Uncertain<*T*>

A First-Order Type for Uncertain Data

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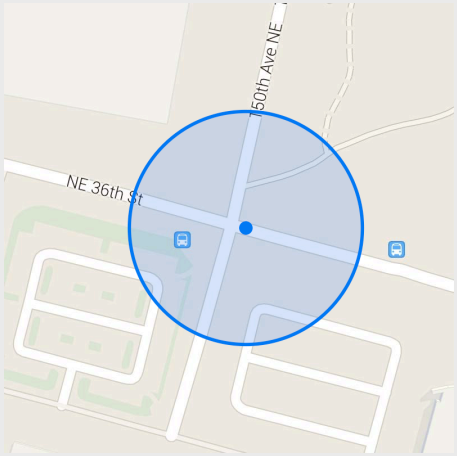
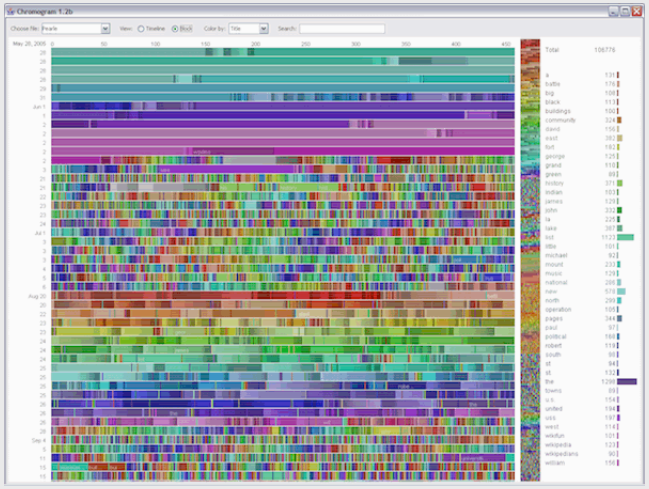


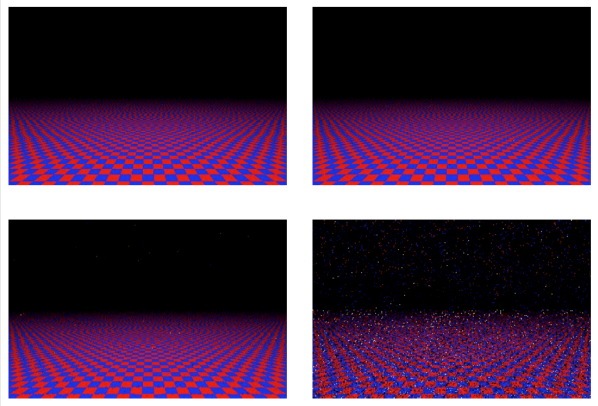
Sensors

Wikipedia



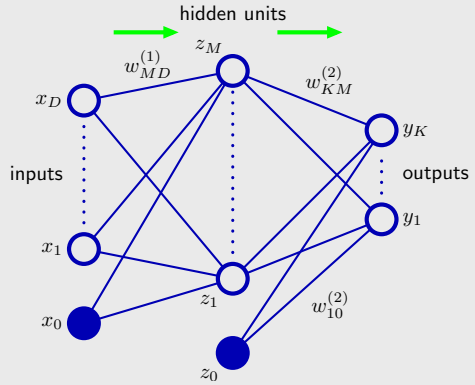
Big data

Sampson et al.



Approximate computing

Bishop



Machine learning

uncertain data

```
struct Geocoordinate {
    double Latitude;
    double Longitude;
}

Geocoordinate Loc = GetGPSLocation();
```

discrete type

uncertain data

\+ discrete type

= ???

uncertain data

\+ discrete type

= who cares?

uncertain data

\+ discrete type

= uncertainty bug

uncertain data

\+ discrete type

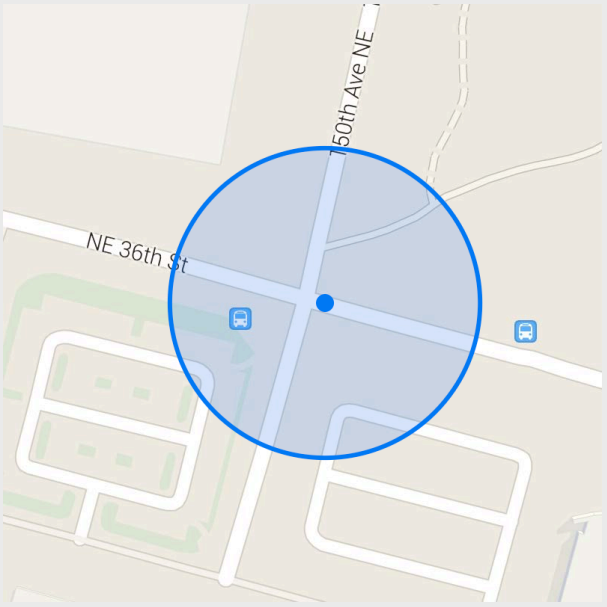
= **uncertainty bug**

errors that occur when applications pretend that uncertain data is certain

treating estimates as facts

```
struct Geocoordinate {
   double Latitude;
   double Longitude;

   double HorizontalAccuracy;
}
```



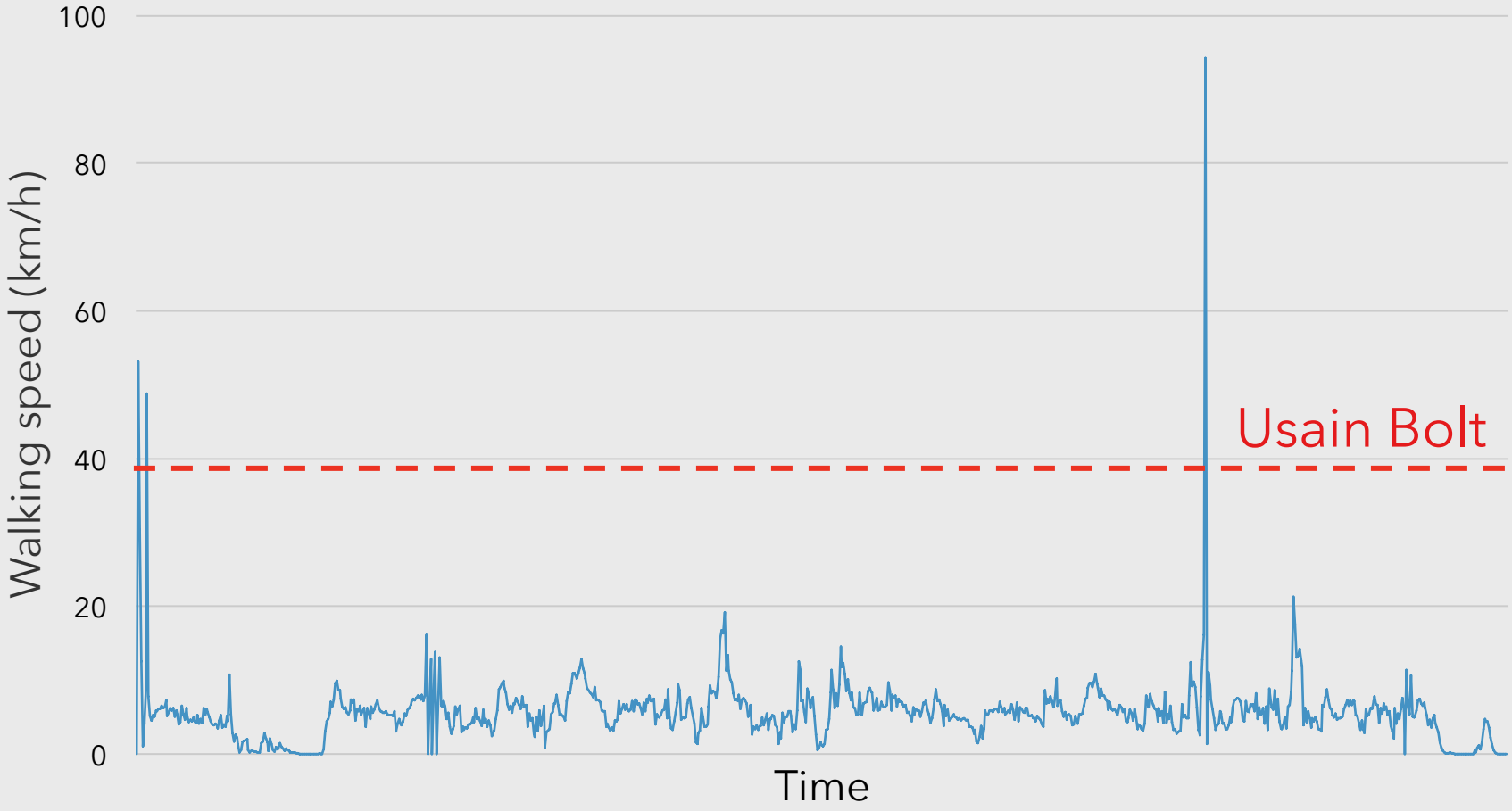
95% of apps ignore accuracy!

