Reducing Sampling Error in Batch Temporal Difference Learning

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Reinforcement Learning Successes

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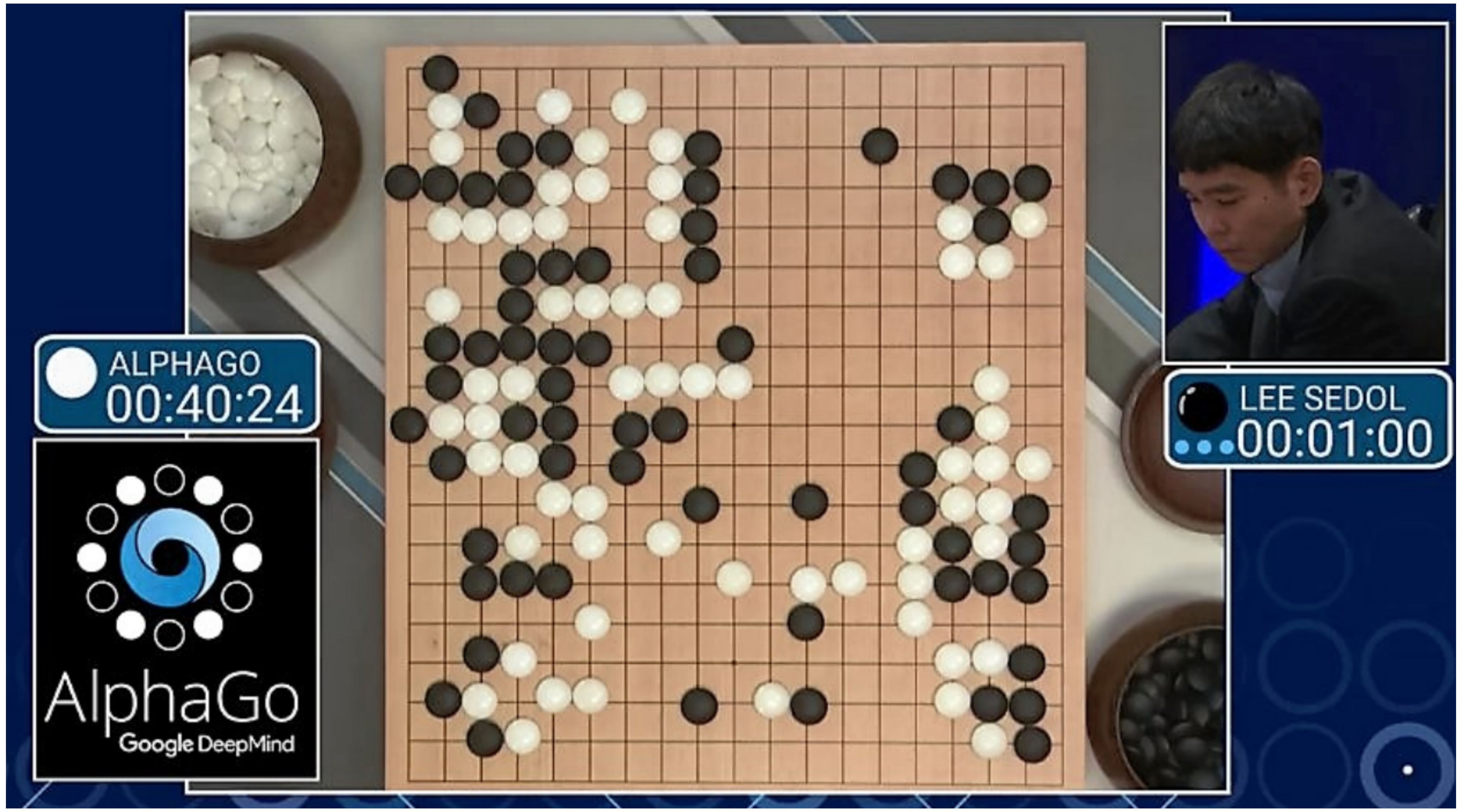

Reinforcement Learning Successes

How can RL agents make the most from a finite amount of experience?

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Learning an accurate estimation of the **value function** with **finite amount data**.

Spotlight Overview

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- With **finite** batch of data, **on-policy** single-step temporal difference learning converges to the value function for the **wrong** policy.

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- With **finite** batch of data, **on-policy** single-step temporal difference learning converges to the value function for the **wrong** policy.
- Propose and prove that a **more efficient** estimator converges to the value function for the **true** policy.

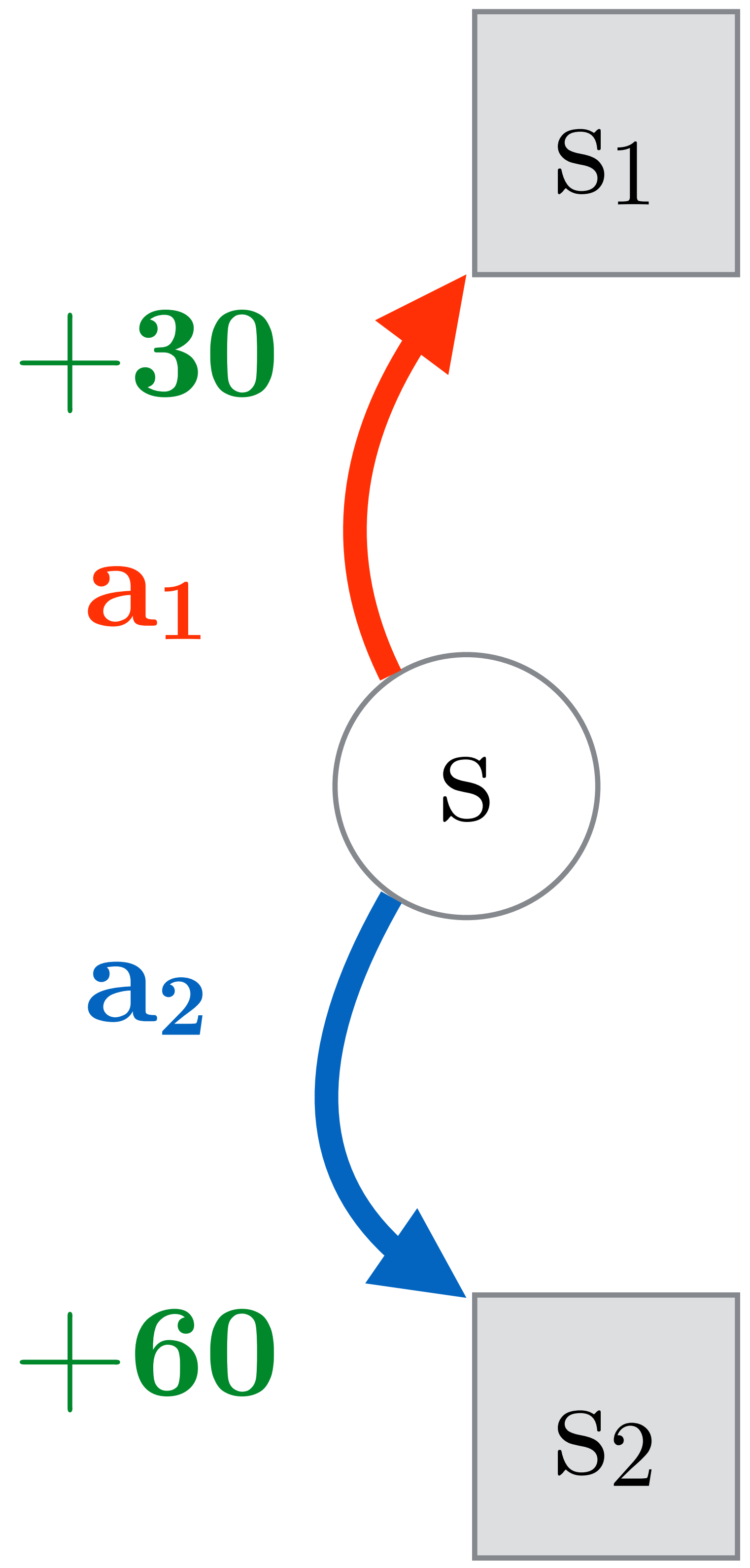
Spotlight Overview: Flaw in Batch TD(0)

True policy

$$\pi(.|s) = \frac{1}{2}$$

True value function

$$v^{\pi}(s) = 45$$



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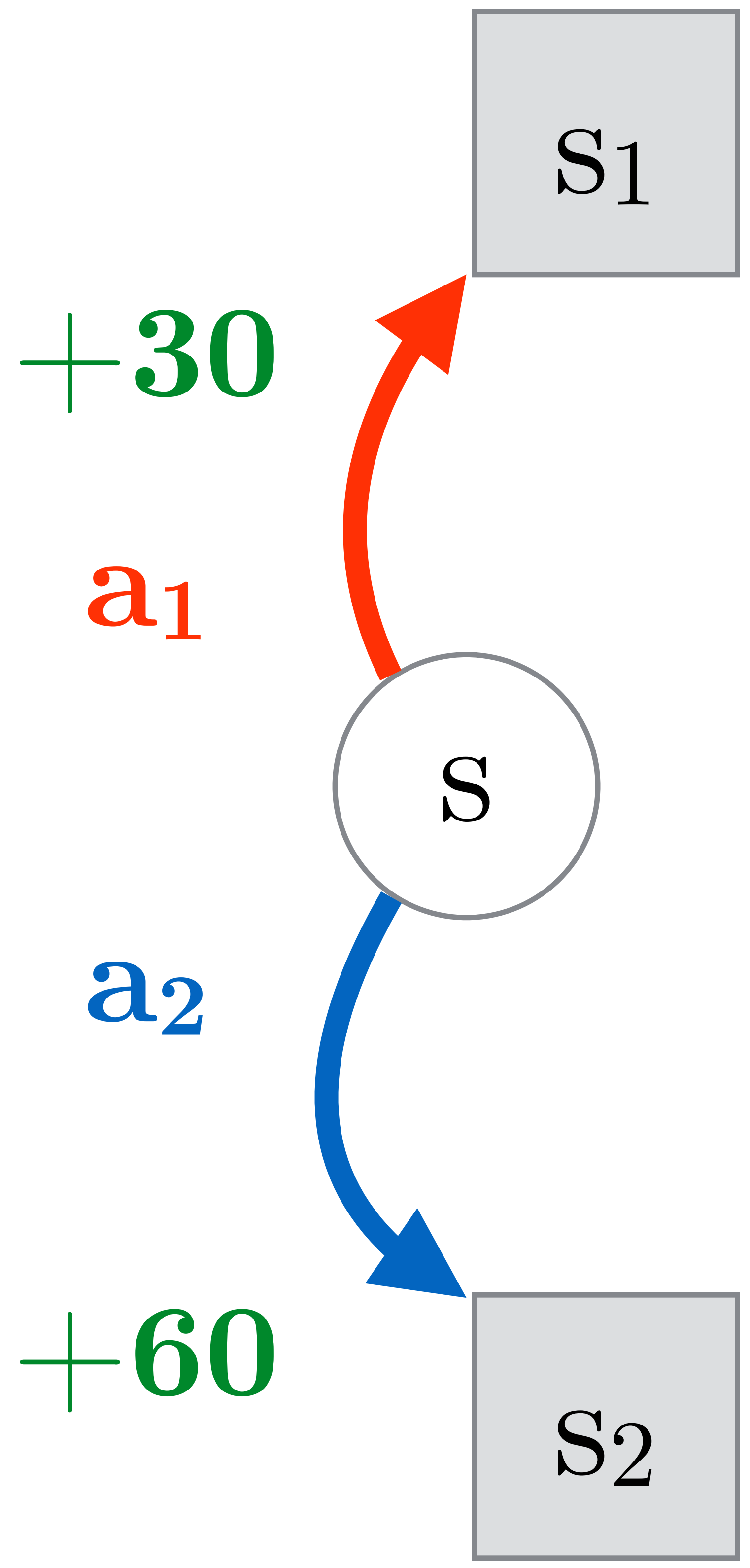

