A. Provide counterexamples disproving the following propositions. (Explain why they are counterexamples). (5 points each).

1. Define $R$ as an equivalence relation on $\mathbb{Z}$ where $aRb \iff |a| = |b|$. Then for all $x \in \mathbb{Z}$, the size (cardinality) of the equivalence class $[x]_R$ is 2.

2. $\forall x[P(x) \lor Q(x)] \equiv [\forall xP(x)] \lor [\forall xQ(x)]$

3. If $R$ is a relation from $A$ to $B$, and $S$ is a relation from $B$ to $A$, then the composition of the relations, $S \circ R$, is an equivalence relation.

B. Enumerate every element of the following sets. (5 points each).

1. $\mathcal{P}(\{a, b\}) \times \mathcal{P}(\emptyset) =$

2. (Recall $\mathbb{N} = \{0, 1, 2, \ldots\}$)
   $\{3^i | i \in \mathbb{N} \land i \leq 4\} =$
C. Prove the following theorems. (10 points each).

1. Theorem: Suppose $R$ is a relation from $A$ to $B$, and $S$ is a relation from $B$ to $C$, and $T$ is a relation from $C$ to $D$. Then $(T \circ S)\circ R \subseteq T \circ (S \circ R)$.
2. Theorem: If $A$ and $B$ are sets, then $(A \times B) \cap (A^C \times B) = \emptyset$. 

D. Decide whether or not the following propositions are true, and then either prove or disprove them. (5 points each).

1. If $R$ is a binary relation on $A$, and $R = R^{-1}$, then $R$ is symmetric.

2. If $R$ is a binary relation on $A$, and $R = R^{-1}$, then $R$ is transitive.
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