computation compounds error

computation compounds error

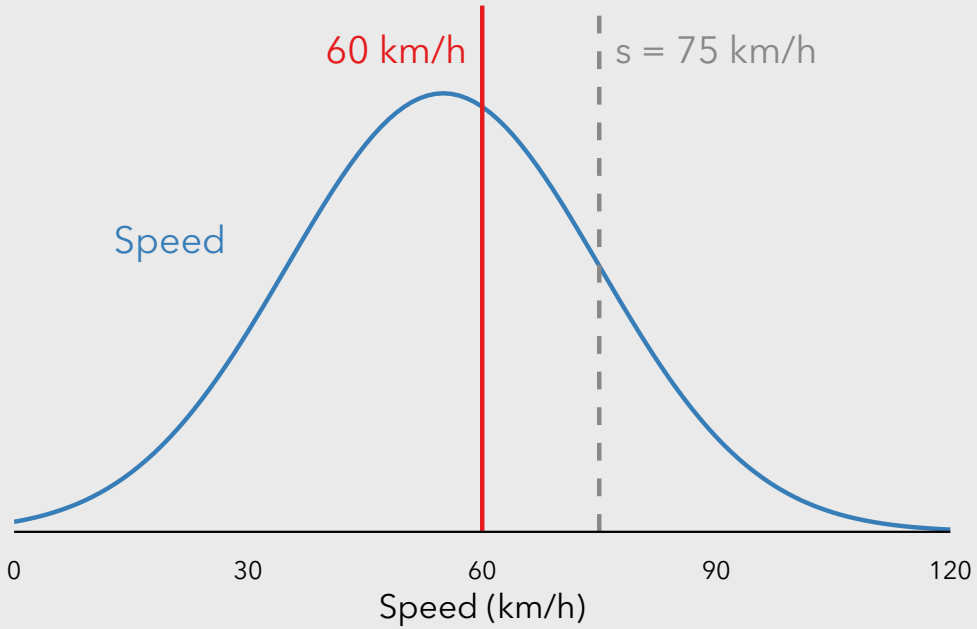
Usain Bolt

computation compounds error



false positives in questions

```
if (Speed > 60)
    IssueSpeedingTicket();
```



uncertainty bugs

Treating estimates as facts

Computation compounds error

False positives in questions

Caused by poor programming language abstractions

Uncertainty should not be abstracted away

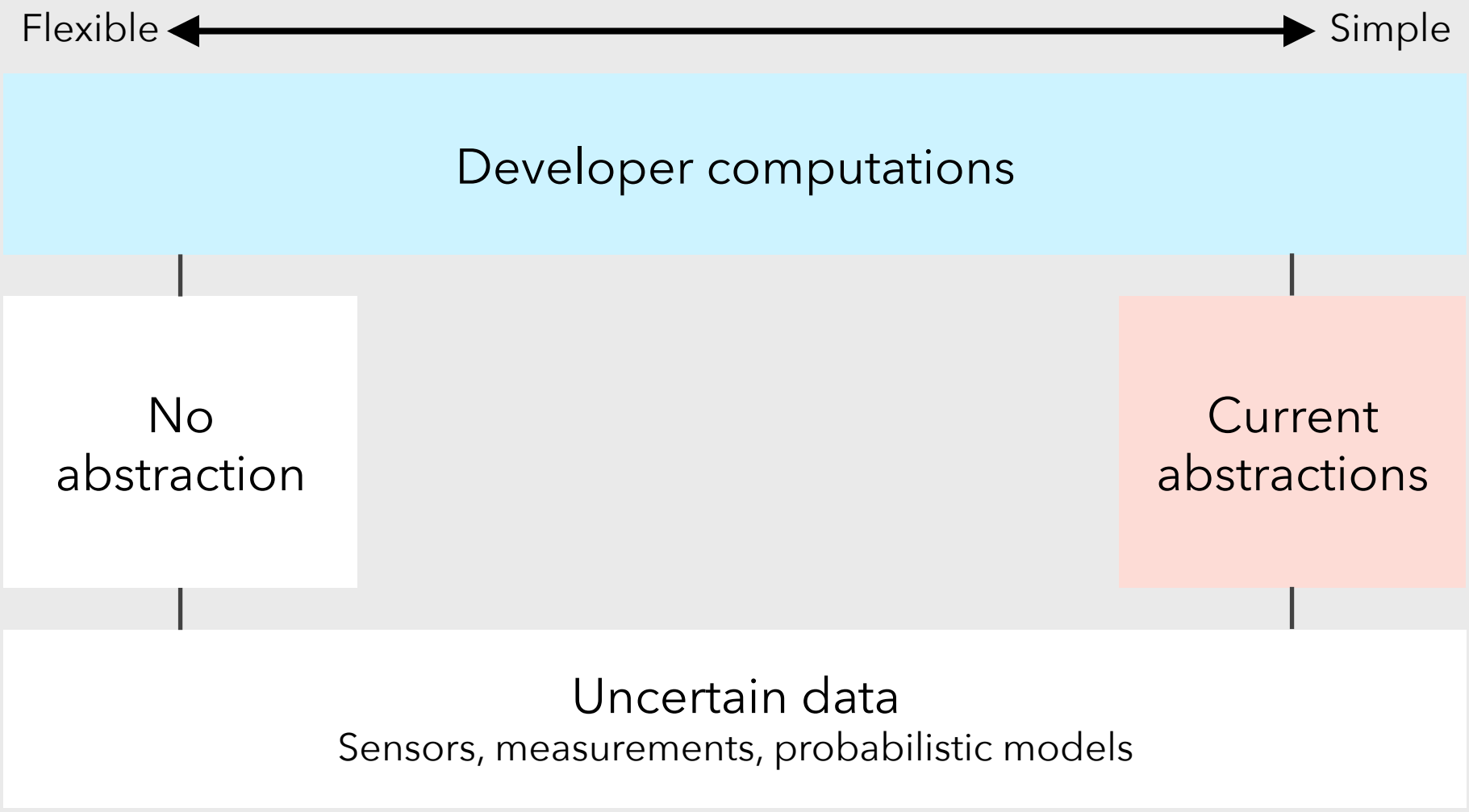
related work

Developer computations

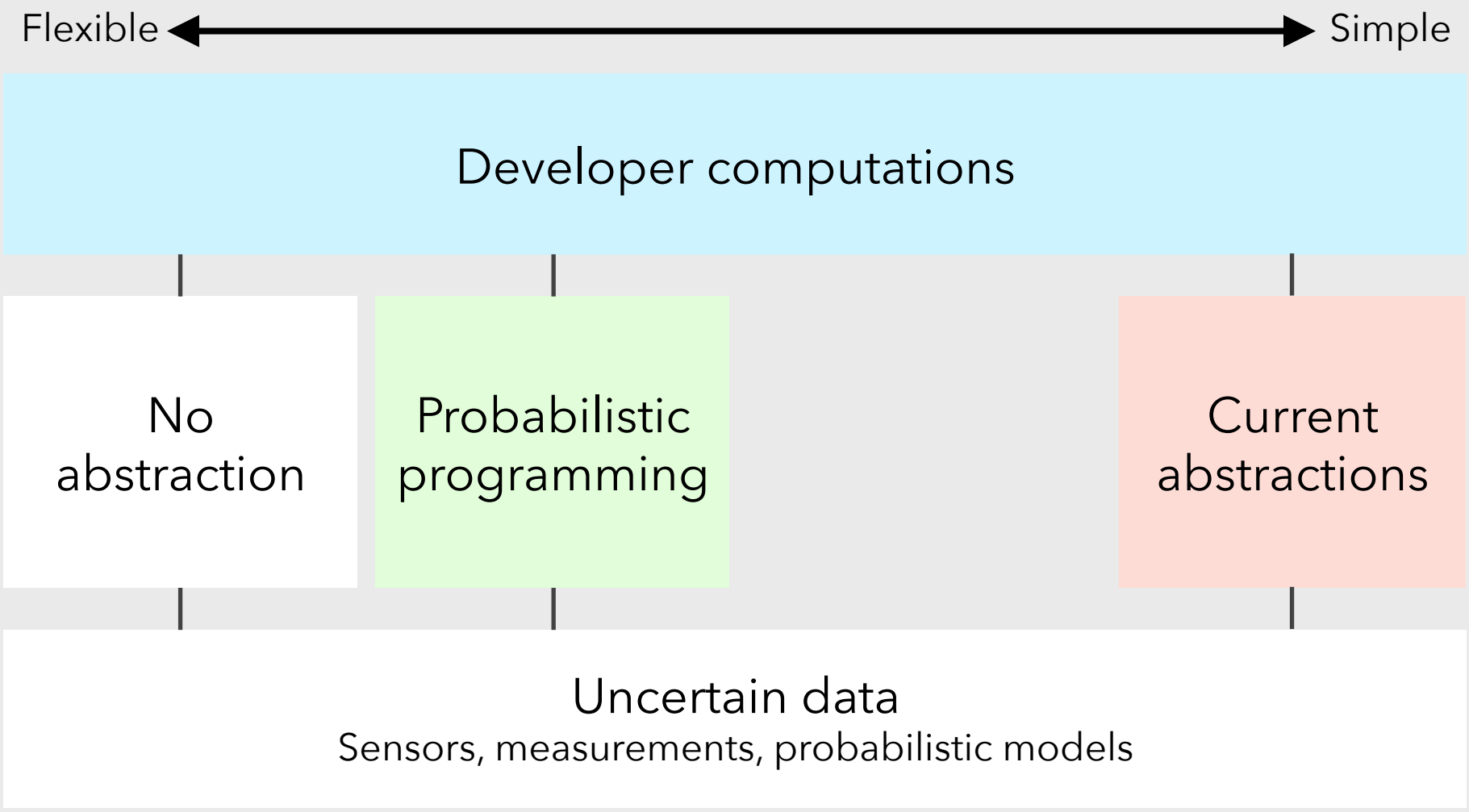
Uncertain data

Sensors, measurements, probabilistic models

related work



related work



probabilistic programming

Reasoning about probabilistic models

```
earthquake = Bernoulli(0.0001)
burglary   = Bernoulli(0.001)
alarm      = earthquake or burglary

if (earthquake)
  phoneWorking = Bernoulli(0.7)
else
  phoneWorking = Bernoulli(0.99)
```

inference

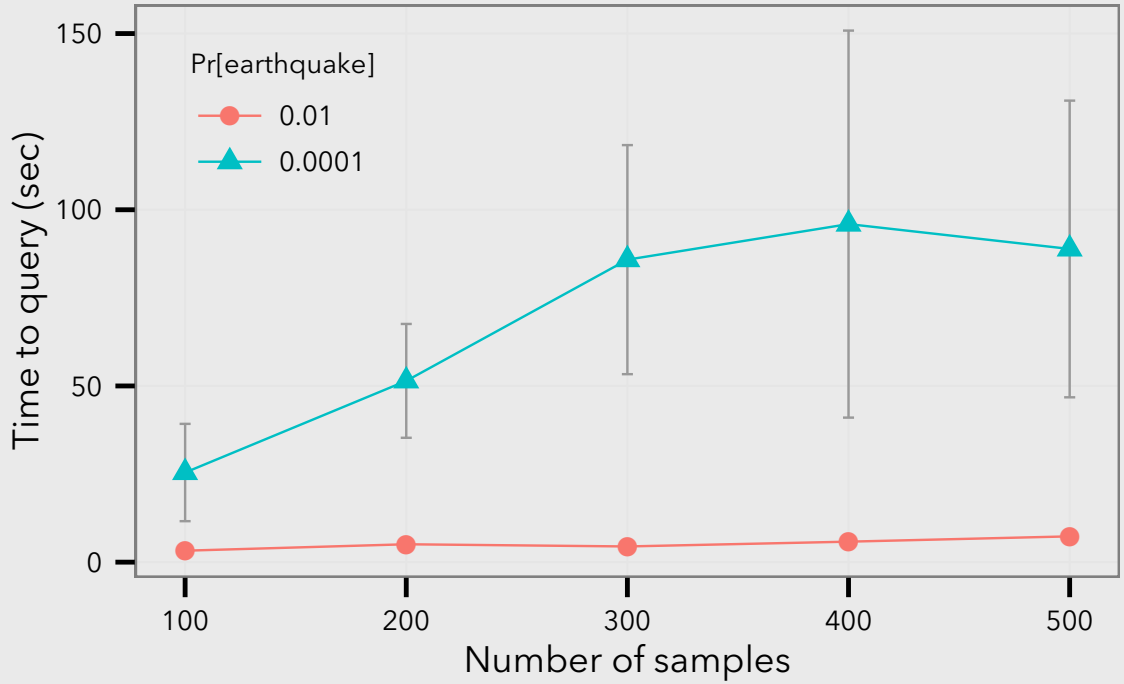
```
earthquake = Bernoulli(0.0001)
burglary   = Bernoulli(0.001)
alarm      = earthquake or burglary
if (earthquake)
  phoneWorking = Bernoulli(0.7)
else
  phoneWorking = Bernoulli(0.99)

observe(alarm=true)
query(phoneWorking)
```

What is Pr[phoneWorking=*v* | alarm=True], for each possible value of *v* (i.e. True and False)?

inference is expensive

Some paths of execution are very unlikely



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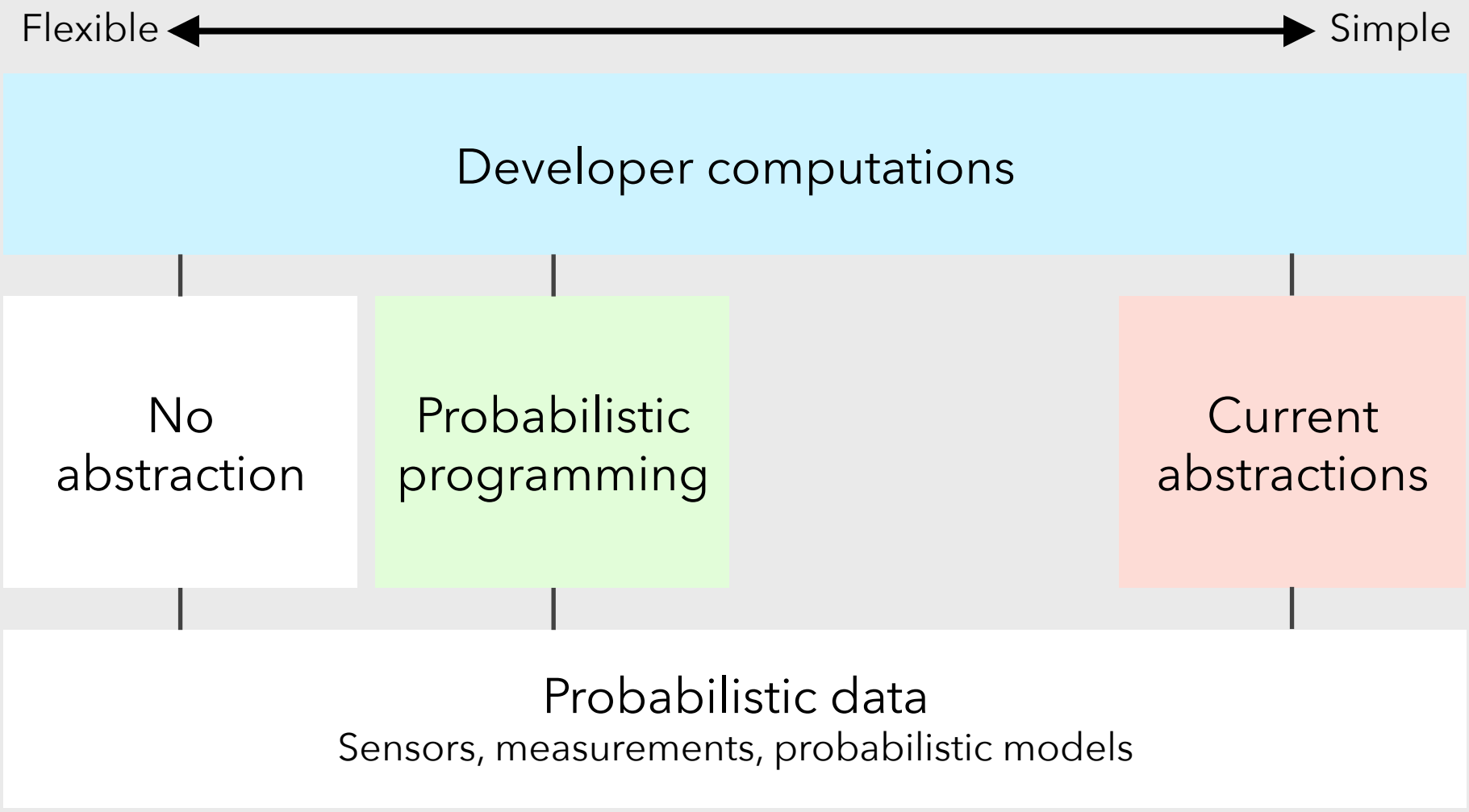