finite-sized batch

$$(s, a_1, s_1)$$
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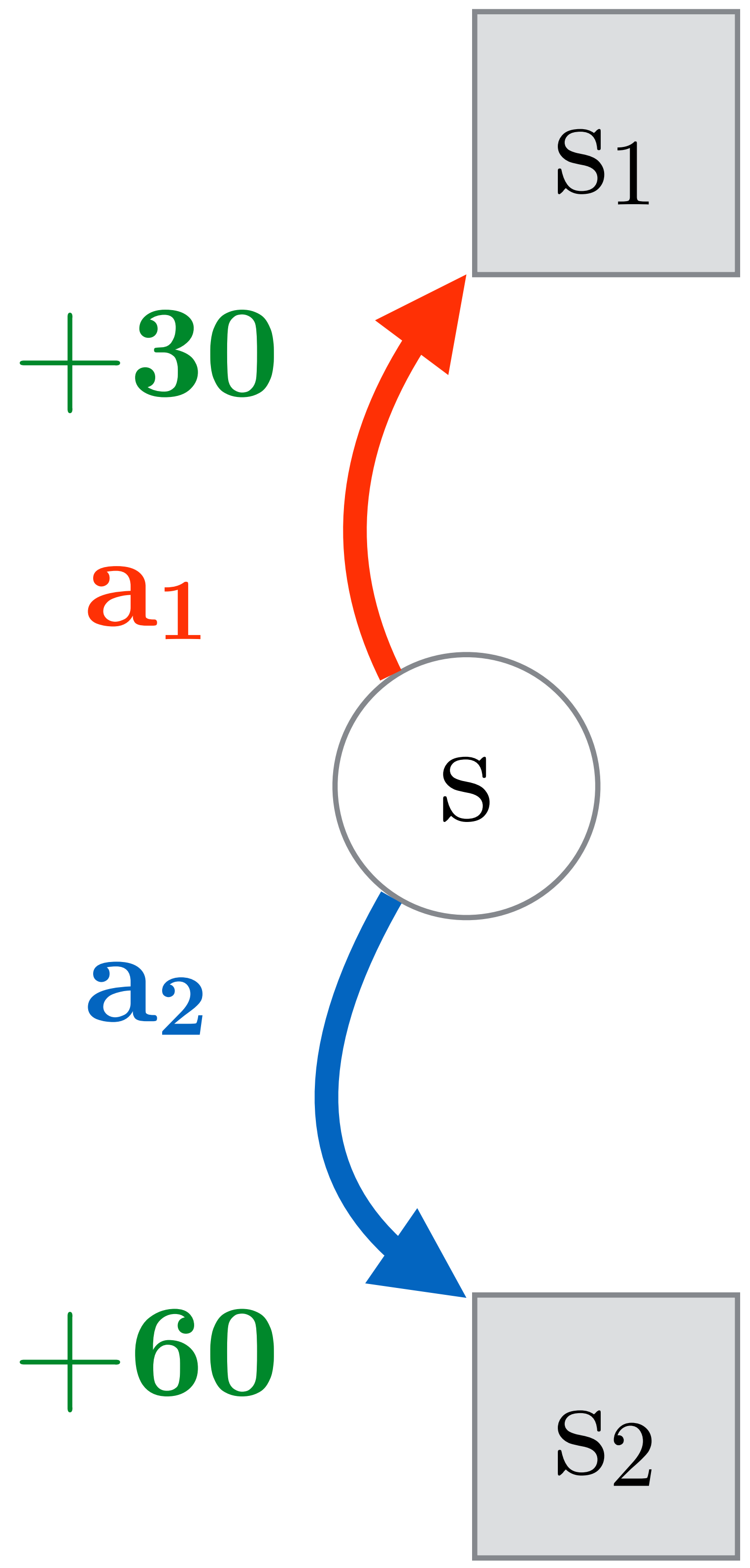

Batch TD(0) computes

$$\hat{v}(s) = 40$$

$$v^\pi \neq \hat{v}$$

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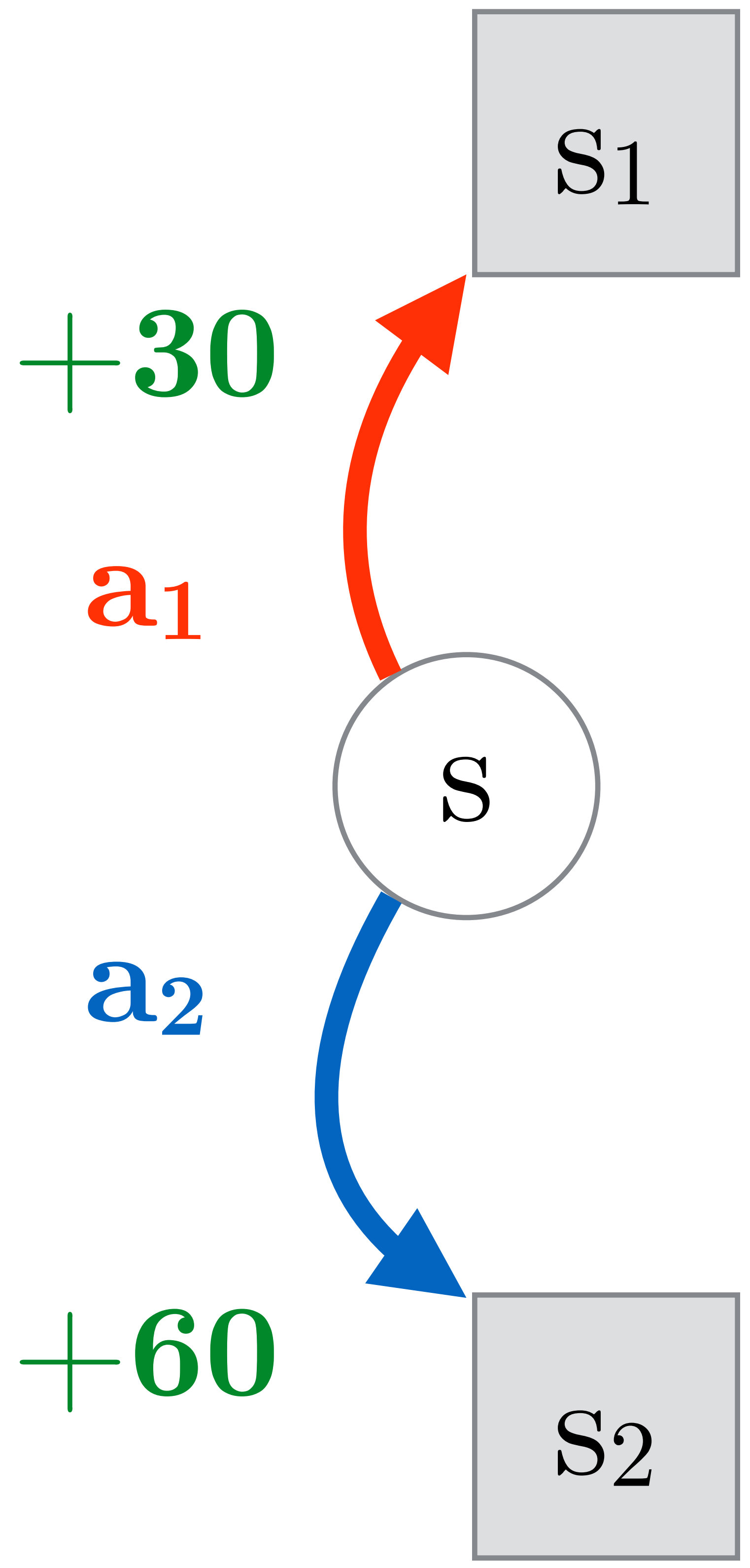

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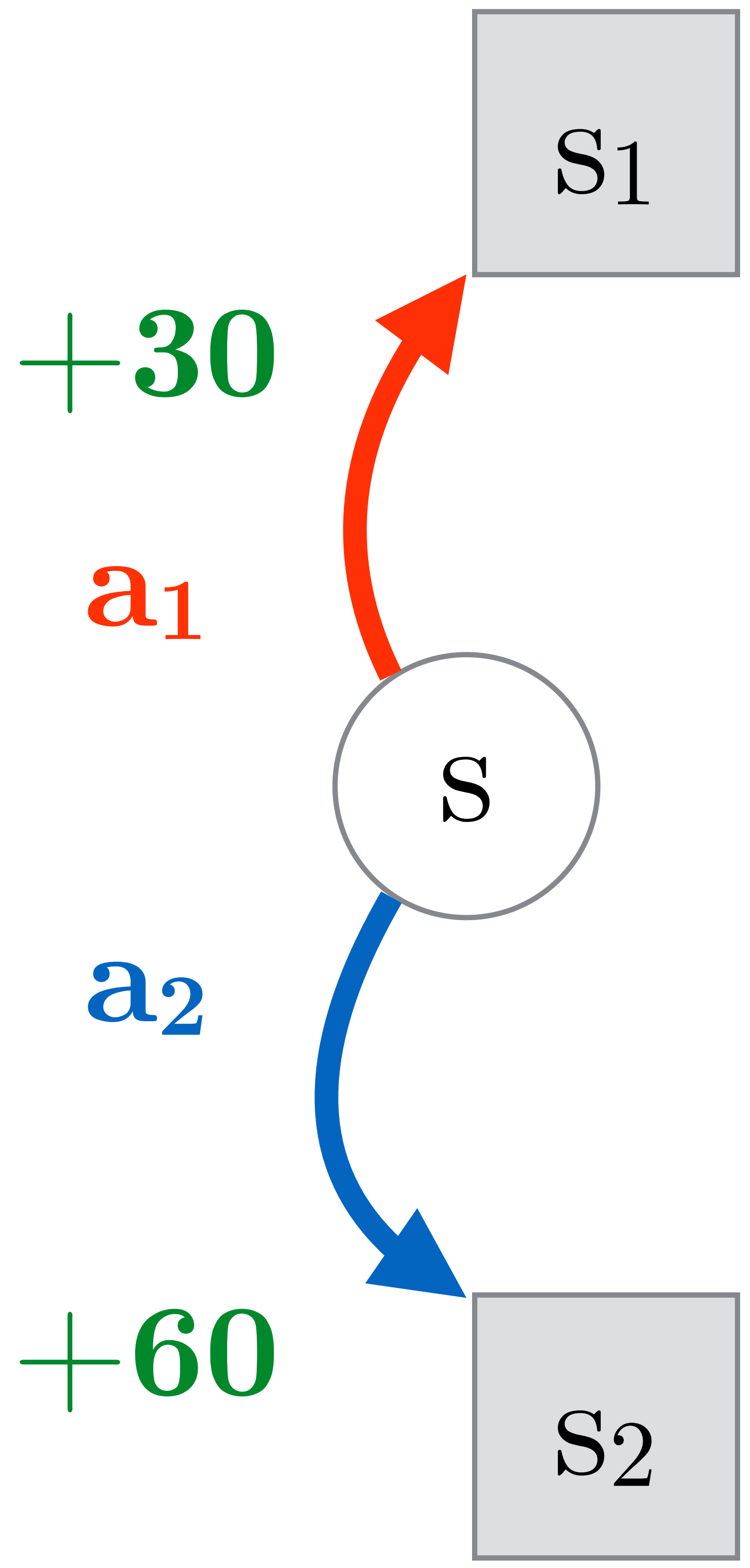

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Batch TD(0) estimates value function for the **wrong** policy!

Our estimator will estimate value function for the **true** policy

Batch Linear* Value Function Learning

*Empirical analysis also considers non-linear TD(0)

Batch Linear* Value Function Learning

Policy and environment transition dynamics:

$$\pi : \mathcal{S} \times \mathcal{A} \to [0, 1] \qquad P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$$

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Generates batch of m episodes:

$$\mathcal{D} := \{\tau_i\}_{i=1}^{m} \quad \text{where} \quad \tau := (s_1, a_1, r_1, ..., s_{L_\tau}, a_{L_\tau}, r_{L_\tau})$$

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Estimate value function:

$$v^\pi(s) := \mathbf{E}_\pi\left[\sum_{k=0}^{L} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right], \forall s \in \mathcal{S}$$

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Assumptions:

1. π is known (policy we want to learn about).
2. P is unknown (model-free).
3. Reward function is unknown.
4. On-policy (focus of talk).

