Midterm 1 Review Problems

A. Are the following propositions? Write yes or no.

1. I like to read.
2. If I clean my room, can I go to the mall?
3. 7 < 3.
4. If I work overtime, I can afford a new computer.
5. \( \forall x \exists y [(x + y)^2] \).
6. Let’s go to the movies and have some fun.
7. There is a way to solve this.

B. Analyze the logical forms of the following statements without using any predicates.

1. Today I’m going to buy groceries, go eat lunch, and then relax at the pool.
2. If you don’t relax, you’re going to have a heart arrack!
3. If aliens do exist, but they are very far away, or they are more primitive than humans, then they will probably never visit earth.
4. Because I could not stop for Death, He kindly stopped for me.

C. Analyze the logical forms of the following statements using predicates for each.

1. Of all the people here, only one will win the prize.
2. If \( a, b \) and \( c \) are lengths of the sides of a triangle, then \( a + b \geq c \).
3. Dogs and cats are of the order \textit{Carnivora}, therefore they are both mammals.
4. The function \( f(x) \) is continuous at \( p \) if and only if for any \( \epsilon > 0 \) there exists a \( \delta > 0 \) such that \( |x - p| < \delta \) implies \( |f(x) - f(p)| < \epsilon \).
D. Make truth tables for the following formulas. You can write compact truth tables if you prefer, in which case you should circle the final column that you fill in.

1. \((A \land \neg B) \leftrightarrow (C \land \neg B)\)
2. \(\neg Q \leftrightarrow (P \lor Q)\)
3. \((A \rightarrow B) \land (\neg A \rightarrow B)\)
4. \(\neg P \rightarrow Q\)
5. \((P \lor Q) \leftrightarrow \neg(P \rightarrow \neg Q)\)

E. Give the truth sets of the following predicates.

1. \(P(x) \equiv [10 > x^2] \text{ for } x \in \mathbb{Z}\).
2. \(P(x) \equiv [x > 5 \land 4 > x] \text{ for } x \in \mathbb{R}\).
3. \(P(x) \equiv \text{“x is even” for } x \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}\).

F. Write linear proofs of the following equivalences.

1. \((P \rightarrow Q) \land \neg(\neg P \land \neg Q) \equiv Q\).
2. \(((A \lor B) \land (C \rightarrow B)) \lor (C \land A) \equiv (A \lor B)\).
3. \((A \leftrightarrow (\neg B \land C)) \rightarrow \neg C \equiv C \rightarrow (A \leftrightarrow B)\)
4. \(\exists x \forall y (P(x, y) \land \neg Q(x, y)) \equiv \forall x \exists y (\neg P(x, y) \lor Q(x, y))\).
G. Use rules of inference to prove the following arguments.

1. 
\[ \neg P \rightarrow R \\
\neg R \rightarrow \neg Q \\
\neg R \\
\therefore P \land \neg Q \]

2. 
\[ P \rightarrow X \\
P \lor R \\
\therefore X \lor R \]

3. 
\[ X \lor M \\
X \rightarrow \neg Q \\
M \rightarrow P \\
\neg (P \land \neg Q) \\
\therefore P \leftrightarrow Q \]

4. 
\[ c \in U \\
P(c) \\
\forall x \in U : (Q(x) \rightarrow R(x)) \\
\forall x \in U : (P(x) \rightarrow Q(x)) \\
\therefore R(c) \]