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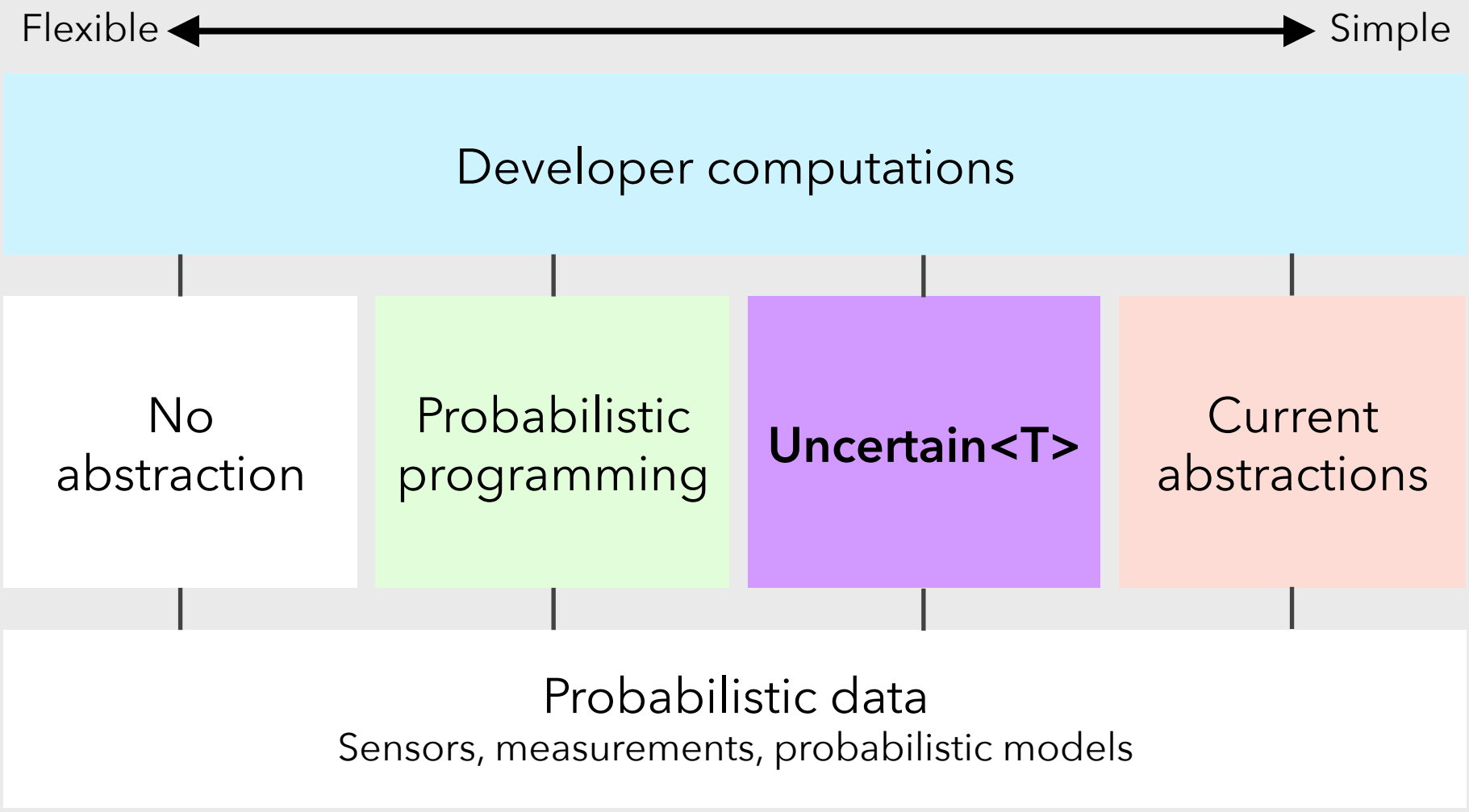


Uncertain<T> is an uncertain type abstraction.

Encapsulates distributions, like prior work.

But focuses on an accessible interface.

For everyday programmers, Uncertain<T> enables programs that are more concise, expressive, and correct.

using Uncertain<T>

Identify the distribution

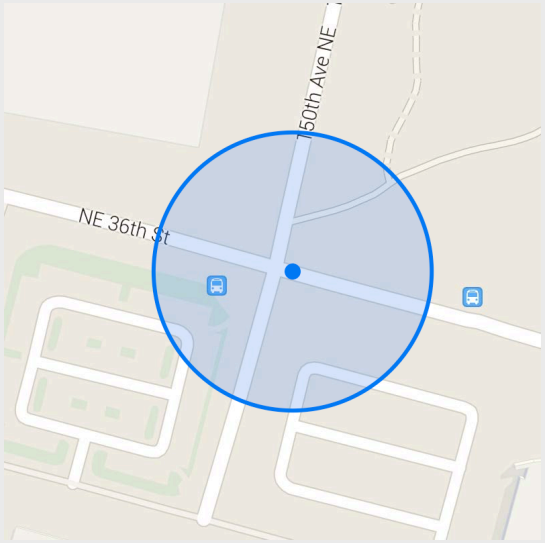
Compute with the distribution

Ask questions using conditionals

Improve the quality of estimates

identifying the distribution

Many library programmers already know the distribution they need to return!



identifying the distribution

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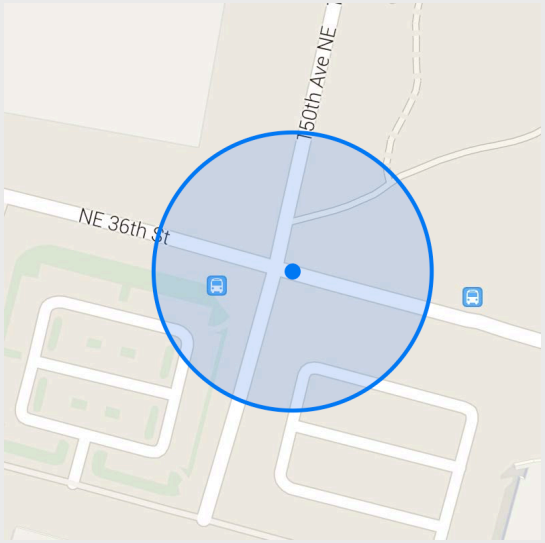

"Get the estimated accuracy of this location, in meters. We define accuracy as the radius of 68% confidence. […] In statistical terms, it is assumed that location errors are random with a normal distribution."

—Android

representing distributions

$$\text{Norm}(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

representing distributions

Store probability density functions?

$$\mathrm{Norm}(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

Two problems:

1. Even simple operations are complex:

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_Y(z-x) f_X(x)\,\mathrm{d}x$$

2. Many interesting distributions don't have PDFs

representing distributions

Store probability density functions?

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2. Many interesting distributions don't have PDFs

representing distributions

Random sampling: two birds with one stone

- Simple operations are simple (e.g., +)
- More distributions can be represented

Later: how to implement random sampling

computing with distributions

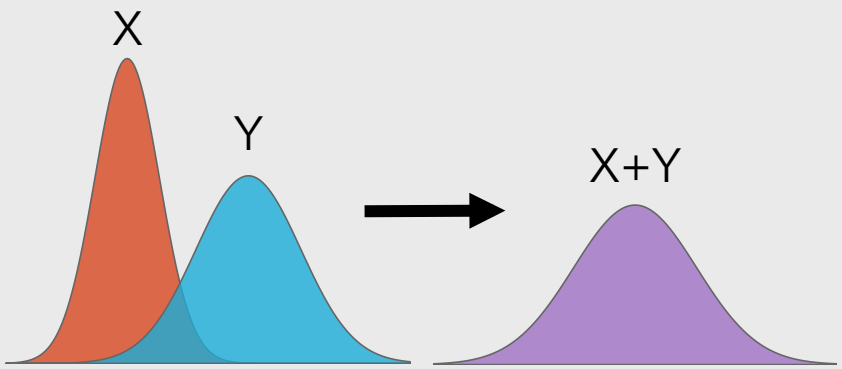
Propagating uncertainty through calculations automatically with operator overloading

A key advantage of random sampling: computation is simply* lifting of the original operators

computing with distributions

Propagating uncertainty through calculations automatically with operator overloading

A key advantage of random sampling: computation is simply* lifting of the original operators



If x a sample of X
and y a sample of Y
then $x+y$ a sample of $X+Y$

computing with distributions

* The caveat is that this only works if the operands are independent

If not, we need to know something about how the variables are related

This is an issue for all probabilistic programming

induced dependencies

$$A = X + Y \quad \text{(X,Y independent)}$$
$$B = A + X$$

induced dependencies

We can distinguish inherent dependencies from programmer-induced dependencies

```
A = X + Y    (X,Y independent)
B = A + X
```

When evaluating B, both operands depend on X, so they are not independent

Lazy evaluation to the rescue!

induced dependencies

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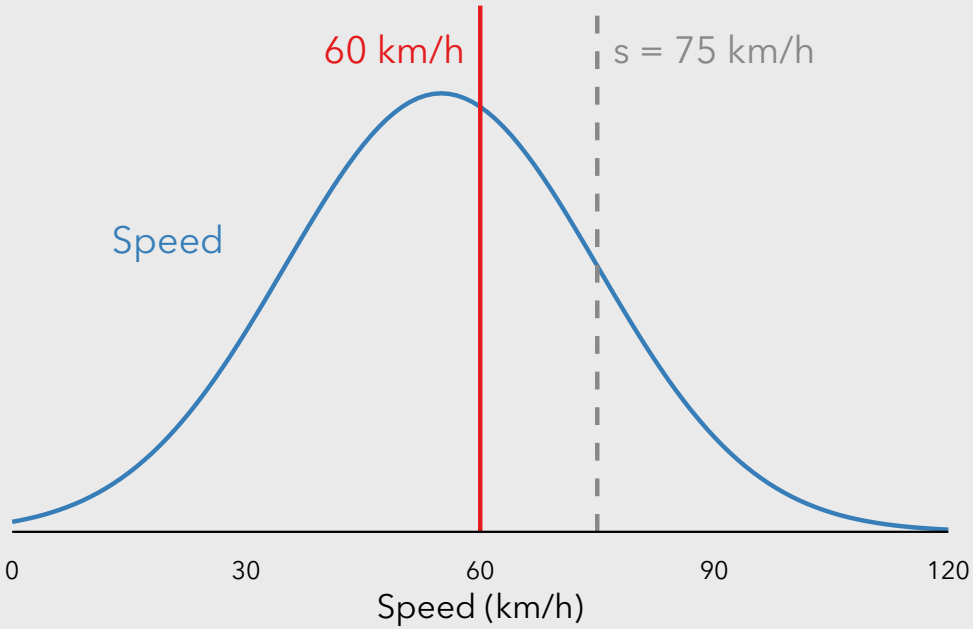
Lazy evaluation to the rescue!

