Tuning the Wall-Following Controller

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September 24, 1999

We would like to tune the parameters of the wall-following controller so that it is as close to critically-damped as possible. That is, it converges to the setpoint as quickly as possible without overshooting.

![Diagram of robot and range sensor](image)

Figure 1: The robot is at position $(x,y)$ and orientation $\theta$. The range sensor in direction $\phi$ senses distance $s_\phi$, but in this paper we assume that $y$ and $\theta$ are sensed directly.

The dynamical model of the robot (Figure 1) is $\mathbf{x}' = f(\mathbf{x}, u)$, with constant forward velocity $v$ and exogenously specified angular velocity $\omega$.

\[
\begin{bmatrix}
x' \\
y' \\
\theta' \\
v'
\end{bmatrix} =
\begin{bmatrix}
v \cos \theta \\
v \sin \theta \\
\omega \\
0
\end{bmatrix}
\] (1)

The Wall-Following Controller

We use a slightly simplified version of the wall-following controller from [2].
The wall-following control law (3) sets angular velocity $\omega$, responding to positional error $e = y - y_{set}$ and orientation error $\theta$, and assuming that forward velocity $v$ is constant.

We want to specify the control law so that the behavior of the system will be described by

$$\ddot{e} + k_\theta \dot{\theta} + k_e e = 0.$$\hspace{1cm} (2)

where the constants $k_e$ and $k_\theta$ are tuned to make the system behave well (i.e., critically damped convergence $e \to 0$).

For small values of $\theta$,

$$\dot{e} = v \sin \theta \approx v \theta$$

and

$$\ddot{e} = v \cos \theta \dot{\theta} \approx v \omega,$$

so we can transform the general system description (2) into a control law: a rule for specifying the value of the controlled variable $\omega$ as a function of the values of the observed variables $e$, $\theta$ and $v$:

$$\omega = \frac{1}{v} [-k_\theta v \theta - k_e e].$$\hspace{1cm} (3)

**Qualitative Behavior**

Any elementary discussion of differential equations and dynamical systems (e.g. [1]) provides the qualitative framework we need to tune this controller.

When obeying the wall-following control law, the robot’s behavior approximates the linear harmonic oscillator, which is a special case of the general linear second-order system:

$$a \ddot{x} + b \dot{x} + cx = 0.$$\hspace{1cm} (4)

The behavior of this system is determined by the roots of its characteristic equation:

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ 

The qualitative behavior $x(t)$ is determined by the qualitative properties of the roots, which are determined in important part by the sign of the discriminant:

$$D = b^2 - 4ac.$$
• If the roots have non-zero imaginary part (i.e., $D < 0$), the behavior oscillates. If the roots are purely imaginary (i.e., $D < 0$ and $b = 0$), the oscillation is periodic.

• If either root has a positive real part, the behavior diverges. In a usefully controlled system, the behavior must converge, so both roots must have negative real parts. If $D < 0$, we require only that $b > 0$. If $D > 0$, we also need to have $b > \sqrt{b^2 - 4ac}$, which therefore requires that

$$0 < c < \frac{b^2}{2a}.$$  

• *Critical damping* occurs at the boundary between oscillatory and non-oscillatory behavior; that is, where $D = 0$, so:

$$c = \frac{b^2}{4a}.  \tag{5}$$  

**Tuning the Controller**

To make the behavior of the system (2) critically damped, we apply (5) and require that

$$k_e = \frac{k^2}{4} \text{ or equivalently, } k_\theta = \sqrt{4k_e}.  \tag{6}$$  

**Experimental Results**

The model rwa112.m implements this controller, with some modifications. In order to avoid divergence when $v \approx 0$ during starting and stopping, the $1/v$ term in (3) is replaced by $\min(v^2, 1/\max(0.01, v))$.

Setting $k_e$ to values in the range $[0.1, 0.6]$ controls how aggressively the controller seeks the setpoint. Setting $k_\theta = \sqrt{4k_e}$ makes the resulting controller critically damped, as expected. High gains at lower velocities results in turns that might be overly aggressive.

**Next Steps**

• Set the gain as a function of forward velocity, to see whether this gives better performance.
• What are the applicability conditions for this controller? How close is $\theta \approx 0$? What happens if the robot is close to facing the wall?

References