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Algorithm 1 Batch Linear TD(0) to estimate v^π

Input: batch $\mathcal{D}$, step-size $\alpha > 0$, convergence threshold $\epsilon > 0$
Initialize: weight vector $\boldsymbol{w}_0$ arbitrarily (e.g.: $\boldsymbol{w}_0 := \boldsymbol{0}$), update aggregation vector $\boldsymbol{u} := \boldsymbol{0}$, batch process counter, $i = 0$
while $|\boldsymbol{w}_{i+1} - \boldsymbol{w}_i| \geq \epsilon\boldsymbol{1}$ **do**
  for each episode, $\tau \in \mathcal{D}$ **do**
    for each transition, $(s, a, r, s') \in \tau$ **do**
      $\boldsymbol{u} \leftarrow \boldsymbol{u} + \left[r + \gamma \boldsymbol{w}_i^T \boldsymbol{x}(s') - \boldsymbol{w}_i^T \boldsymbol{x}(s)\right] \boldsymbol{x}(s)$
    end for
  end for
  $\boldsymbol{w}_{i+1} \leftarrow \boldsymbol{w}_i + \alpha \boldsymbol{u}$ {batch update}
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fixed finite batch as input

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while $|\boldsymbol{w}_{i+1} - \boldsymbol{w}_i| > \epsilon\boldsymbol{1}$ **do**
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for each transition