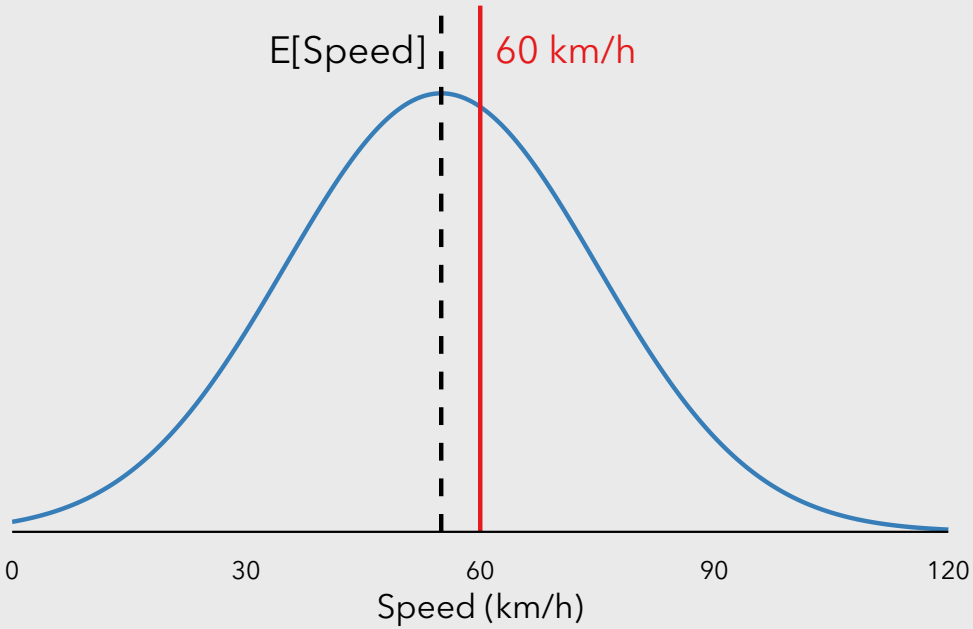
asking questions

```
if (Speed > 60)
    IssueSpeedingTicket();
```



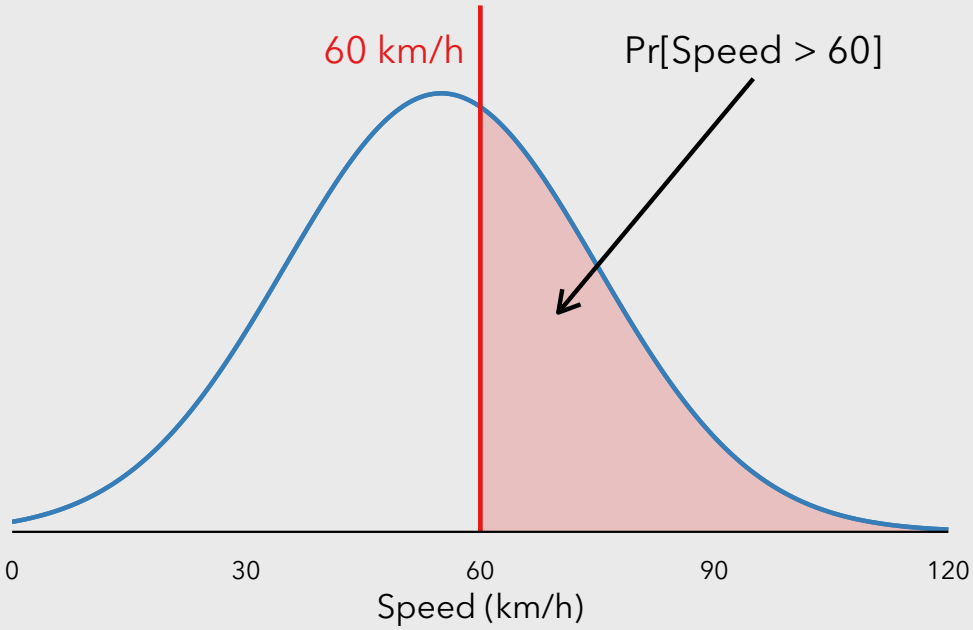
comparing means

```
if (Speed.E() > 60)
    IssueSpeedingTicket();
```



comparing evidence

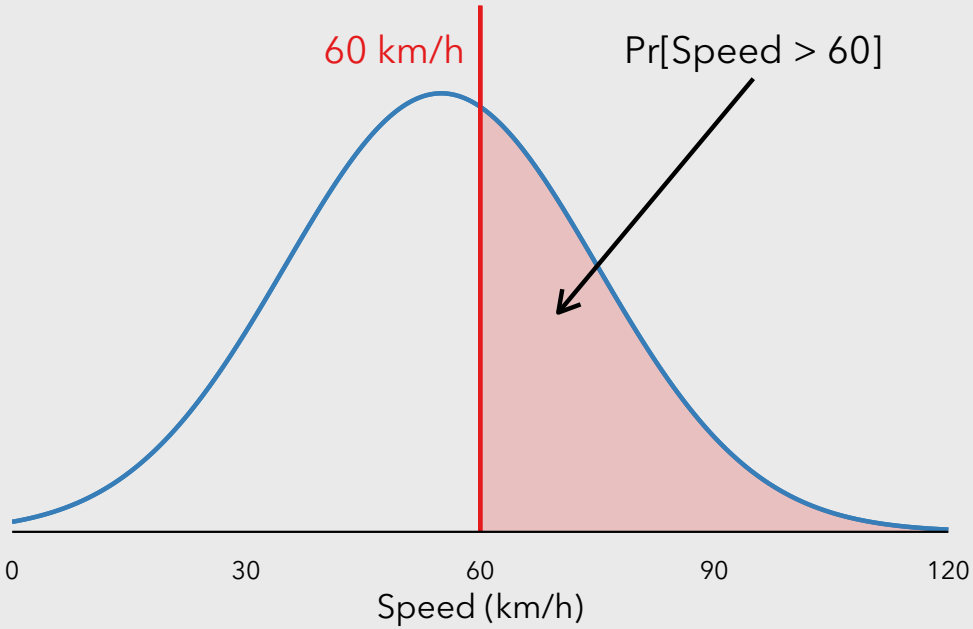
```
if ((Speed > 60).E() > 0.95)
    IssueSpeedingTicket();
```



comparing evidence

> is a lifted operator

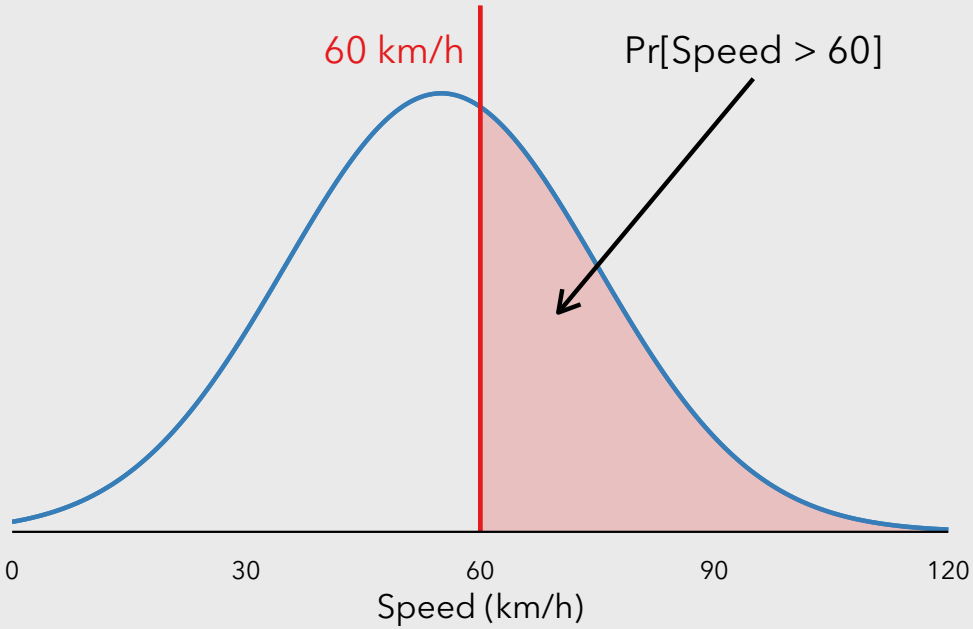
```
if ((Speed > 60).E() > 0.95)
    IssueSpeedingTicket();
```



comparing evidence

type Uncertain<bool>

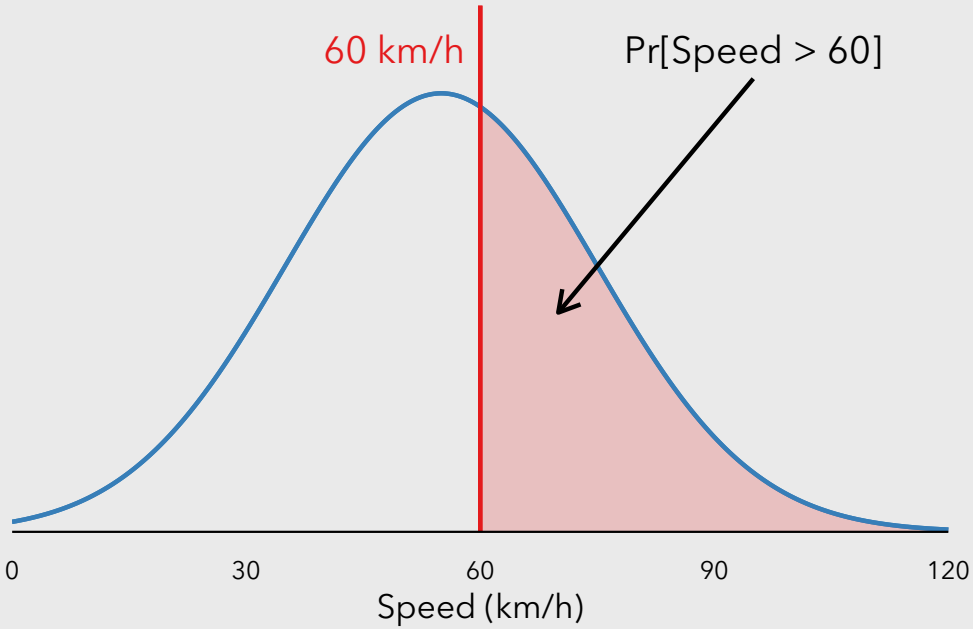
```
if ((Speed > 60).E() > 0.95)
    IssueSpeedingTicket();
```



comparing evidence

mean of Uncertain<bool>

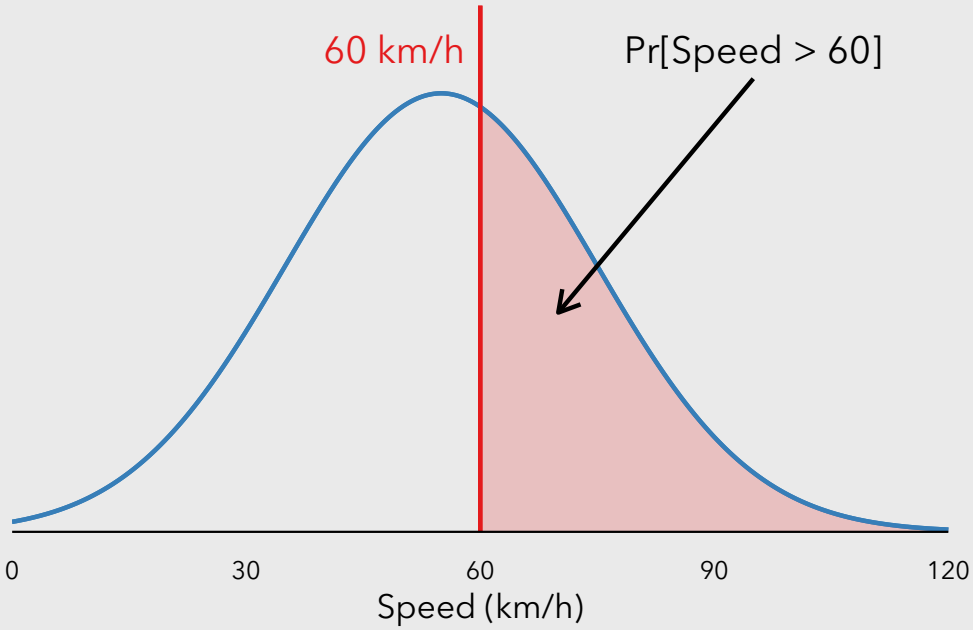
```
if ((Speed > 60).E() > 0.95)
    IssueSpeedingTicket();
```



comparing evidence

= number in [0,1]

