Midterm 1 for CS301K Fall 2011

A. Are the following propositions? Write yes or no (1 point each).

1. I am the king of France.
2. Who is the president?
3. \( \forall x \exists z [x + 7 < z] \).
4. If this car is flying, then we’re in the future, or I’m dreaming.
5. \((x + 3)^2(y - 9)^3\).
6. Give me the keys so I can start the car.
7. All the money in the world can’t buy happiness.
8. If the blip swips the bloq, then we won’t be able to wesp the tiq.

B. Analyze the logical forms of the following statements without using any predicates (3 points each).

1. If you love me, then we should get married.

2. If Jim likes Kate and Kate likes John, but John doesn’t like Kate, then neither Jim, John nor Kate will get married.

3. Either get in the car now, or we won’t reach the theater in time.

4. Water boils only when it is hot enough, and not before then.
C. Analyze the logical forms of the following statements using predicates for each (3 points each).

1. If $a$, $b$ and $c$ are lengths of the sides of a right triangle, and $c$ corresponds to the hypotenuse, then $a^2 + b^2 = c^2$.

2. Not all that glitters is gold.

3. If Jim likes Kate and Kate likes John, but John doesn’t like Kate, then neither Jim, John nor Kate will get married.

4. Point $x$ is in the Pareto front of $F$ iff there does not exist a $y$ in $F$ that dominates $x$. 
D. Give the truth sets of the following predicates (3 points each).

1. $P(x) \equiv [x = x^2]$ for $x \in \mathbb{Z}$.

2. $P(x) \equiv [x > 5]$ for $x \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

3. $P(x) \equiv [x^2 < 0]$ for $x \in \mathbb{R}$.

4. $P(x) \equiv [4 \leq x \leq 5]$ for $x \in \mathbb{Z}$.
E. Make truth tables for the following formulas. You can write compact truth tables if you prefer, in which case you should circle the final column that you fill in (4 points each).

1. $P \rightarrow \neg Q$

2. $\neg Q \leftrightarrow \neg P$

3. $(A \land B) \lor (\neg A \rightarrow \neg B)$

4. $(P \rightarrow Q) \leftrightarrow \neg (\neg P \land Q)$

5. $(\neg A \lor \neg B) \rightarrow (\neg C \land \neg B)$
F. Write linear proofs of the following equivalences (6 points each).

1. \( P \land ((P \lor Q) \land \neg (\neg P \land \neg Q)) \equiv P. \)

2. \( (\neg A \leftrightarrow B) \rightarrow (\neg C \rightarrow (A \lor \neg B)) \equiv A \lor C \lor \neg B. \)
3. $\forall x \forall y (P(x, y) \rightarrow Q(x, y)) \equiv \neg \exists x \exists y (P'(x, y) \land \neg Q(x, y))$.

G. Use rules of inference to prove the following arguments (6 points each).

1.

\[
\begin{align*}
P \rightarrow \neg Q \\
\neg Q \rightarrow \neg R \\
R \\
\therefore \neg P
\end{align*}
\]
2. 

\[ P \leftrightarrow Q \]
\[ \neg X \rightarrow Q \]
\[ \neg (X \lor M) \]
\[ \therefore P \]

3. 

\[ c \in U \]
\[ \forall x \in U : P(x) \]
\[ \forall x \in U : (P(x) \rightarrow Q(x)) \]
\[ \therefore Q(c) \]