1. Solve the following systems of equations. For each,
   - Write it in matrix form as an augmented system.
   - Perform Gaussian Elimination with the augmented system.
   - Solve the system.
   - Give $L$ and $A$ so that $A = LU$.
   - Solve $Ax = b$ via two triangular solves (with $L$ and $U$).

(a)
\[
\begin{align*}
2\chi_0 + (-1)\chi_1 &+ 0\chi_2 = -3 \\
-2\chi_0 + (-1)\chi_1 &+ 1\chi_2 = \ 3 \\
-8\chi_0 + 10\chi_1 + (-4)\chi_2 &= 10
\end{align*}
\]

(b)
\[
\begin{align*}
2\chi_0 + (-1)\chi_1 &+ -2\chi_2 = -6 \\
6\chi_0 + (-5)\chi_1 + (-5)\chi_2 &= -19 \\
-2\chi_0 + (-7)\chi_1 + 8\chi_2 &= 12
\end{align*}
\]

2. With octave, generate a nontrivial (e.g., not diagonal) system of four equations in four unknowns with the following properties:
   - All coefficients are integers.
   - The right-hand side consist of integers.
   - The solution consists of integers.

3. Challenge question: In class I said that one can generate a matrix $A$ so that if one does Gaussian Elimination one never runs into a fraction by making $A = LU$ and picking $L$ and $U$ to only have integers (and have their special structure). Explain why this works. (You may illustrate it with a $3 \times 3$ system, but then you would want to explain it for general size).