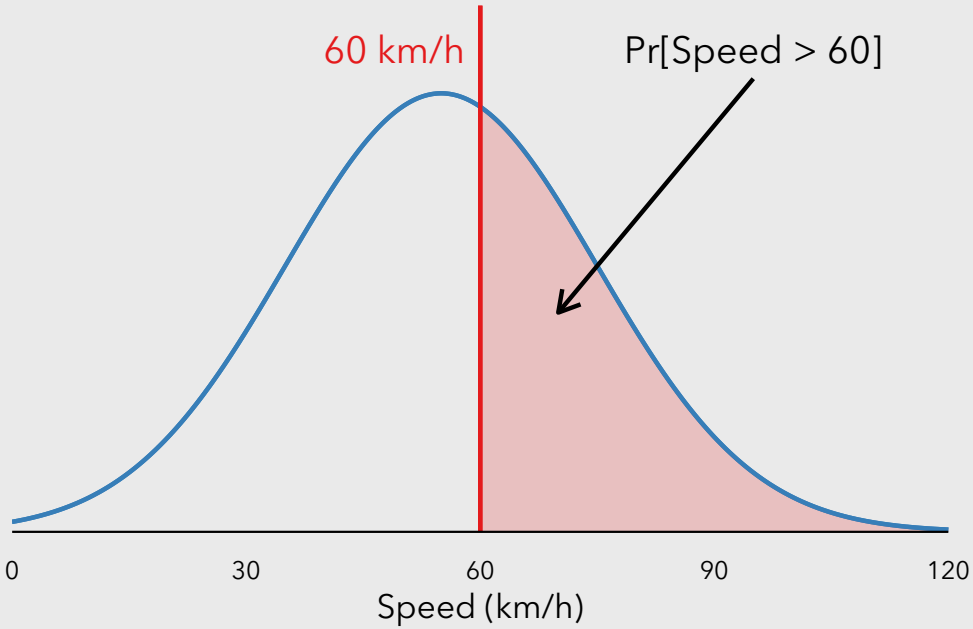
```
if ((Speed > 60).E() > 0.95)
    IssueSpeedingTicket();
```



comparing evidence

% of True instances

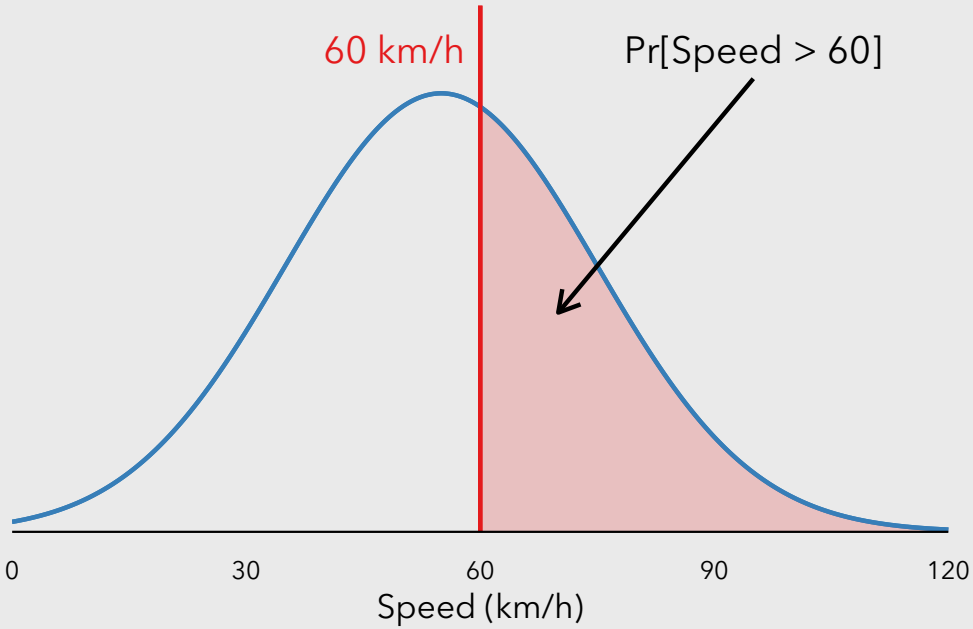
```
if ((Speed > 60).E() > 0.95)
    IssueSpeedingTicket();
```



comparing evidence

is there a >95% chance that Speed > 60?

```
if ((Speed > 60).E() > 0.95)
    IssueSpeedingTicket();
```



comparing evidence

```
if ((Speed > 60).E() > 0.95)
   IssueSpeedingTicket();
```

The threshold allows the programmer to balance false positives and false negatives

Higher thresholds give fewer false positives, but more false negatives

improving estimates

Uncertain<T> is Bayesian: error distributions track degrees of belief about the value of a variable

$$\Pr[H|E] = \frac{\Pr[E|H]\Pr[H]}{\Pr[E]}$$

Bayes' theorem: use prior knowledge to improve estimates

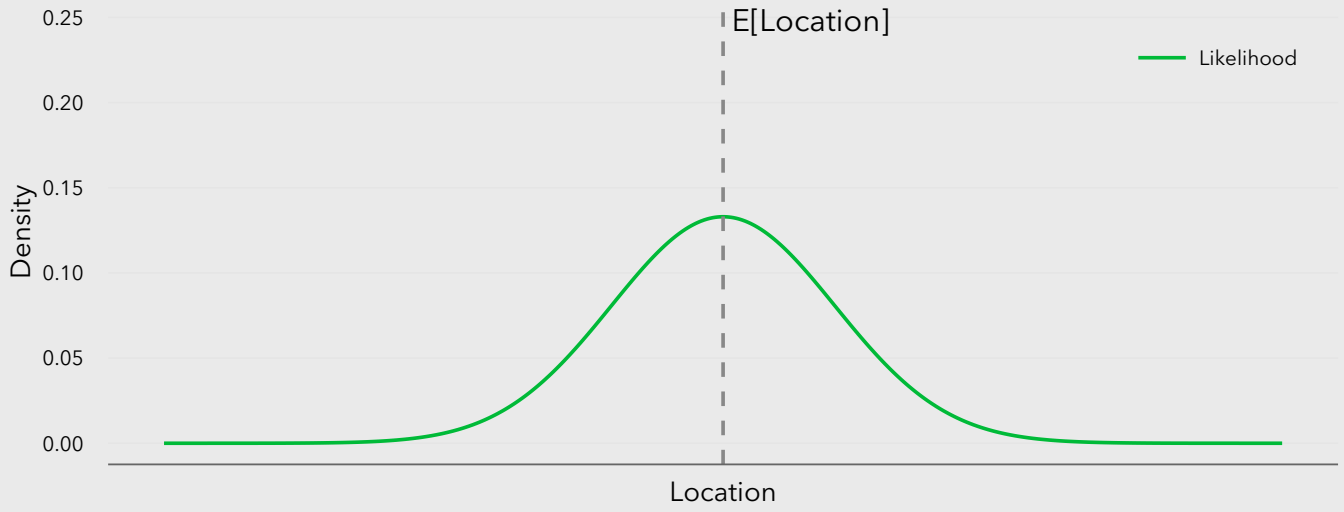
improving estimates

$$\Pr[H|E] = \frac{\Pr[E|H]\Pr[H]}{\Pr[E]}$$

improving estimates

likelihood

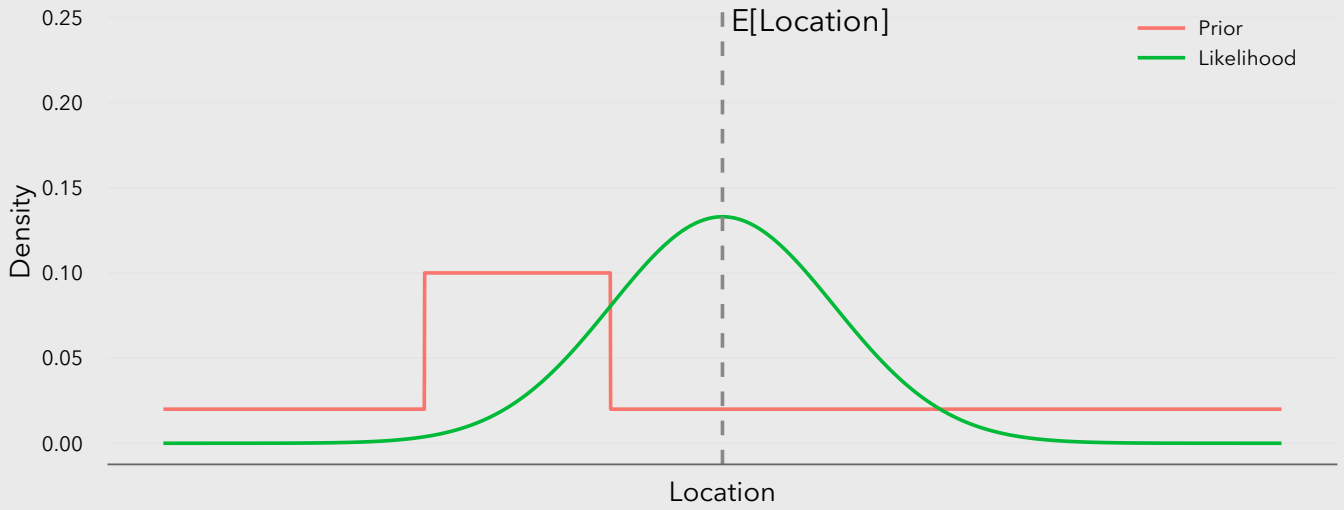
$$\Pr[H|E] = \frac{\Pr[E|H]\Pr[H]}{\Pr[E]}$$