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end while

accumulate computed TD error

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```
Input: batch 𝒟, step-size α > 0, convergence threshold ε > 0
Initialize: weight vector w_0 arbitrarily (e.g.: w_0 := 0), update aggregation vector u := 0, batch process counter, i = 0
while |w_{i+1} − w_i| ≥ ε1 do
    for each episode, τ ∈ 𝒟 do
        for each transition, (s, a, r, s′) ∈ τ do
            u ← u + [r + γ w_i^T x(s′) − w_i^T x(s)] x(s)
        end for
    end for
    w_{i+1} ← w_i + αu {batch update}
    u ← 0 {clear aggregation}
    i ← i + 1 {update batch process counter}
end while
```

make aggregated update to weights

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clear aggregation

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end while

until convergence

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Batch TD(0) Value Function

finite-sized $\mathcal{D}$ $\longrightarrow$ batch TD(0) $\longrightarrow$ $\hat{v}(s)$

Batch TD(0) Value Function

finite-sized $\mathcal{D}$ ⟶ batch TD(0) ⟶ $\hat{v}(s)$

certainty-equivalence estimate for MDP*

$$\hat{v}(s_j) = \sum_{a \in \widehat{\mathcal{A}}} \hat{\pi}(a|s_j) \left(\bar{R}(s_j, a) + \gamma \sum_{k \in \widehat{\mathcal{S}}} \widehat{P}(s_k|s_j, a)\hat{v}(s_k) \right), \forall s_j, s_k \in \widehat{\mathcal{S}}$$

