Homework 5

CS 323E

Due: 11/12/2001

In this homework, we experiment with different ways of implementing matrix-matrix multiplication.

1. Let $x$ and $y$ be vectors of length $n$. Recall that the inner-product (dot product) is defined by $x^T y = \sum_{i=1}^n x_i y_i$, where $x_i$ and $y_i$ equal the $i$th entries of $x$ and $y$, respectively. Compute

   (a) $\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} =$

   (b) $\begin{pmatrix} 2 \\ 0 \\ 1 \\ -1 \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} =$

   (c) $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} =$

2. Compute

   (a) $\begin{pmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} =$

   (Hint: use 1a, 1b, and 1c)

   (b) $\begin{pmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix}^T \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix} =$

   (c) $\begin{pmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} =$

3. Compute $\begin{pmatrix} -1 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & -1 \end{pmatrix} =$

   (Hint: use 2a, 2b, and 2c).

4. From the class web page pick up M-file matrix_script.m.

   (a) Use it to time the cost of $C = AB$ where $A$, $B$, and $C$ are all $100 \times 100$ matrices.

   (b) Write a function

       \[
       \text{function } C = \text{mmult_mv}(A, B) \]

       Then test the function using

       \[
       \text{mmult_mv}(A, B) \]

       where $A$, $B$, and $C$ are $100 \times 100$ matrices.
that computes the product of matrices \( A \) and \( B \) by columns using the observations used to compute part 3 above. You may assume that all three matrices are square and of equal dimensions.

Hints:

<table>
<thead>
<tr>
<th>( n = \text{length}( A ) )</th>
<th>assigns the row dimension of ( A ) to ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C = \text{zeros}( n, n ) )</td>
<td>creates an ( n \times n ) matrix ( C ) filled with zeroes.</td>
</tr>
<tr>
<td>( C(:, j) )</td>
<td>equals the ( j )th column of ( C ).</td>
</tr>
</tbody>
</table>

(c) Time the resulting matrix-matrix multiply by commenting out the obvious lines in \texttt{mmult_script}.

(d) Plot the cost (compute times) for the MATLAB matrix multiplication and \texttt{mmult_mv} by commenting out the obvious lines.

5. (a) Write a function

\[
\text{function } y = \text{mvmult}( A, x )
\]

that computes the matrix-vector \( y = Ax \) by doing dot products of rows of \( A \) with \( x \).

Hints:

\[
A( i, : ) * x \] computes the dot product of the \( i \)th row of \( A \) with \( x \).

(b) Modify \texttt{mmult_mv} so that the computation of the \( j \)th column of \( C \) uses \texttt{mvmult}. Call the resulting function \texttt{mmult_mvmult} and store in \texttt{mmult_mvmult.m}.

(c) Modify \texttt{mmult_script} by uncommenting the obvious lines so that it times and plots the cost of doing a matrix multiply using the MATLAB matrix multiplication \( C = A * B \), \texttt{mmult_mvmult}, and \texttt{mvmult}.

**Remember:** A function must be stored in an M-file that has the same name as the function.

The purpose of this exercise is to learn something about how different implementations will yield different performance. Thus, take a moment to look carefully at the performance graphs.

E-mail the resulting \texttt{mmult_mv}, \texttt{mvmult}, and \texttt{mmult_mvmult} functions to Paolo.