1. Assume a finite domain that includes just the specific cities mentioned here. Let $R$ be the reflexive, symmetric, transitive closure of:
   - (Austin, Dallas), (Dallas, Houston), (Dallas, Amarillo), (Austin, San Marcos),
   - (Philadelphia, Pittsburgh), (Philadelphia, Paoli), (Paoli, Scranton),
   - (San Francisco, Los Angeles), (Los Angeles, Long Beach), (Long Beach, Carmel)
(a) Draw $R$ as a graph.
(b) List the elements of the partition defined by $R$ on its domain.

2. Let $R$ be a relation on the set of positive integers. Define $R$ as follows:
\[ \{(a, b) : (a \mod 2) = (b \mod 2)\} \] In other words, $R(a, b)$ iff $a$ and $b$ have the same remainder when divided by 2.
(a) Consider the following example integers: 1, 2, 3, 4, 5, 6. Draw the subset of $R$ involving just these values as a graph.
(b) How many elements are there in the partition that $R$ defines on the positive integers?
(c) List the elements of that partition and show some example elements.

3. Consider the language $L$, over the alphabet $\Sigma = \{a, b\}$, defined by the regular expression
\[ a^* (b \cup \varepsilon) a^* \]
Let $R$ be a relation on $\Sigma^*$, defined as follows:
$R(x, y)$ iff both $x$ and $y$ are in $L$ or neither $x$ nor $y$ is in $L$. In other words, $R(x, y)$ if $x$ and $y$ have identical status with respect to $L$.
(a) Consider the following example elements of $\Sigma^*$: $\varepsilon$, b, aa, bb, aabaaa, bab, bbaabb. Draw the subset of $R$ involving just these values as a graph.
(b) How many elements are there in the partition that $R$ defines on $\Sigma^*$?
(c) List the elements of that partition and show some example elements.

Solutions

1. (b) [cities in Texas], [cities in Pennsylvania], [cities in California]

2. (b) Two
   (c) [even integers] Examples: 2, 4, 6, 106
   [odd integers] Examples: 1, 3, 5, 17, 11679

3. (a) (Hint: $L$ is the language of strings with no more than one b.)
   (b) Two
   (c) [strings in $L$] Examples: $\varepsilon$, aa, b, aabaaa
   [strings not in $L$] Examples: bb, bbaabb, bab