*Sutton (1988) proved a similar result for a Markov reward process

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maximum-likelihood estimates (MLE) computed from $\mathcal{D}$

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Problem! $\hat{\pi} \neq \pi$ $\hat{P} \neq P$ policy and transition dynamics *sampling error*

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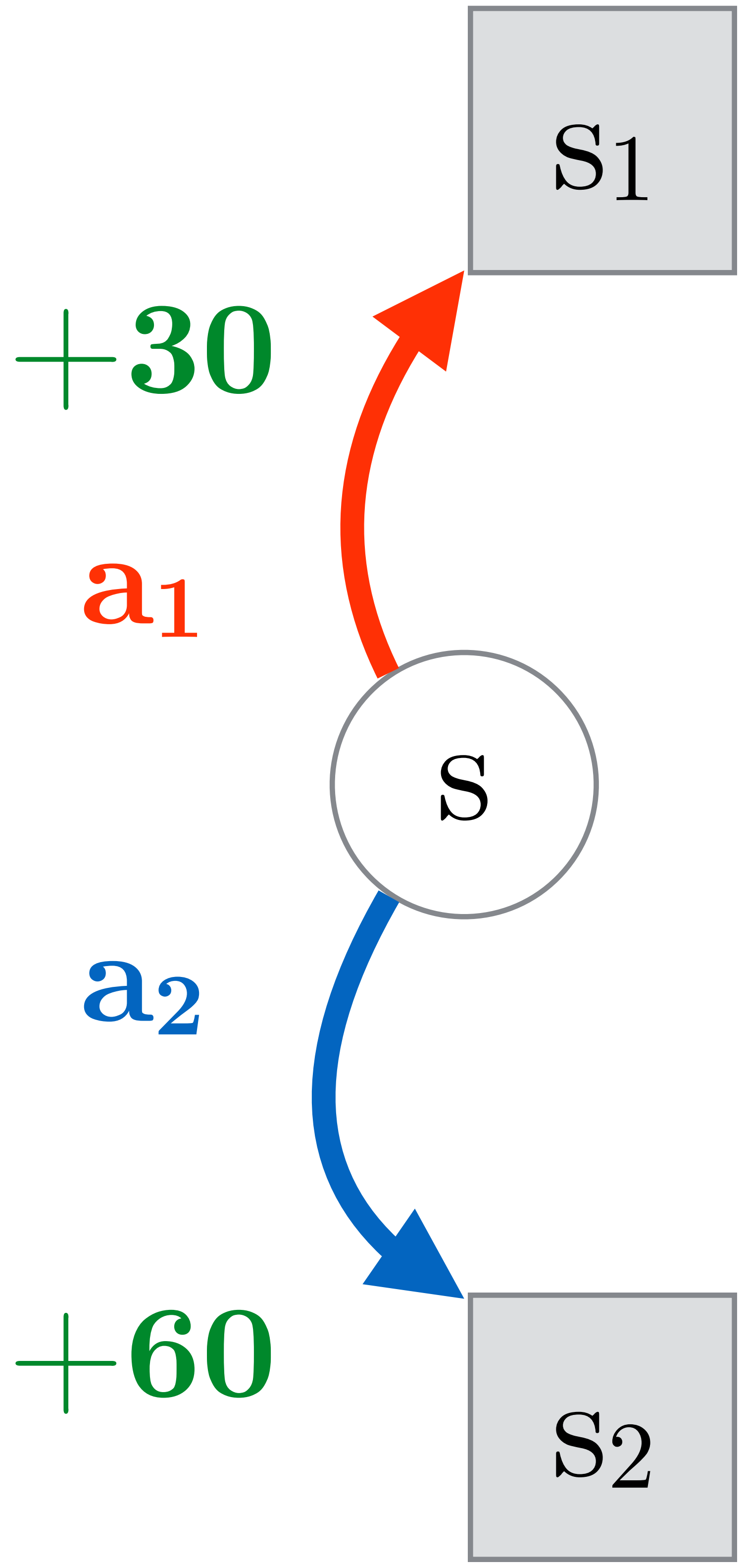

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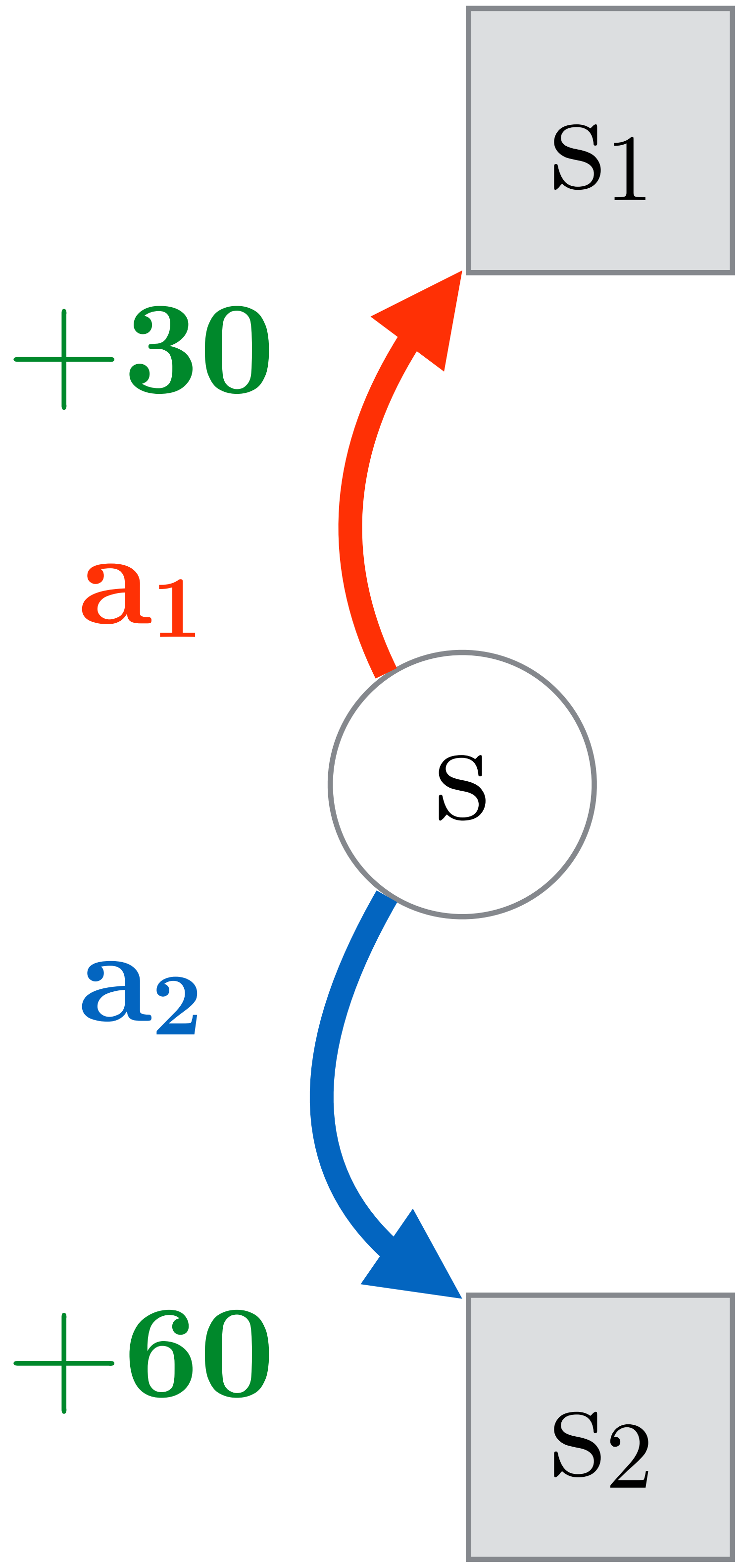

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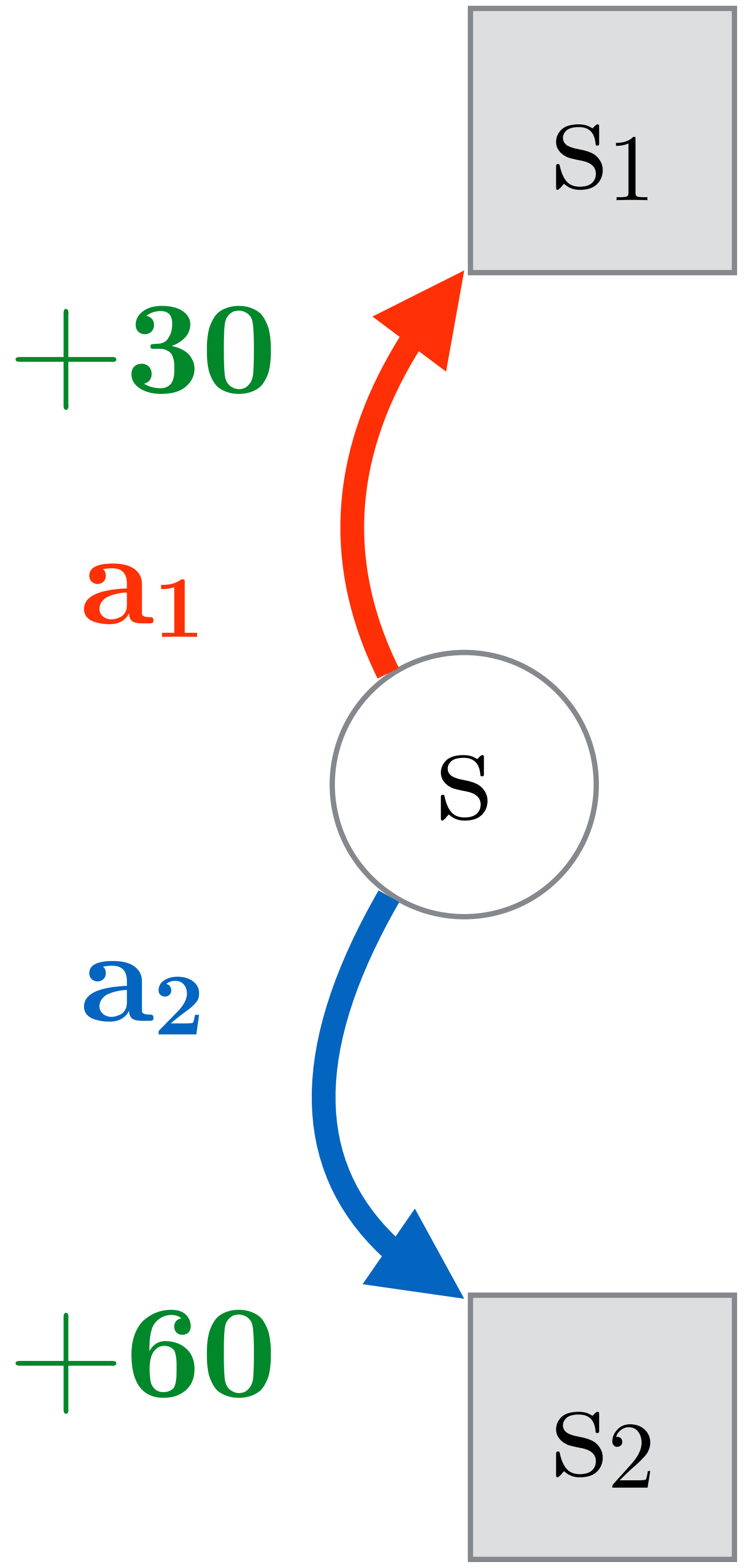

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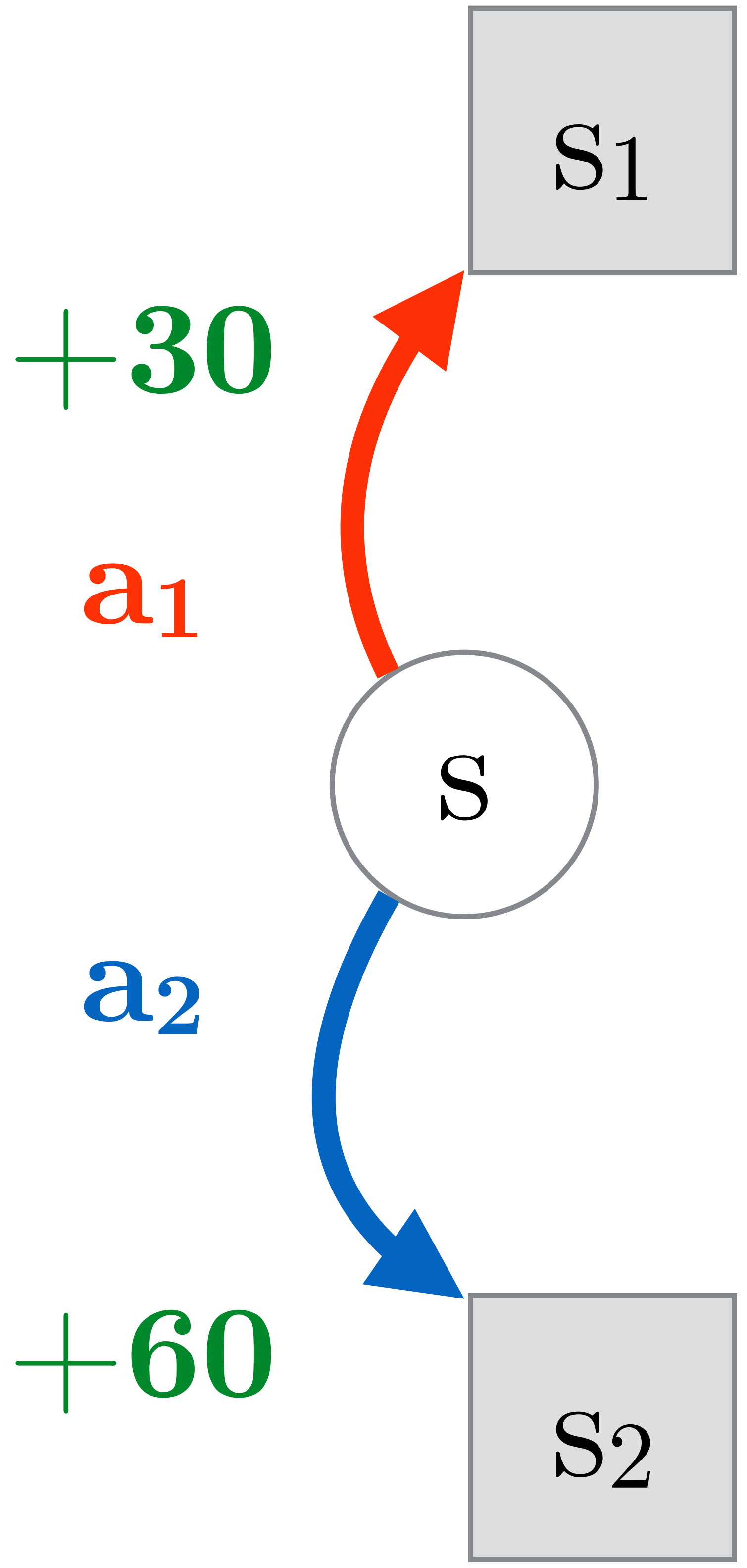

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$$\pi \neq \hat{\pi}$$

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Policy Sampling Error Corrected-TD(0)

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True policy distribution is assumed to be known.

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Correct learning from the **MLE** policy distribution to the **true** policy distribution.

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An **off-policy-styled** correction for an **on-policy** algorithm.

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PSEC ratio (importance sampling [Precup et al., 2000, Ghiassian et al., 2018]):

