Always assume that \( \mathbb{N} \) includes 0.

**A. Are the following propositions? Write yes or no. (2 points each)**

1. \( x \in (A \cap B = \emptyset) \).
2. \( \forall x, y \in \mathbb{R} \; [x + y] \).
3. Pluto is a planet.
4. \( x \in A \cap x \in B \).
5. \( \{(a, b) \mid a \in A \land b \in B\} \).
6. Are you struggling?
7. \( \forall x [(x \notin B) \rightarrow (x \in B)] \).

**B. Are the following propositions true or false? (2 points each)**

1. \( \forall a \in \mathbb{R} \; \exists b \in \mathbb{R} \; [a + b = 0] \).
2. The \( < \) relation is a partial order on \( \mathbb{R} \).
3. \( \exists x \in \mathbb{Z} \forall y \in \mathbb{Z} \; [x/y = 0] \).
4. For any set \( U \), the \( \subseteq \) relation on \( \mathcal{P}(U) \) is anti-symmetric.
5. For sets \( A \) and \( B \), \( (A = B) \leftrightarrow (\forall x [x \in A \iff x \in B]) \).
6. \( A \land (A \lor B) \equiv A \).
7. \( \bigcap_{i=0}^{\infty} \{x \in \mathbb{R} \mid -i \leq x \leq i\} = \mathbb{R} \).

**C. Make a truth table for the following formula. You can write a compact truth table if you prefer. Circle the final column that you fill in. (3 points)**

\( (A \land (C \leftrightarrow B)) \rightarrow \neg B \)
D. Provide counterexamples disproving the following propositions. Briefly explain why they are counterexamples. (4 points each)

1. Define \( A = \{a, b, c\} \). No function \( f : A \to A \) can be an equivalence relation on \( A \).

2. \( \forall a \in \mathbb{R} \exists b \in \mathbb{R} [ab = 1] \).

3. Relation \( R \) on \( \mathbb{R} \), defined as \( (aRb \leftrightarrow (a^b \in \mathbb{Z})) \), is transitive.

4. For sets \( A, B, \) and \( C \), \( A - (B \cup C) = (A - B) \cup (A - C) \).
E. What are these expressions equal to? Enumerate every element of the set in your answers. (3 points each)

1. $\mathcal{P}(\{a, \emptyset\}) =$

2. $\{(1, a), (2, c), (3, b)\} \circ \{(1, 2), (1, 3), (2, 3)\} =$

3. $\{2^i | i \in \mathbb{N} \land i \leq 5\} =$

4. Define relation $R$ on $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ as
   $aRb \leftrightarrow (a - 5 \leq 0 \land b - 5 \leq 0)$
   Then $[5]_R =$

5. $\bigcup_{i=0}^{10} \{x \in \mathbb{Z} | x^2 = i\} =$

6. Given $f : \mathbb{N} \to \mathcal{P}(\mathbb{N})$ defined as $f(n) = \{n\}$,
   what is the image of the set $\{1, 5, 7\}$ under the function $f$? In other words,
   $f(\{1, 5, 7\}) =$
F. Prove the following theorems. (10 points each)

1. Suppose $R$ is the “divides” relation, i.e. $aRb \iff a|b$. Show that 0 is greatest on the set $\mathbb{N}$ according to $R$ (i.e. $R$-greatest).
2. Define \( f : [0, 1] \to [1, 3] \) as \( f(x) = 2x + 1 \). Prove that \( f \) is onto.
3. For all $n \in \mathbb{N}$, given sets $A_0, A_1, \ldots, A_n$, and $X$, the following set equality holds: $X - \left( \bigcap_{i=0}^{n} A_i \right) = \bigcup_{i=0}^{n} (X - A_i)$. 
4. $\forall n \in \mathbb{N} \left[ \sum_{i=0}^{n} 1/2^i = 2 - (1/2^n) \right]$. 
5. For integers $a$, $b$, and $c$, assume $ab = c$ and $c$ is odd. Then at least one of $a$ and $b$ is odd as well.
6. \((A \land \neg B) \leftrightarrow (A \lor B) \equiv \neg B\) (Prove via linear equivalence proof)