improving estimates

likelihood prior

$$\Pr[H|E] = \frac{\Pr[E|H]\Pr[H]}{\Pr[E]}$$



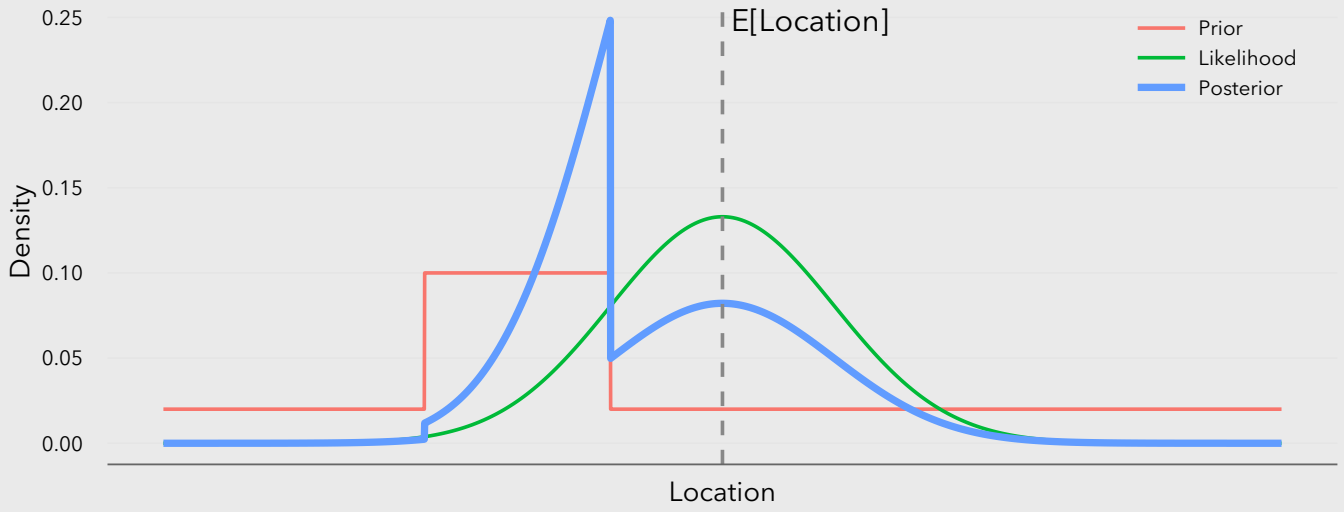
improving estimates

posterior

likelihood

prior

$$\Pr[H|E] = \frac{\Pr[E|H]\Pr[H]}{\Pr[E]}$$



improving estimates

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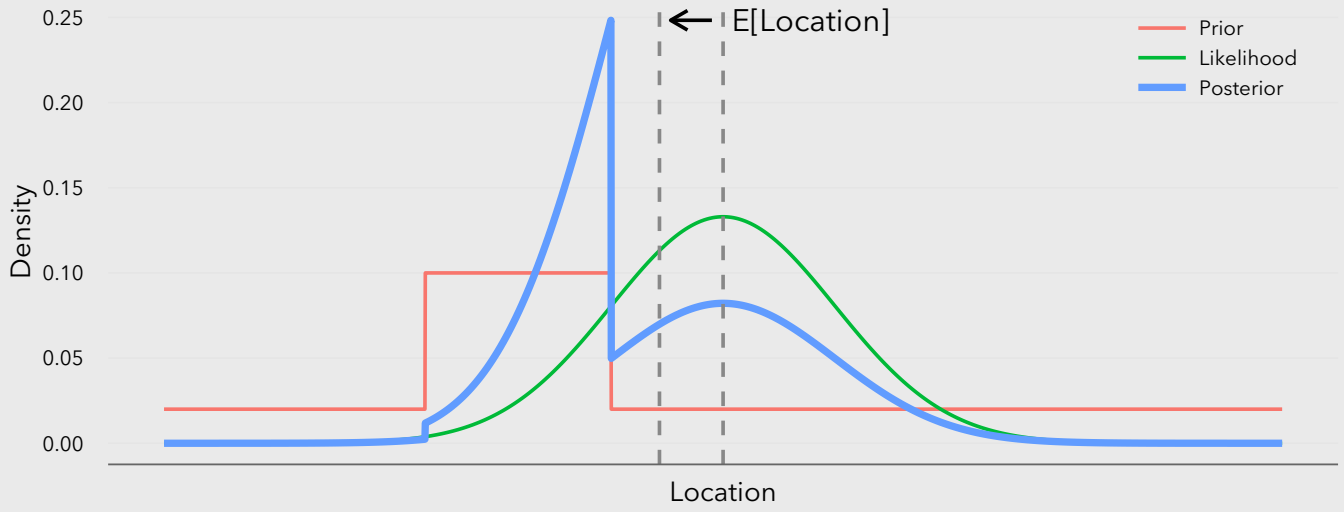

implementing Uncertain<T>

Two key insights in the design inform an efficient implementation

1. Distributions are random samples

   Suggests lazy evaluation

2. All evaluations end up in expected values

   Suggests hypothesis tests

lazy evaluation

Uncertain<T> uses random sampling, but how?

Option 1: store a vector of N samples

A	5.6	2.8	6.4	4.9	4.9	5.1	4.3	5.0	…	4.6

lazy evaluation

Uncertain<T> uses random sampling, but how?

Option 1: store a vector of N samples

A	5.6	2.8	6.4	4.9	4.9	5.1	4.3	5.0	…	4.6
+										
B	4.0	3.2	1.1	3.5	3.9	3.4	4.7	3.8	…	2.2

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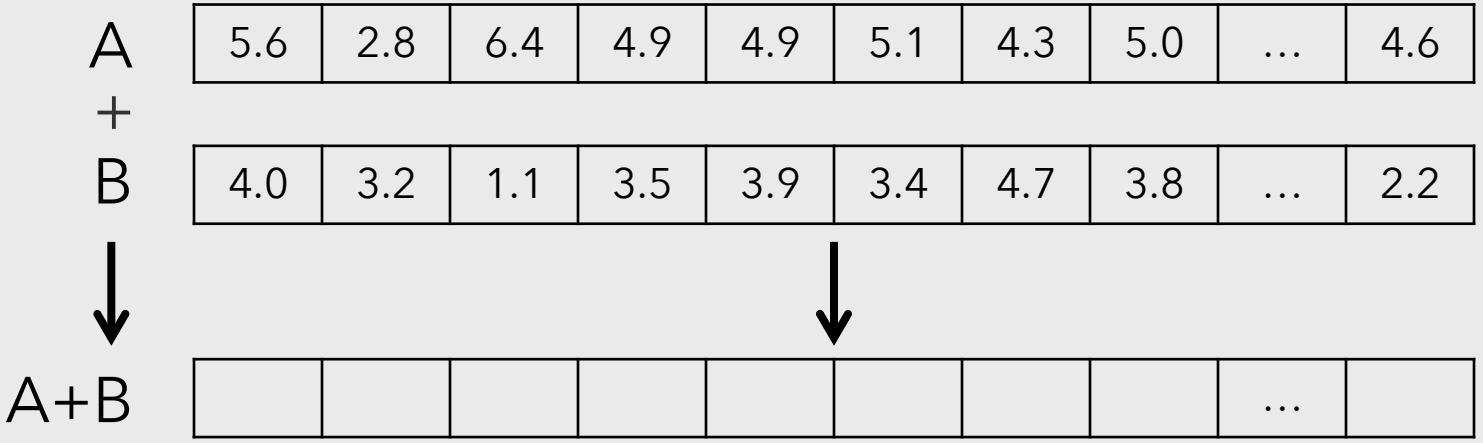

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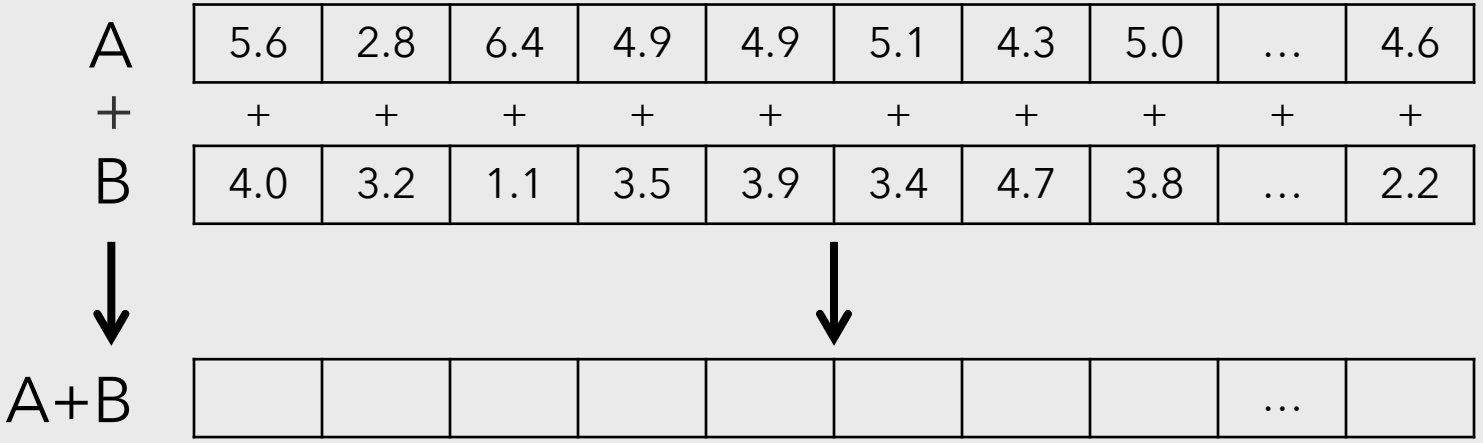

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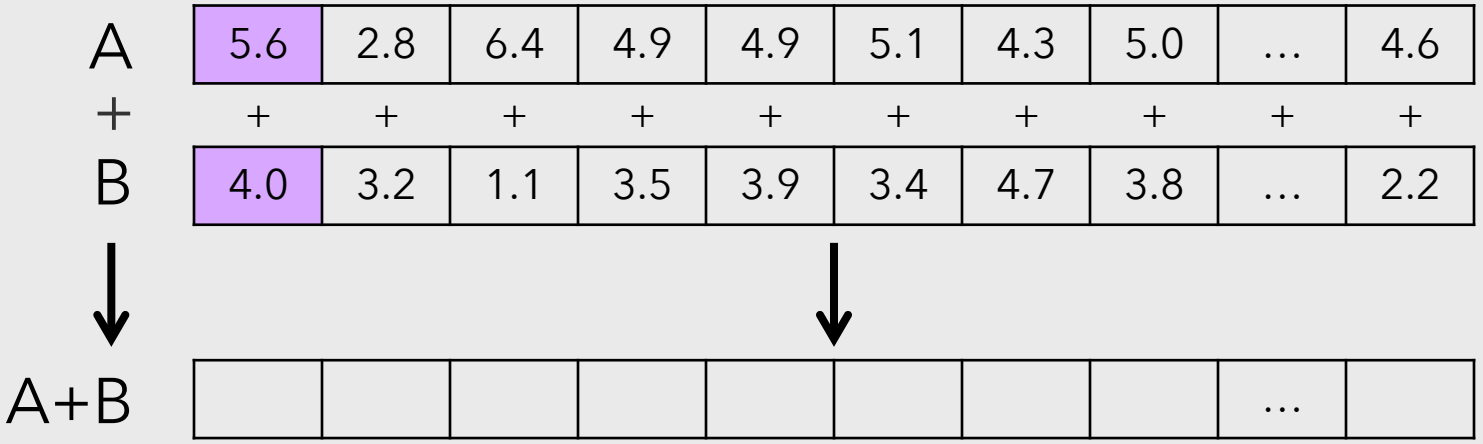

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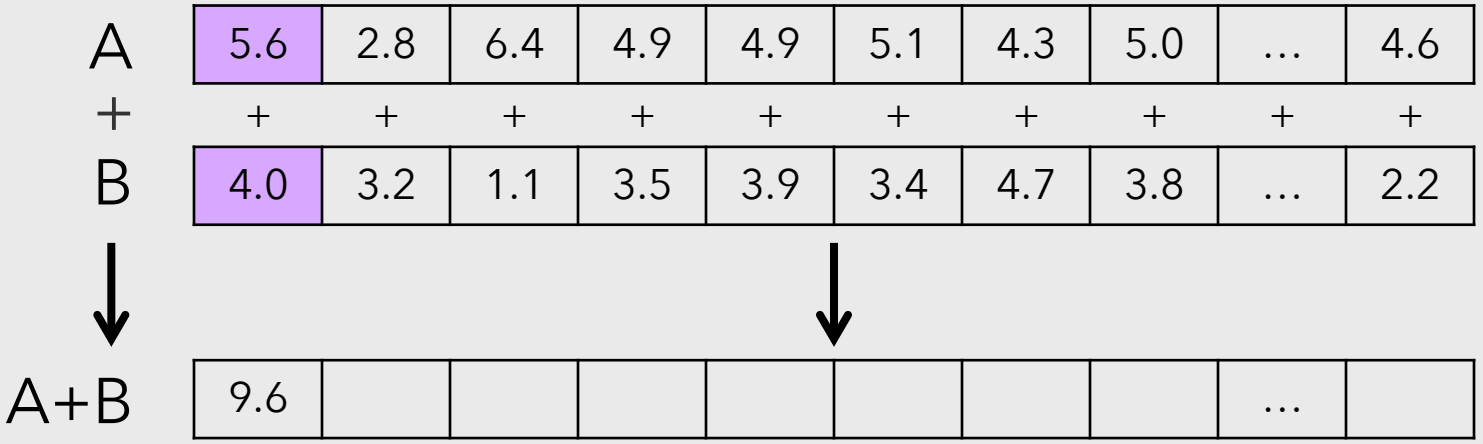

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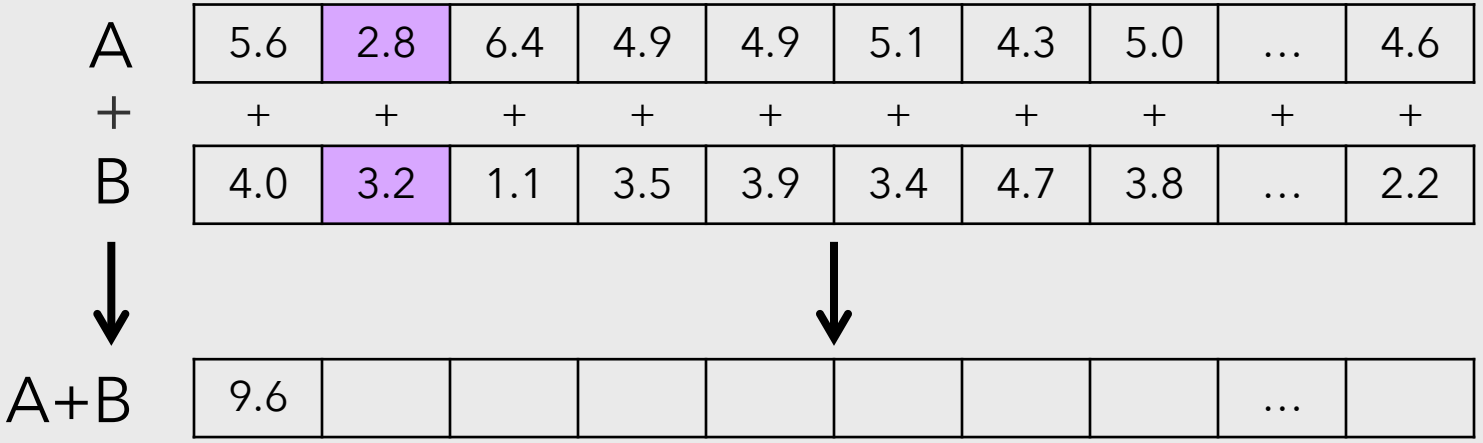