$$\boldsymbol{u} \leftarrow \boldsymbol{u} + \left[r + \gamma \boldsymbol{w}_i^T x(s') - \boldsymbol{w}_i^T \boldsymbol{x}(s)\right] \boldsymbol{x}(s)$$

On-policy TD(0) Update

$$\boldsymbol{u} \leftarrow \boldsymbol{u} + \left[\frac{\pi(a|s)}{\hat{\pi}(a|s)}(r + \gamma \boldsymbol{w}_i^T \boldsymbol{x}(s')) - \boldsymbol{w}_i^T \boldsymbol{x}(s)\right] \boldsymbol{x}(s)$$

On-policy PSEC-TD(0) Update

Batch PSEC-TD(0)

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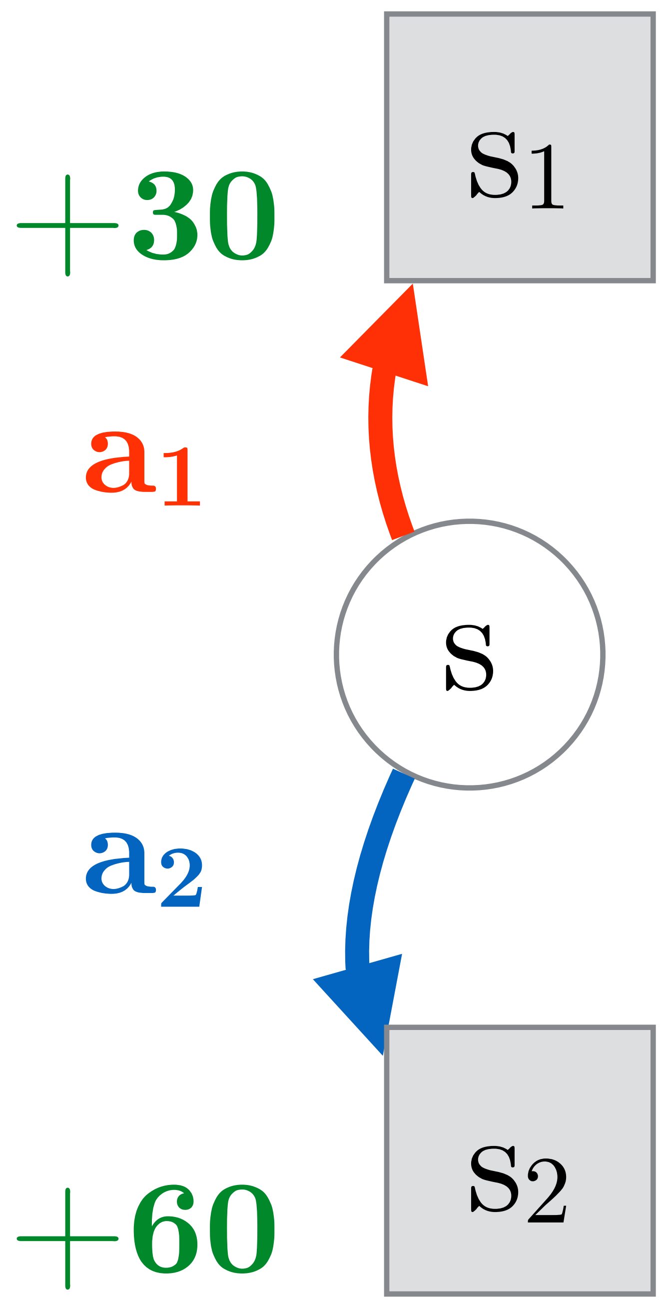

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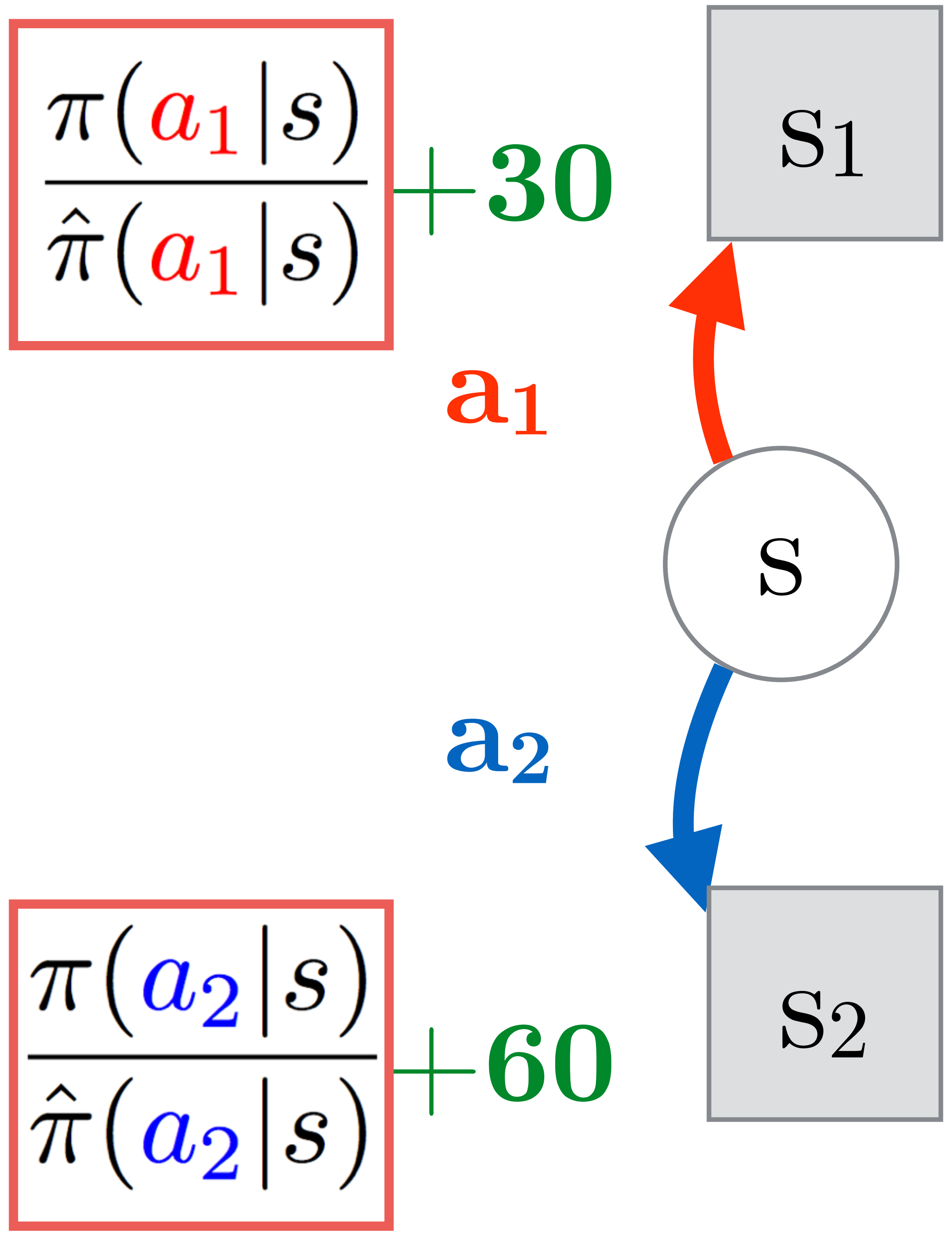

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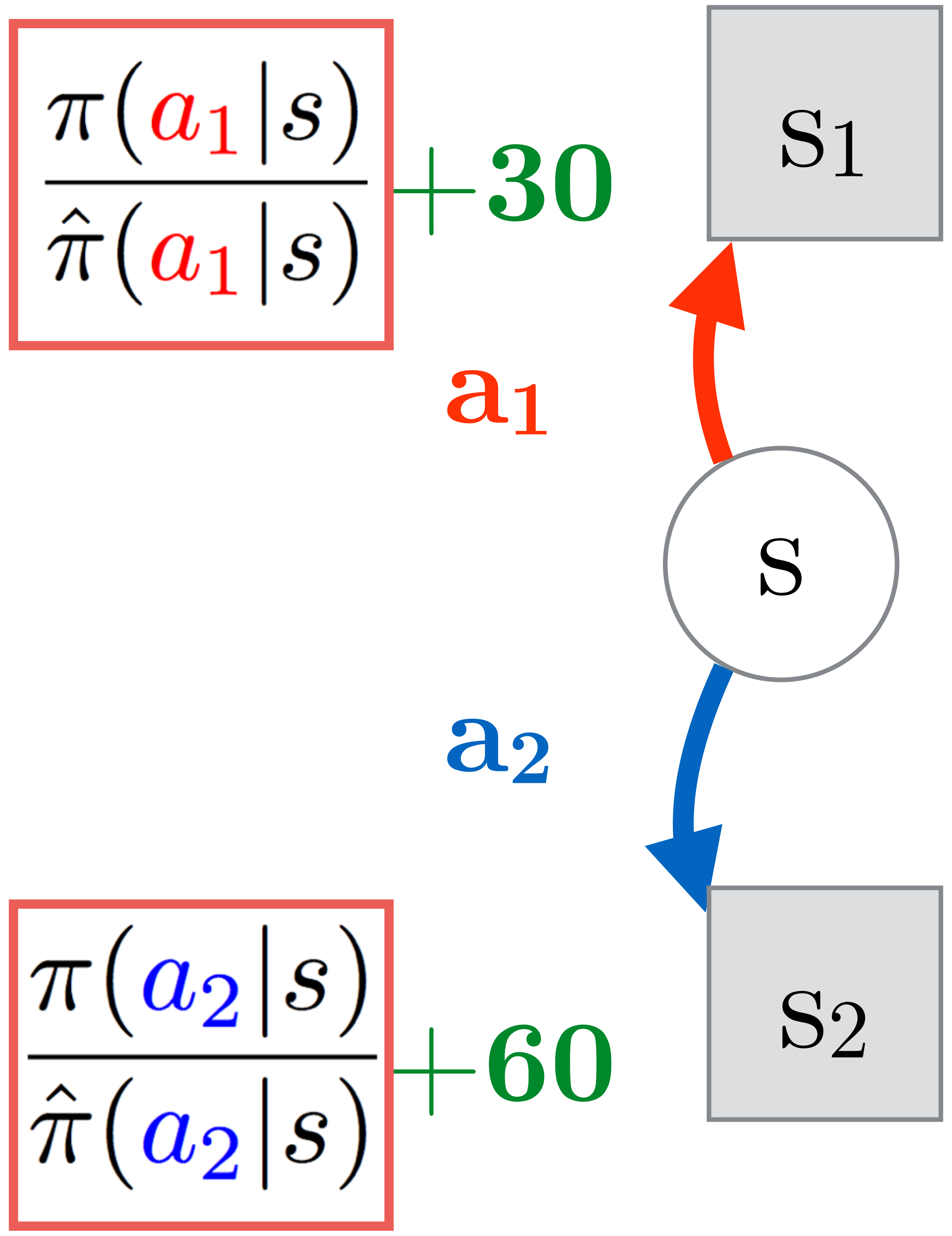
True value function

$$v^{\pi}(s) = 45$$



Batch TD(0) computes

$$\hat{v}(s) = 40$$

MLE policy

$$\hat{\pi}(a_1|s) = \frac{2}{3}$$

$$\hat{\pi}(a_2|s) = \frac{1}{3}$$

finite-sized batch

(s, a_1, s_1)

(s, a_1, s_1)

(s, a_2, s_2)

Batch PSEC-TD(0) computes

$$\hat{v}(s) = 45$$

PSEC-TD(0) Update

$$\boldsymbol{u} \leftarrow \boldsymbol{u} + \left[\frac{\pi(a|s)}{\hat{\pi}(a|s)}(r + \gamma \boldsymbol{w}_i^T \boldsymbol{x}(s')) - \boldsymbol{w}_i^T \boldsymbol{x}(s)\right] \boldsymbol{x}(s)$$

Batch PSEC-TD(0) Value Function

