; 1900];
k = [136; 16];
c = [0; 176];
[t, y] = ode45 (@spring, [0 5], y0, [], m, k, c);
plot (t, y(:, 1), t, y(:, 2));
```

(c) Repeat the solution with \(c_2\) reduced by a factor of 5.

```matlab
C(2) = c(2)/5;
[t, y] = ode45 (@spring, [0 5], y0, [], m, k, c);
plot (t, y(:, 1), t, y(:, 2));
```