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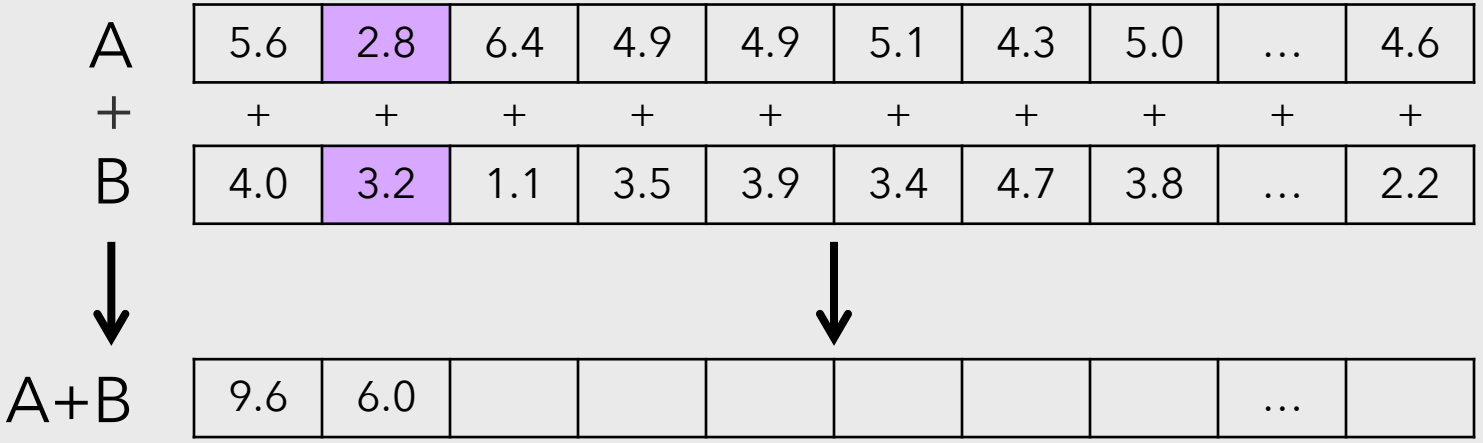

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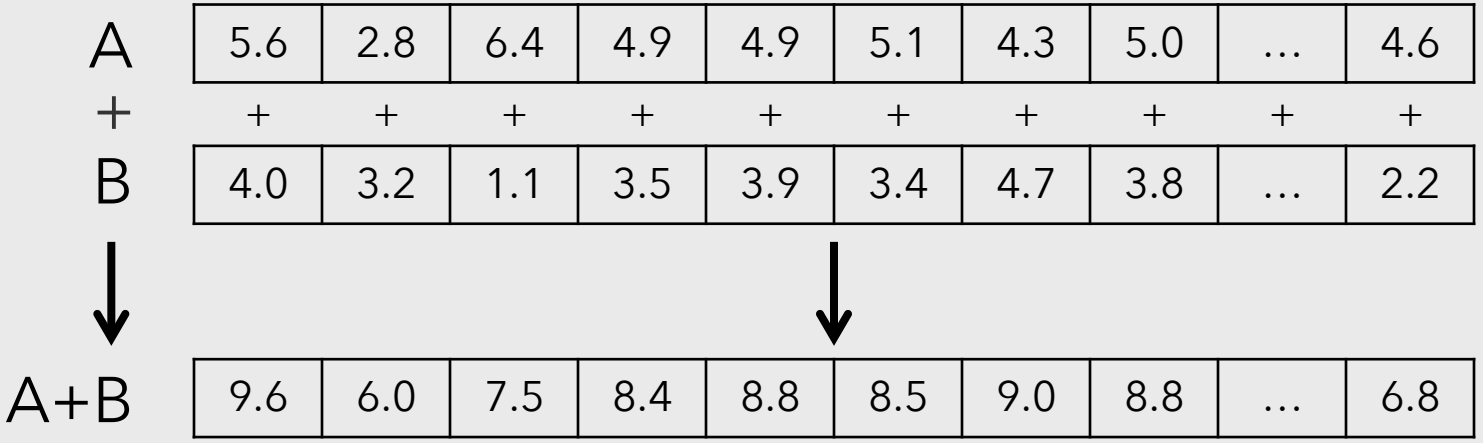


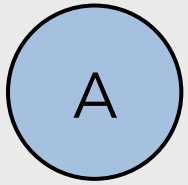
lazy evaluation

Suppose an oracle tells us the “right” sample size for a particular operation (we’ll invent this oracle shortly!) How do we satisfy this sample size?

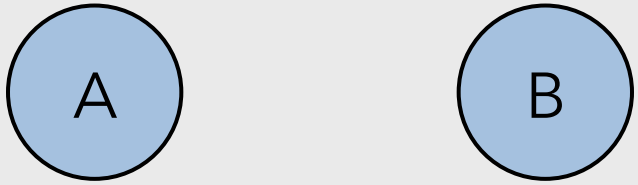
Uncertain<T> represents distributions with sampling functions, returning a new sample on each invocation

Operators combining distributions are lazy, constructing a symbolic expression tree

evaluating expression trees



evaluating expression trees




```
var A = GetReading()
var B = GetReading()
var Sum = A + B
if ((Sum > 10).E() > 75%):
  Alert()
```

evaluating expression trees

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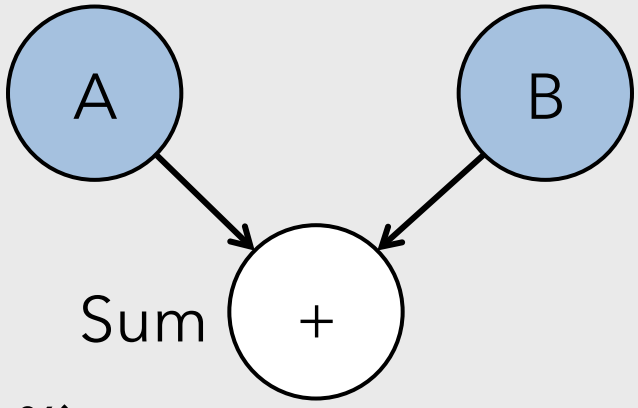

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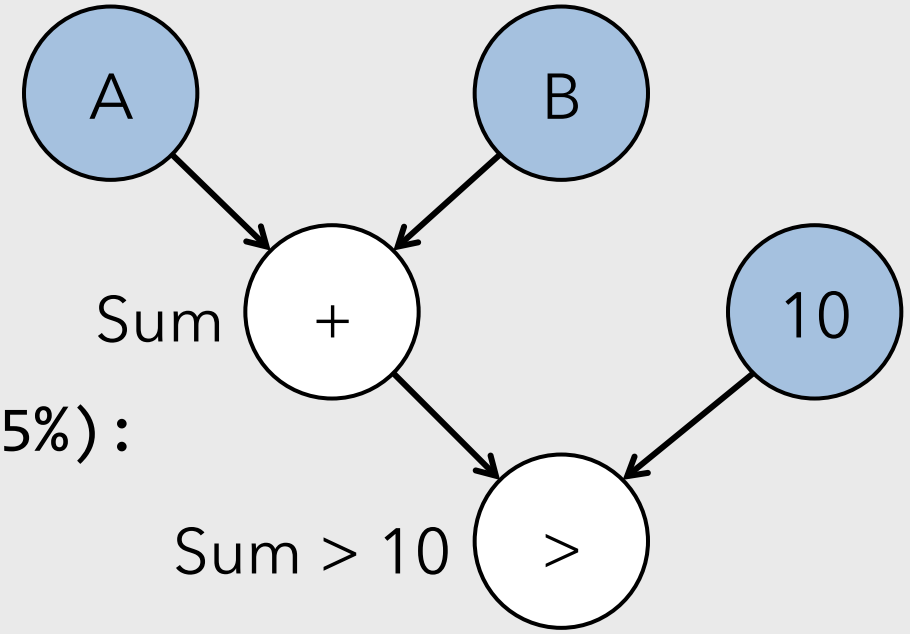

hypothesis tests

How do we decide the “right” sample size for a particular operation?

Distributions only evaluated at conditionals, so use hypothesis tests to address sampling error

hypothesis tests

```
if (Speed.E() > 60)
   IssueSpeedingTicket();
```

This code implicitly performs a hypothesis test

Start with a base sample size

Continue increasing the sample size until either

1. The null hypothesis is rejected; or
2. A maximum sample size limit is reached (to ensure termination)

smartphone GPS sensors

Many smartphone apps use GPS to calculate distances and speeds

How can Uncertain<T> improve these apps?

```
int dt = 1;

Geocoordinate LastLocation =
  GPSLib.GetGPSLocation();
while (true) {
    Sleep(dt); // wait for dt seconds

    Geocoordinate Location =
      GPSLib.GetGPSLocation();
    double Speed =
      GPSLib.Distance(Location, LastLocation) / dt;

    Display(Speed);
    if (Speed > 5)
        GoodJobMessage();

    LastLocation = Location;
}
```

```
  int dt = 1;

★Geocoordinate LastLocation =
    GPSLib.GetGPSLocation();
  while (true) {
      Sleep(dt); // wait for dt seconds

    ★Geocoordinate Location =
        GPSLib.GetGPSLocation();
    ★double Speed =
        GPSLib.Distance(Location, LastLocation) / dt;

      Display(Speed);
      if (Speed > 5)
          GoodJobMessage();

      LastLocation = Location;
  }
```

```
  int dt = 1;

★Uncertain<Geocoordinate> LastLocation =
    GPSLib.GetGPSLocation();
  while (true) {
      Sleep(dt); // wait for dt seconds

    ★Uncertain<Geocoordinate> Location =
        GPSLib.GetGPSLocation();
    ★Uncertain<double> Speed =
        GPSLib.Distance(Location, LastLocation) / dt;

      Display(Speed);
      if (Speed > 5)
          GoodJobMessage();

      LastLocation = Location;
  }
```

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int dt = 1;

Uncertain<Geocoordinate> LastLocation =
  GPSLib.GetGPSLocation();
while (true) {
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      GPSLib.GetGPSLocation();
    Uncertain<double> Speed =
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    if (Speed > 5)
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int dt = 1;

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  GPSLib.GetGPSLocation();
while (true) {
    Sleep(dt); // wait for dt seconds

    Uncertain<Geocoordinate> Location =
      GPSLib.GetGPSLocation();
    Uncertain<double> Speed =
      GPSLib.Distance(Location, LastLocation) / dt;

  ★Display(Speed.E().Project());
    if (Speed > 5)
        GoodJobMessage();

    LastLocation = Location;
}
```

```
int dt = 1;

Uncertain<Geocoordinate> LastLocation =
  GPSLib.GetGPSLocation();
while (true) {
    Sleep(dt); // wait for dt seconds

    Uncertain<Geocoordinate> Location =
      GPSLib.GetGPSLocation();
    Uncertain<double> Speed =
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    Display(Speed.E().Project());
  ★if (Speed > 5)
        GoodJobMessage();

    LastLocation = Location;
}
```

```
int dt = 1;