Theorem 3 (Batch Linear PSEC-TD(0) Convergence). *For any batch whose observation vectors $\{\boldsymbol{x}(s) | s \in \widehat{\mathcal{S}}\}$ are linearly independent, there exists an $\epsilon > 0$ such that, for all positive $\alpha < \epsilon$ and for any initial weight vector, the predictions for linear PSEC-TD(0) converge under repeated presentations of the batch with weight updates after each complete presentation to the fixed-point (6).*

Definition 3. *PSEC Markov Decision Process Certainty Equivalence Estimate (PSEC-MDP-CEE) Value Function. The PSEC-MDP-CEE is the value function, $\hat{v}^\pi$, that, $\forall s_j, s_k \in \widehat{\mathcal{S}}$, satisfies:*

$$\hat{v}^\pi(s_j) = \sum_{a \in \widehat{\mathcal{A}}} \pi(a|s_j)[\bar{R}(s_j, a) + \gamma \sum_{k \in \widehat{\mathcal{S}}} \widehat{P}(s_k|s_j, a)\hat{v}^\pi(s_k))] \tag{6}$$

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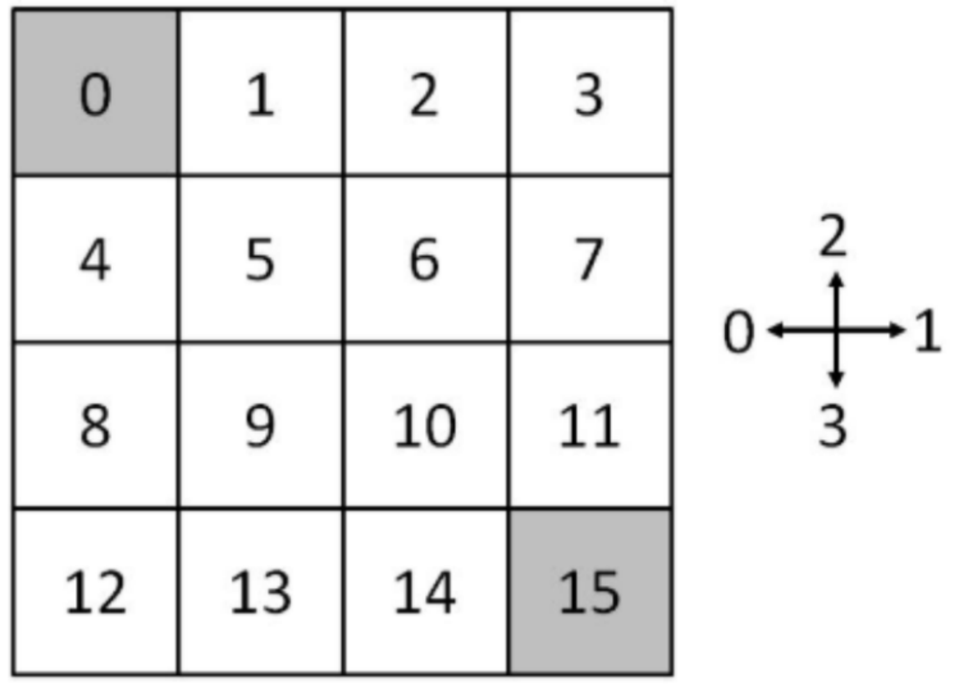
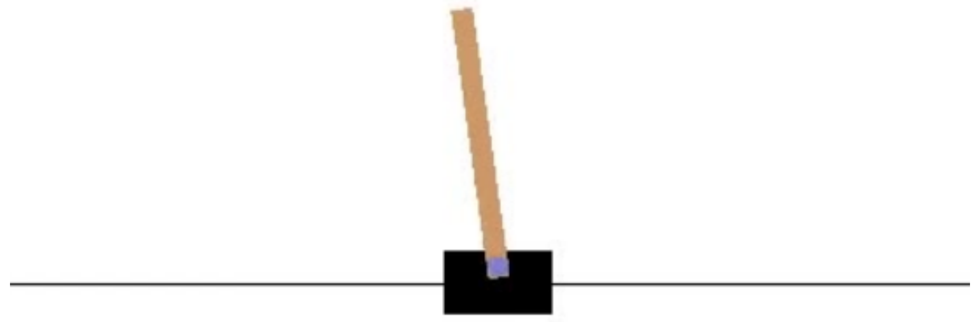
Experimental Results



Gridworld [link]
Discrete States and Actions



CartPole [Brockman et al., 2016]
Continuous States and
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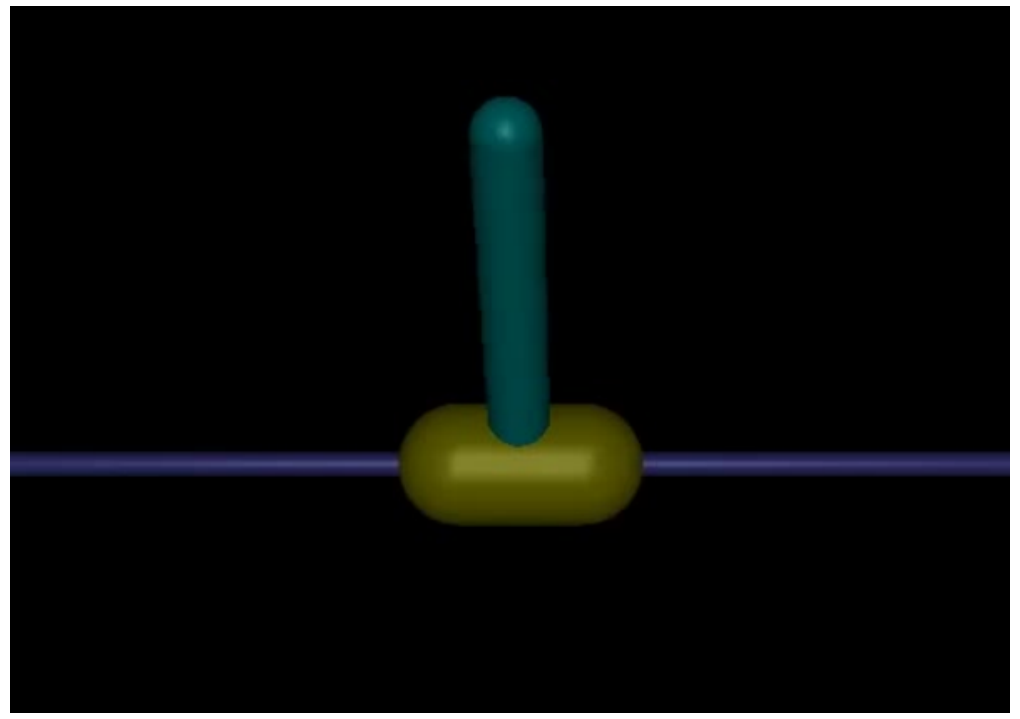

InvertedPendulum
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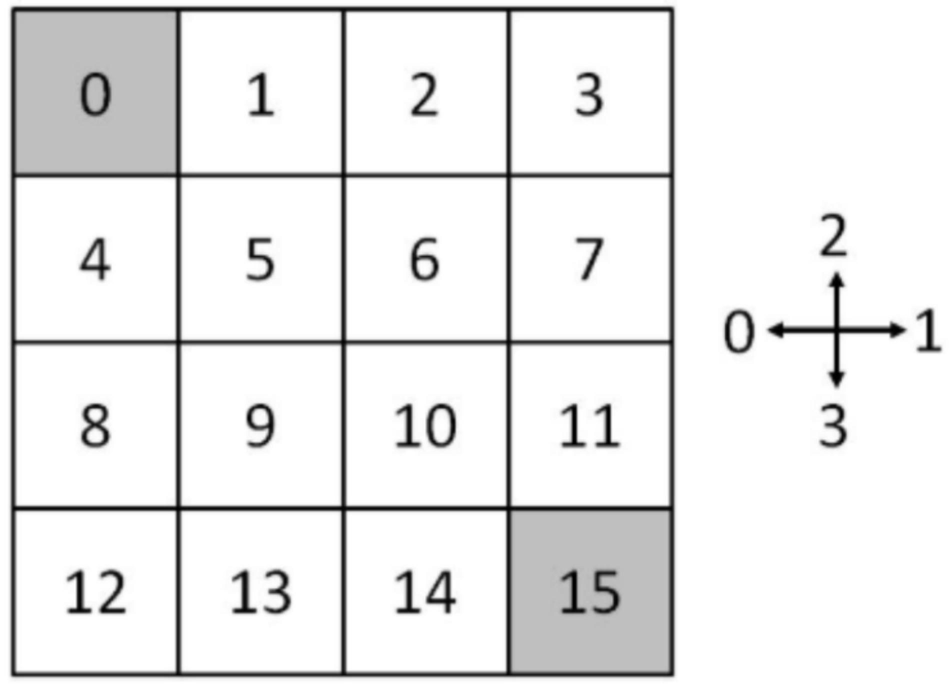

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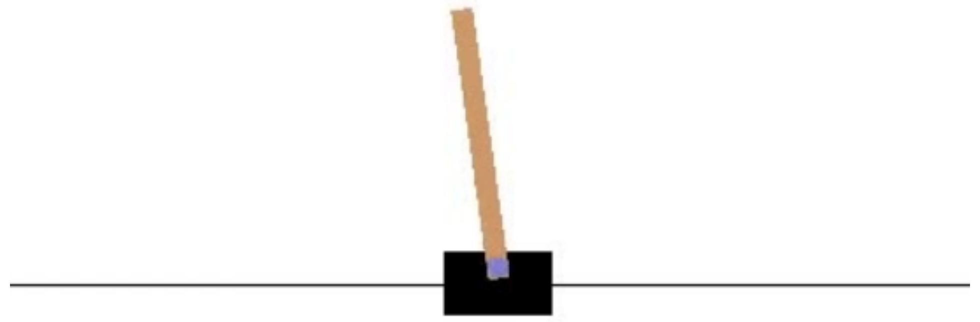

