Exam 1 for CS301K Spring 2014

A. For each item below, circle either TRUE, FALSE, or NOT A PROPOSITION. (2 points each)

1. This test is a dragon.
   FALSE

2. $\exists x \in \mathbb{R} \forall y \in \mathbb{R} [xy = y]$ 
   TRUE

3. Do you need help?
   NOT A PROPOSITION

4. $\forall x, y \in \mathbb{Z} \exists z \in \mathbb{Z} [x + y = z]$
   TRUE

B. Analyze the logical forms of the following statements without using any predicates. Define your propositional variables. (4 points each)

1. This table is pretty, but it is not sturdy enough.
   
   P = “Table is pretty”
   S = “Table is sturdy enough”
   
   $P \land \neg S$

2. If Kevin doesn’t apologize, then I’ll never forgive him.
   
   A = “Kevin apologizes”
   F = “I forgive Kevin”
   
   $\neg A \rightarrow \neg F$
C. Analyze the logical form of the following statement assuming the universe for all variables is “animals” and using only the following predicates:

\[ D(x) = \text{"x is a dinosaur"} \]
\[ B(x) = \text{"x is a bird"} \]
\[ L(x, y) = \text{"y is a descendant of x"} \]

All birds are descended from dinosaurs (4 points)

\[ \forall y(B(y) \rightarrow \exists x(D(x) \land L(x, y))) \]

D. Write a linear proof of the following equivalence. (10 points)

\[ B \land (A \rightarrow (\neg B \rightarrow C)) \equiv B. \]

Proof:
1. \[ B \land (A \rightarrow (\neg B \rightarrow C)) \]
2. \[ \equiv B \land ((A \land \neg B) \rightarrow C) \text{ {exportation}} \]
3. \[ \equiv B \land (\neg (A \land \neg B) \lor C) \text{ {implication}} \]
4. \[ \equiv B \land ((\neg A \lor B) \lor C) \text{ {De Morgan, double negation}} \]
5. \[ \equiv B \land (B \lor (\neg A \lor C)) \text{ {\lor commutativity, \lor associativity}} \]
6. \[ \equiv B \text{ {absorption}} \]
E. Translate the following argument into logical form (define propositional variables and then arrange them in a logical argument), and then use rules of inference and logical identities to prove it. (10 points)

Argument: If you drive a car or ride a bike, then you will get to work on time. You did not get to work on time. Therefore, you did not drive your car.

Propositional variables:
X = “You drive a car”
Y = “You ride a bike”
Z = “You get to work on time”

Translation:

\[(X \lor Y) \rightarrow Z\]
\[\neg Z\]
\[\therefore \neg X\]

Proof:
1. \((X \lor Y) \rightarrow Z\) \{Given\}
2. \(\neg Z\) \{Given\}
3. \(\neg(X \lor Y)\) \{Modus Tollens: 1,2\}
4. \(\neg X \land \neg Y\) \{De Morgan: 3\}
5. \(\neg X\) \{Simplification: 4\}
F. Suppose \( x \in \mathbb{Z} \). Prove that if \( x^2 \) is odd, then so is \( x \). (10 points)

1. Prove by contrapositive: \((x \text{ even}) \rightarrow (x^2 \text{ even})\)
2. Assume \( x \) even
3. \( \exists k \in \mathbb{Z}[x = 2k] \) \{Def. even: 2\}
4. \( x^2 = xx \) \{meaning of “squared”\}
5. \( = 2k2k \) \{substitution: 3\}
6. \( = 2(k2k) \) \{associativity of multiplication\}
7. Define \( q = k2k \), so \( x^2 = 2q \)
8. \( q \in \mathbb{Z} \) \{\( \mathbb{Z} \) closed under multiplication\}
9. \( \exists q \in \mathbb{Z}[x^2 = 2q] \) \{Existential generalization: 7,8\}
10. \( x^2 \) is even \{Def. even: 9\}