3.8 Additional Exercises

1. Solve

(a) \[
2x - y = 5 \\
y - 2z = -3 \\
3z = 3
\]

(b) \[
\chi_0 - \chi_1 + 2\chi_2 - 2\chi_3 = -1 \\
2\chi_1 - 3\chi_2 - \chi_3 = -9 \\
-2\chi_2 + \chi_3 = 0 \\
\chi_3 = 2
\]

(c) \[
2x = 2 \\
-x - 2y = -5 \\
x - y + z = -2
\]

(d) \[
2\chi_0 - 1\chi_1 + 2\chi_2 = -4 \\
-4\chi_0 + 5\chi_1 - 5\chi_2 = 13 \\
2\chi_0 - 7\chi_1 + 2\chi_2 = -10
\]

(e) \[
2\chi_0 - 1\chi_1 + 2\chi_2 = -2 \\
4\chi_0 + 1\chi_1 + 3\chi_2 = 3 \\
-4\chi_0 - 1\chi_1 - 5\chi_2 = -1
\]

(f) \[
-2\chi_0 - 1\chi_1 + 2\chi_2 = 2 \\
-4\chi_0 - 1\chi_1 + 3\chi_2 = 3 \\
4\chi_0 + 1\chi_1 - 1\chi_2 = 3
\]

2. If you need more practice, here is how you can generate “easy” problems (meaning problems that only involve, for example, integers) with Matlab or Octave:

- Create any unit lower triangular matrix \( L \):
  \[
  \text{octave:1}> L = [ \\
  1 0 0 \\
  -1 1 0 \\
  2 3 1 \\
  ]
  \]

- Create any upper triangular matrix \( U \) with same size:
  \[
  \text{octave:2}> U = [ \\
  2 -1 3 \\
  0 1 -2 \\
  0 0 3 \\
  ]
  \]
• Create a vector of “appropriate length”:
  octave:3> x = [1
  2
  3
]

• Set $A = LU$ and the right-hand side $b = Ax$:
  octave:4> A = L * U
  A =
    2  -1  3
    -2  2  -5
    4   1  3

  octave:5> b = A * x
  b =
    9
   -13
    15

• Now, use the coefficients of matrix $A$ and the right-hand side $b$ in your linear system:
  \[
  \begin{align*}
  2\chi_0 - 1\chi_1 + 3\chi_2 &= 9 \\
  -2\chi_0 + 2\chi_1 - 5\chi_2 &= -13 \\
  4\chi_0 + \chi_1 + 3\chi_2 &= 15
  \end{align*}
  \]

• As long as you only choose integers for coefficients in $L$, $U$, and $x$, you will only encounter integers (as long as you don’t swap equations)!