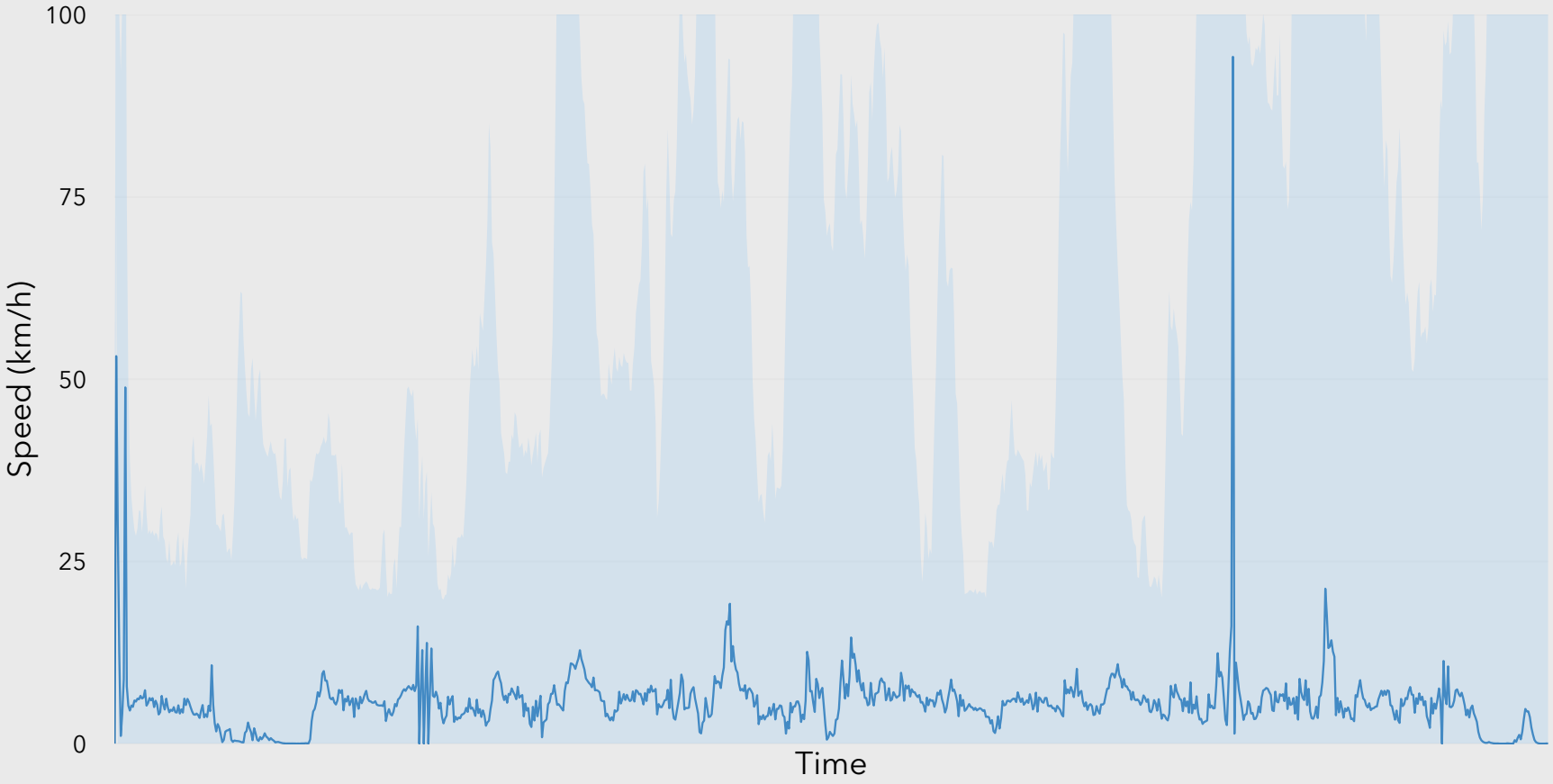
Uncertain<Geocoordinate> LastLocation =
  GPSLib.GetGPSLocation();
while (true) {
    Sleep(dt); // wait for dt seconds

    Uncertain<Geocoordinate> Location =
      GPSLib.GetGPSLocation();
    Uncertain<double> Speed =
      GPSLib.Distance(Location, LastLocation) / dt;

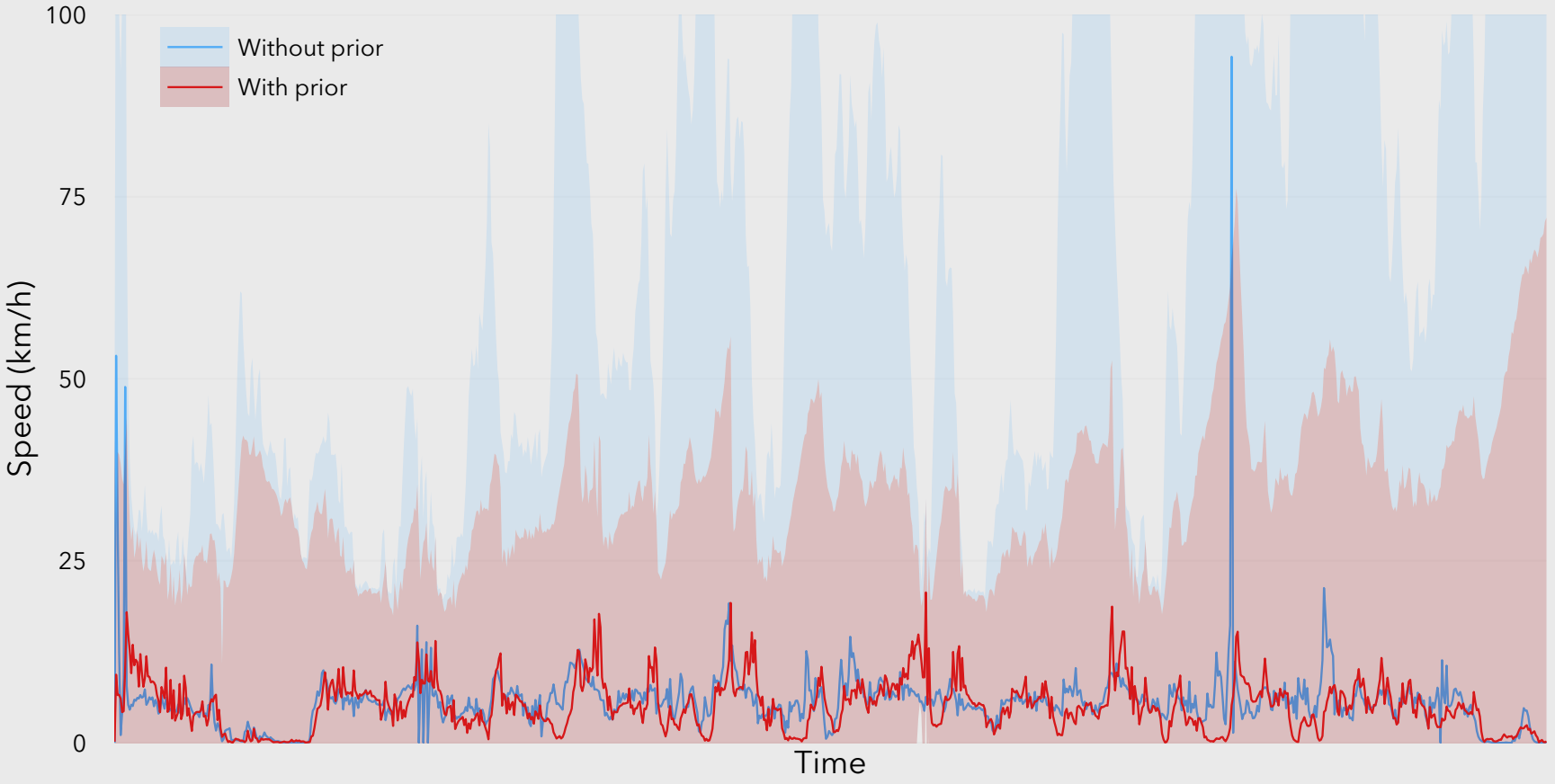
    Display(Speed.E().Project());
  ★if ((Speed > 5).E() > 0.75)
        GoodJobMessage();

    LastLocation = Location;
}
```

walking speeds



improved walking speeds



approximate computing

Recent work uses neural networks to approximate functions, trade accuracy for performance

How to reason about the error this induces?

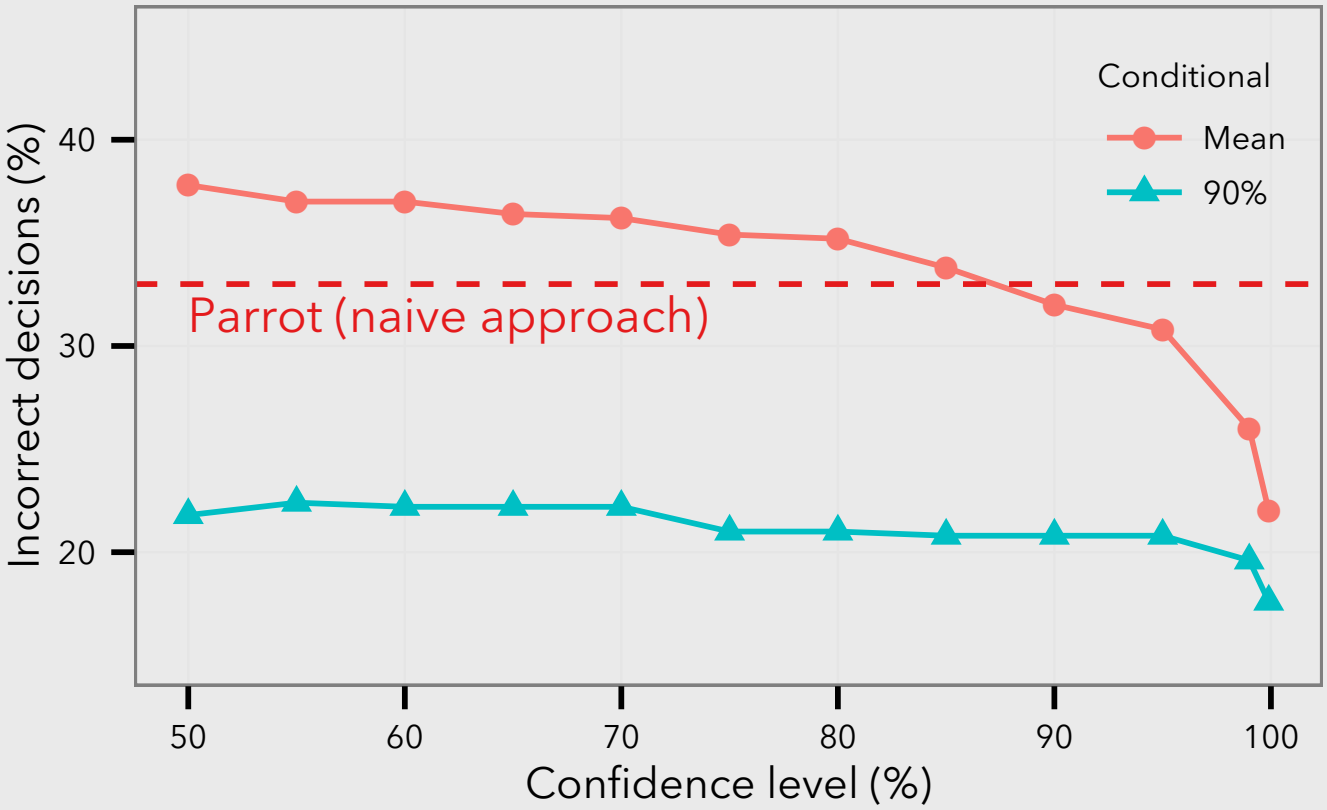
Neural networks: posterior predictive distribution

evaluation

Approximate the Sobel operator $s(p)$, calculating gradient of image intensity at a pixel

Evaluate the conditional $s(p) > 0.1$, with and without Uncertain<T>

evaluation


Conditional
Mean
90%
Parrot (naive approach)
Incorrect decisions (%)
40
30
20
50
60
70
80
90
100
Confidence level (%)

future work

Sensor applications

- Less accurate sensors to save power

A programming model for uncertainty

- Machine learning for non-experts

Optimisation

- Lazy evaluation a promising target

Uncertainty is a growing problem for non-expert programmers. Existing abstractions are inadequate.

Other solutions are either inefficient or inaccessible.

Uncertain<T> focuses on accessibility to non-experts, while still being expressive and efficient.

Programmers can make principled decisions under uncertainty.

With Uncertain<T>, non-expert programmers can build programs that are more concise, expressive, and correct.