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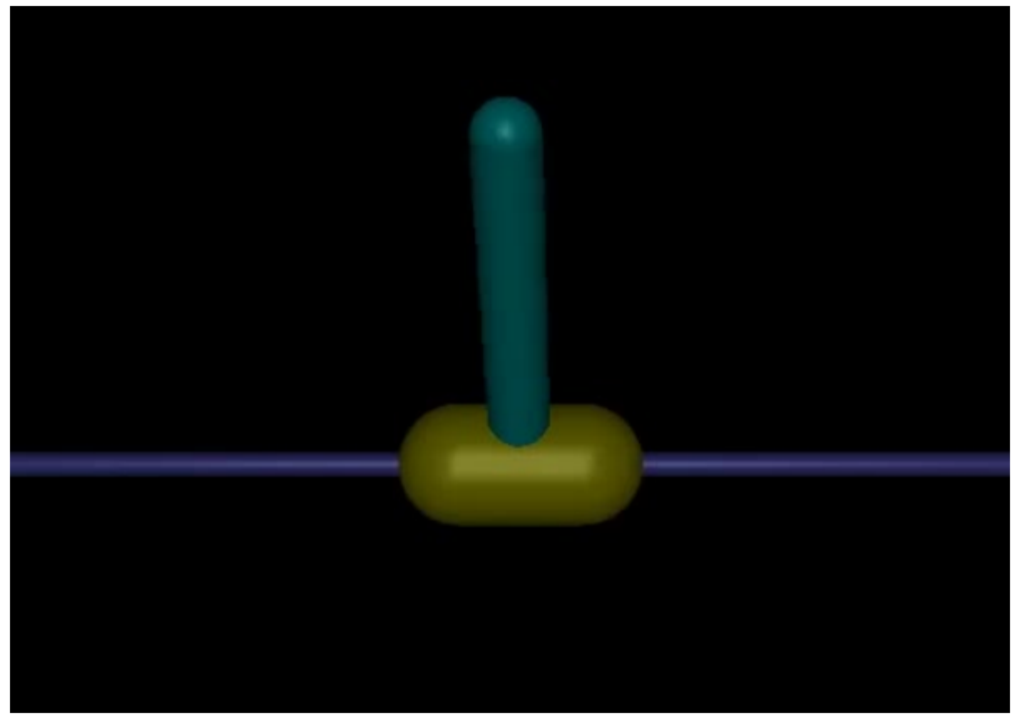

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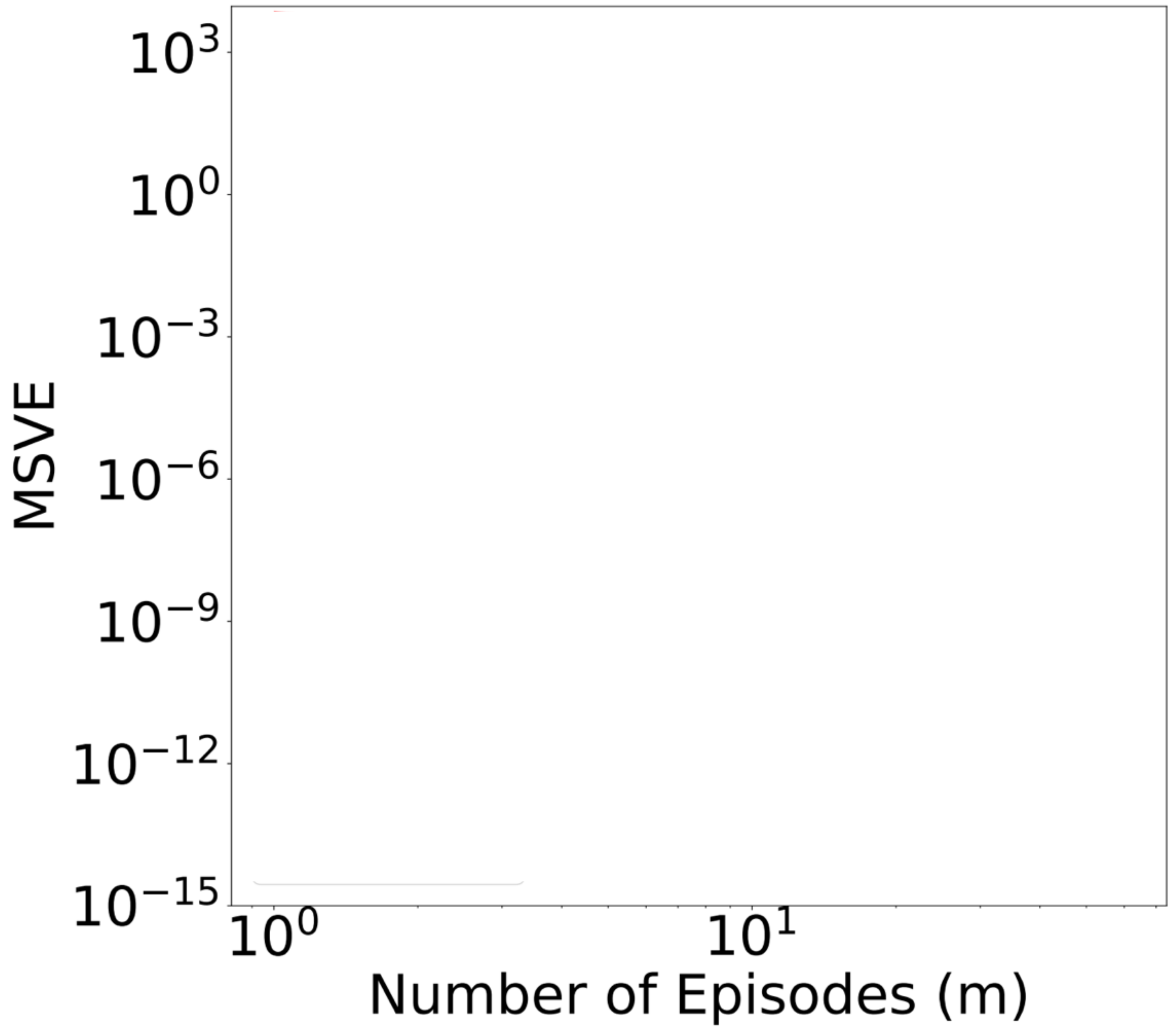
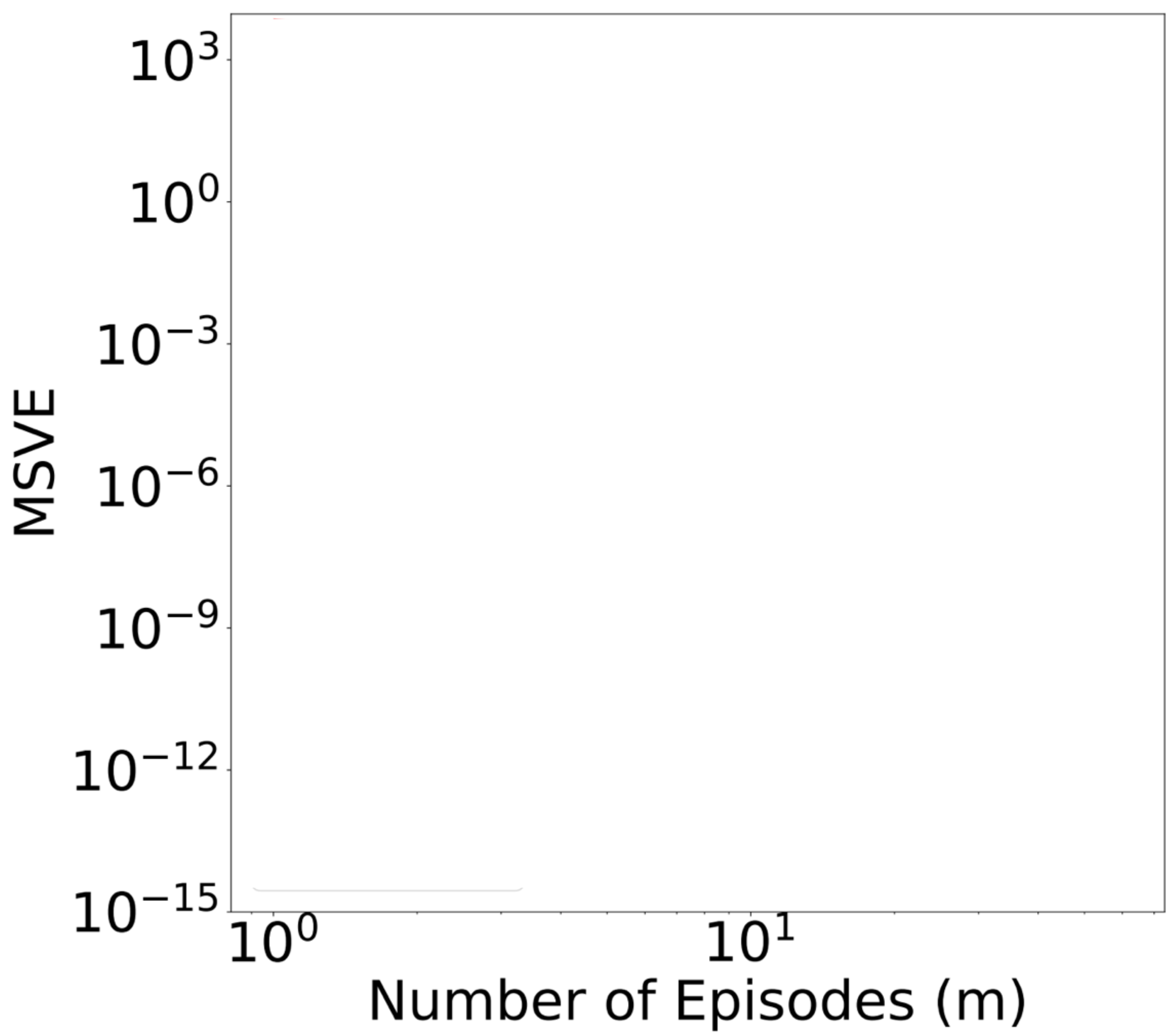
Evaluation Metric
(weighted error):

$$\text{MSVE}(\boldsymbol{w}) := \sum_{s \in \mathcal{S}} d_\pi(s) \Big(v^\pi(s) - \hat{v}(s) \Big)^2, \forall s \in \mathcal{S}$$


MSVE
10^3
10^0
10^−3
10^−6
10^−9
10^−12
10^−15
10^0
10^1
Number of Episodes (m)

Empirical Results: Deterministic Gridworld



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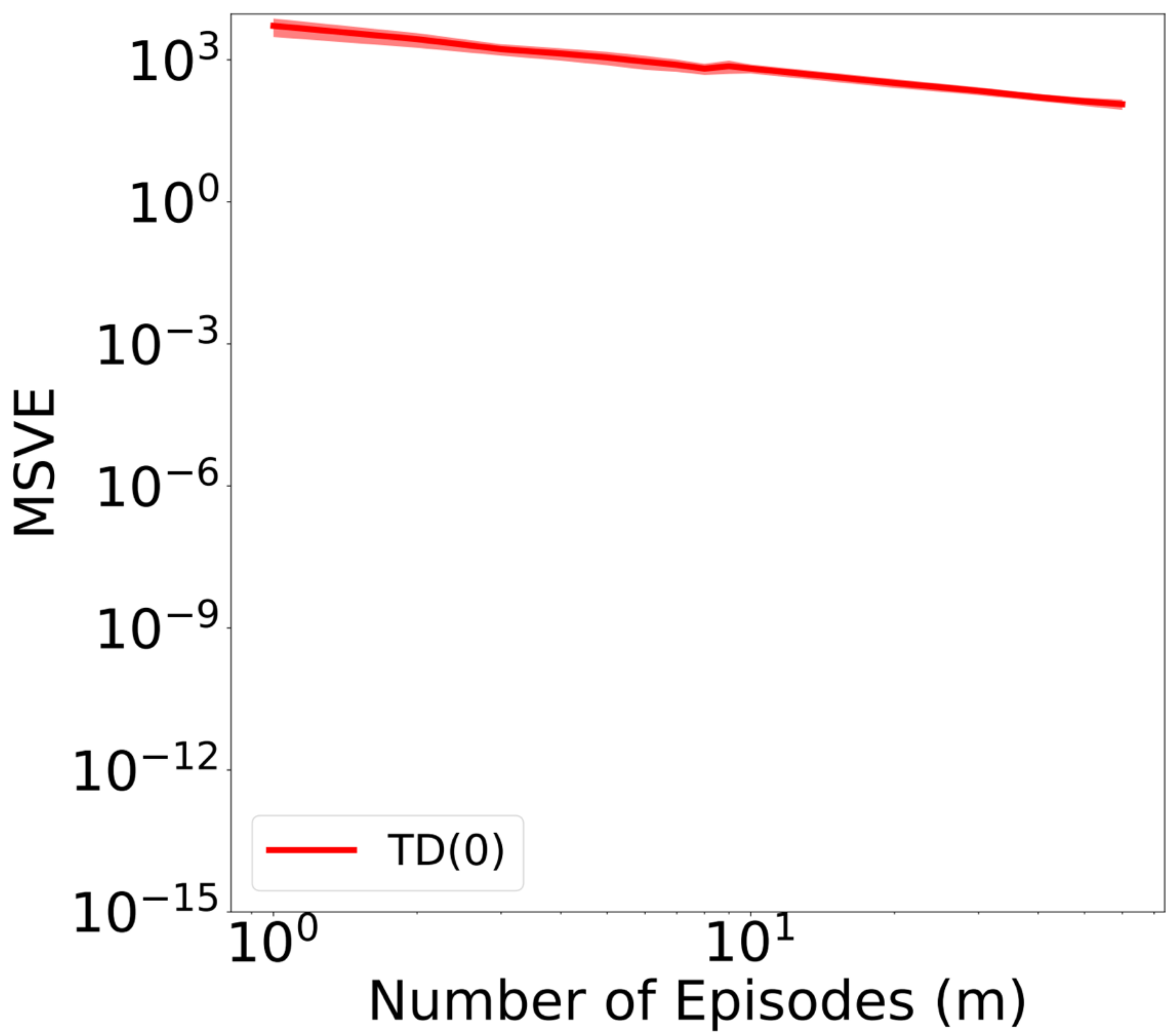

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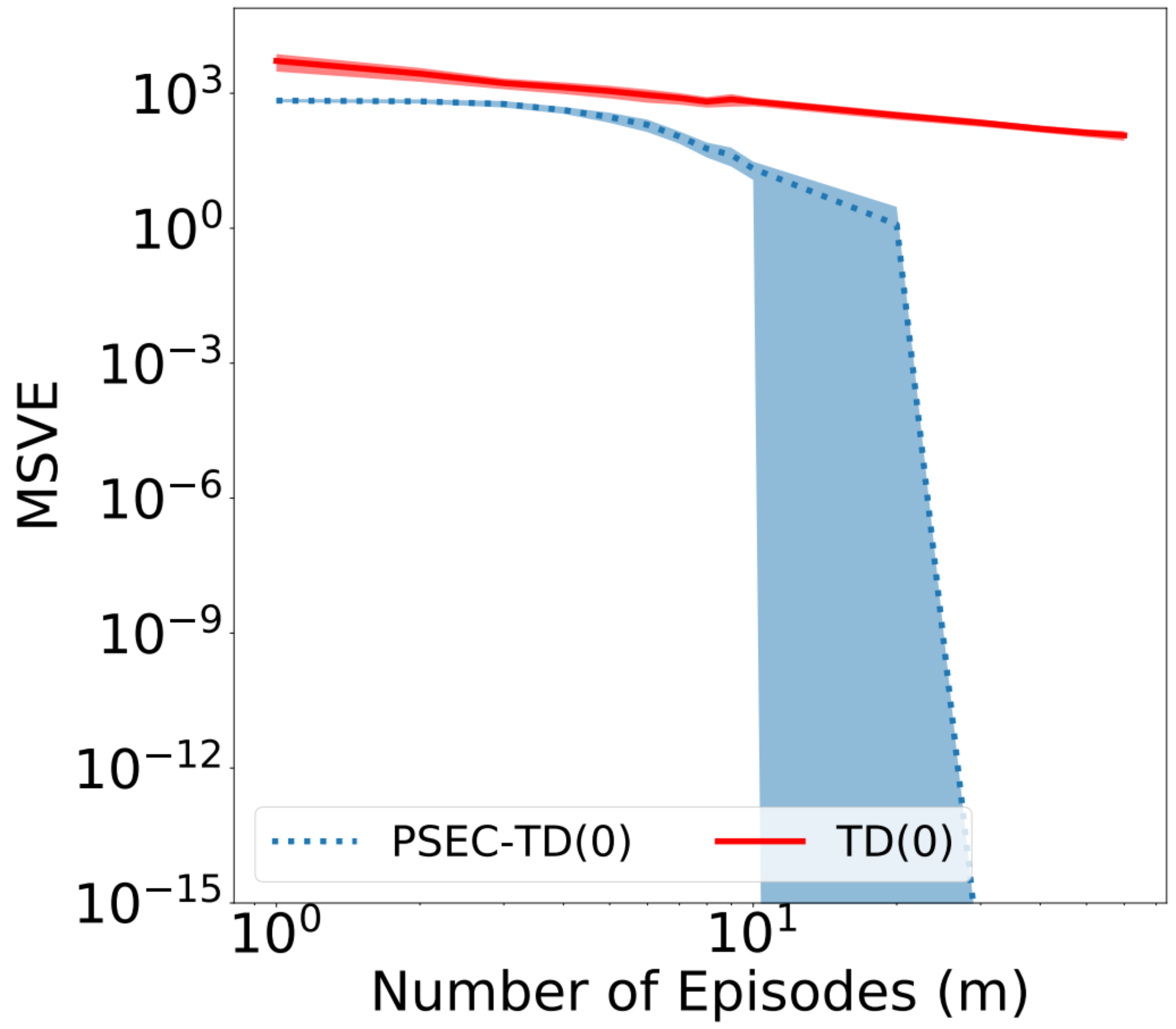

PSEC-TD(0) vs. TD(0)

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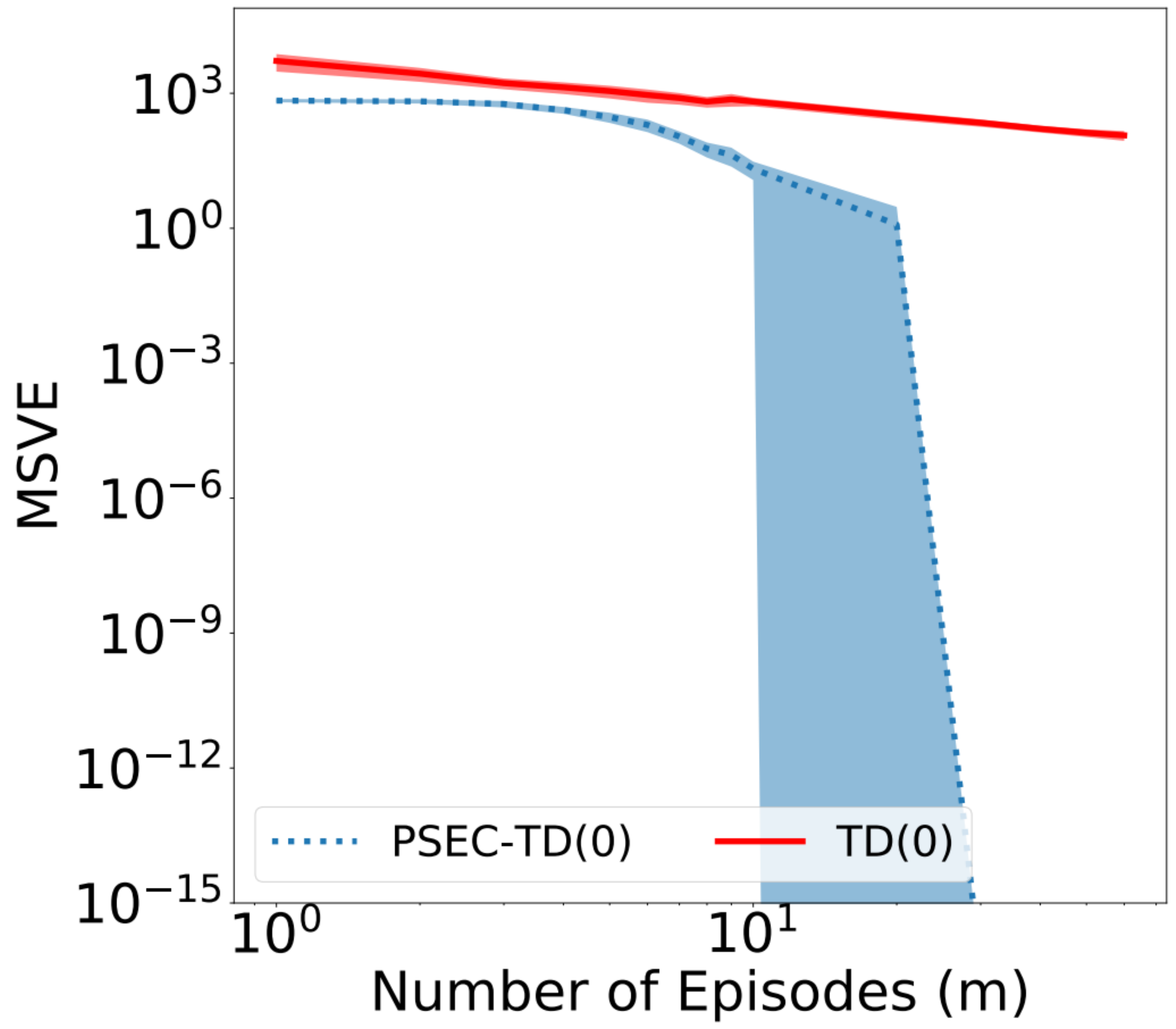

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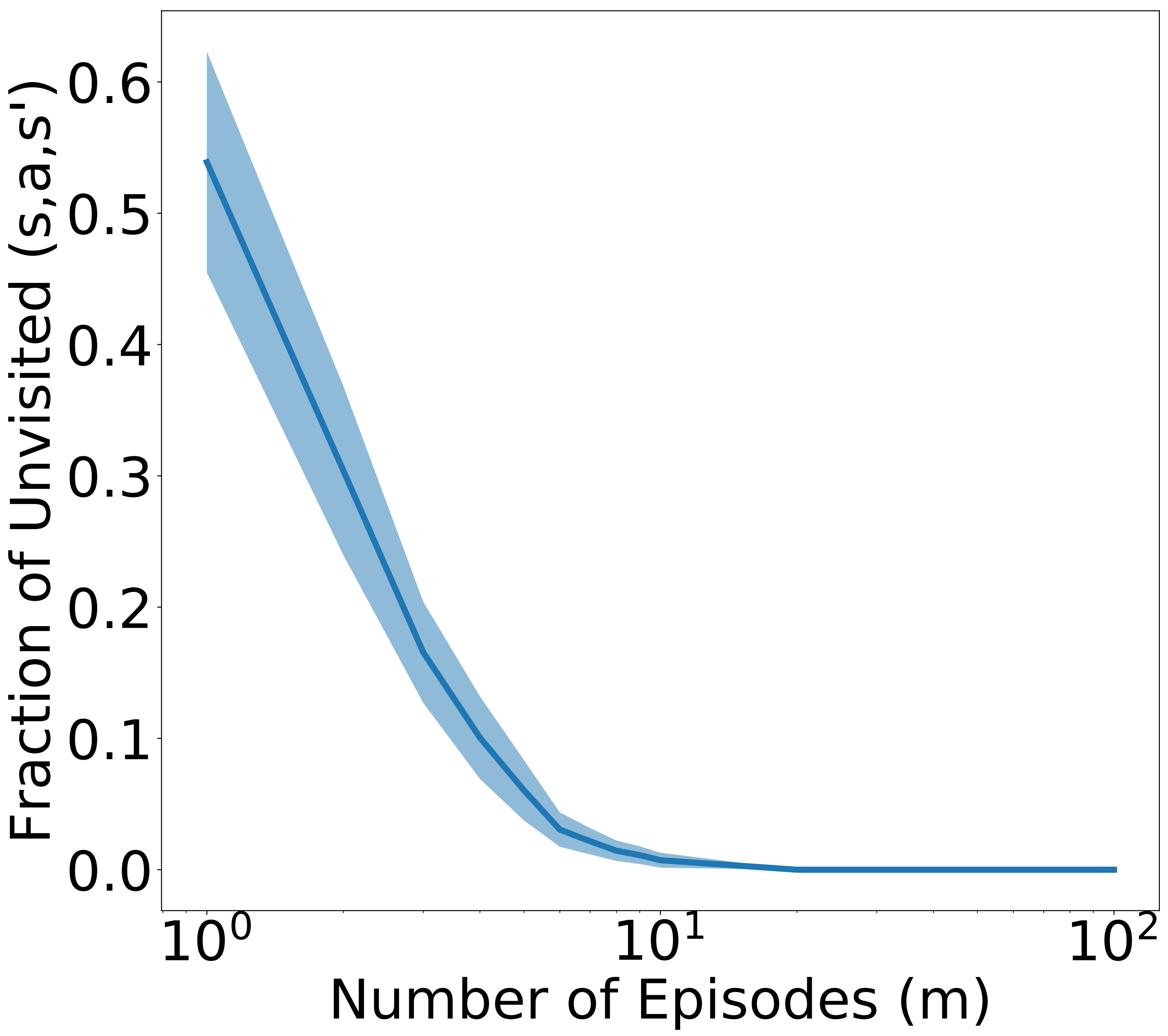

Unvisited (s,a,s') tuples

Empirical Results: Function Approximation

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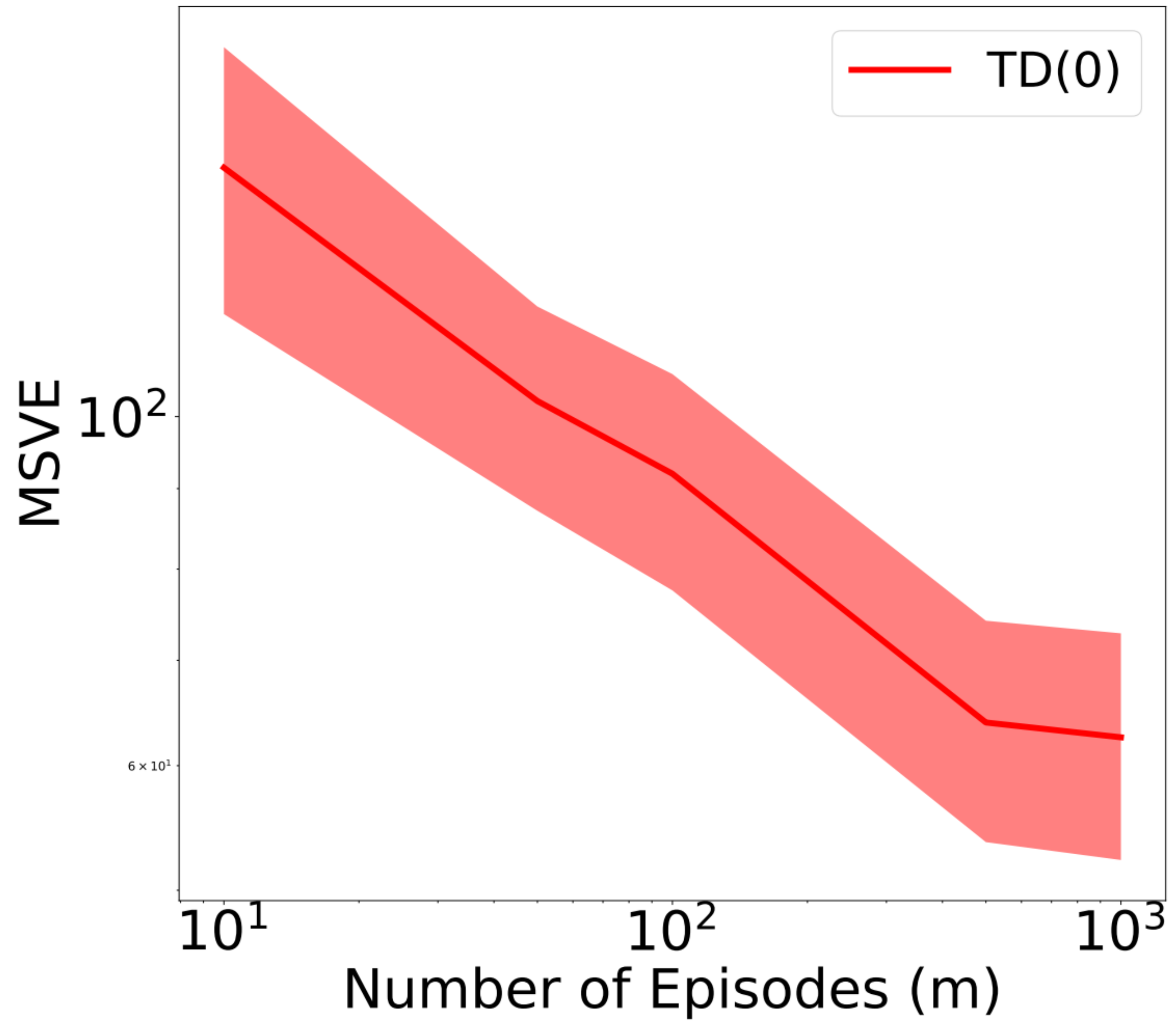

CartPole*

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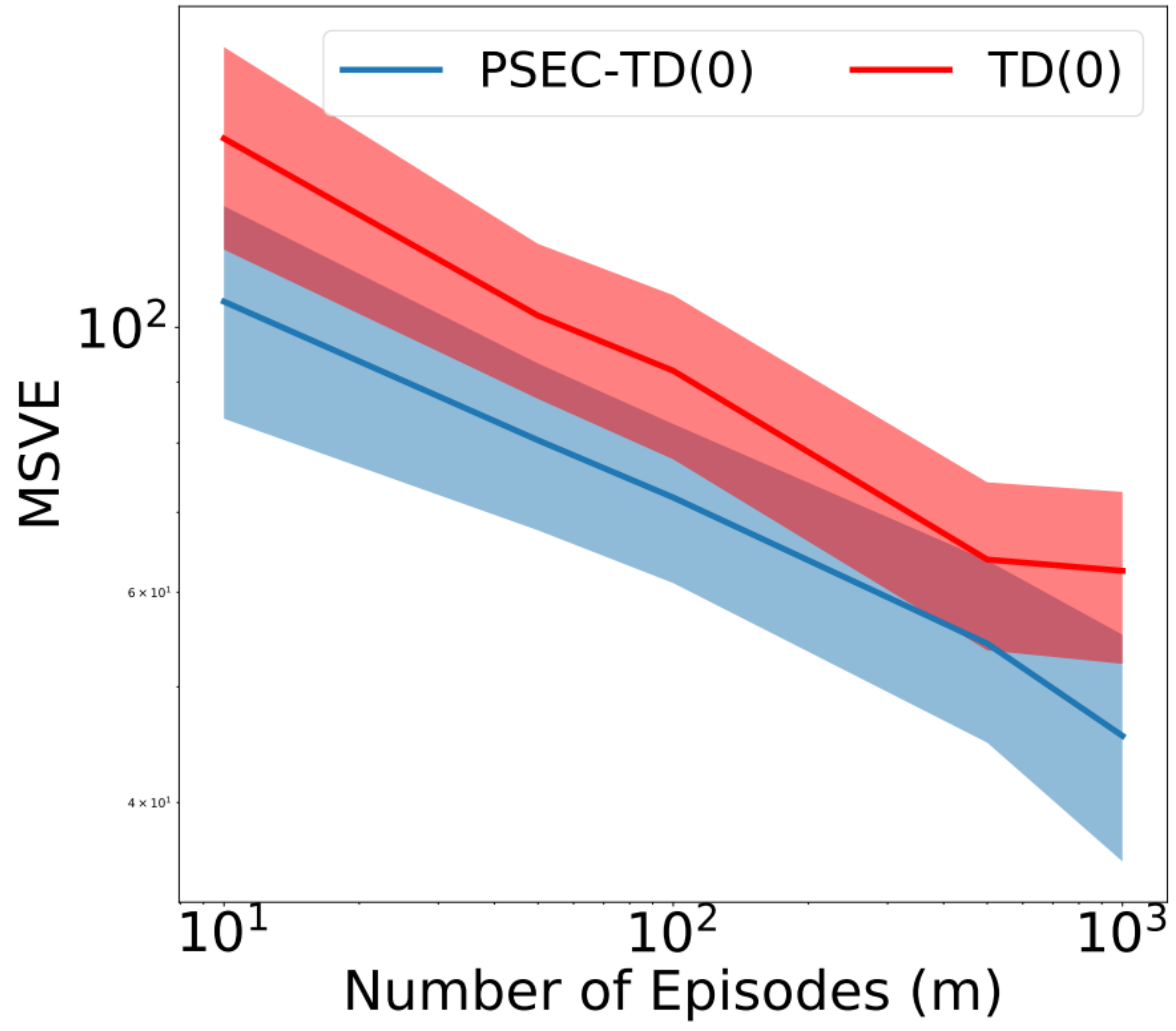

CartPole*

* Statistically significant according to Welch's test [Welch 1947]

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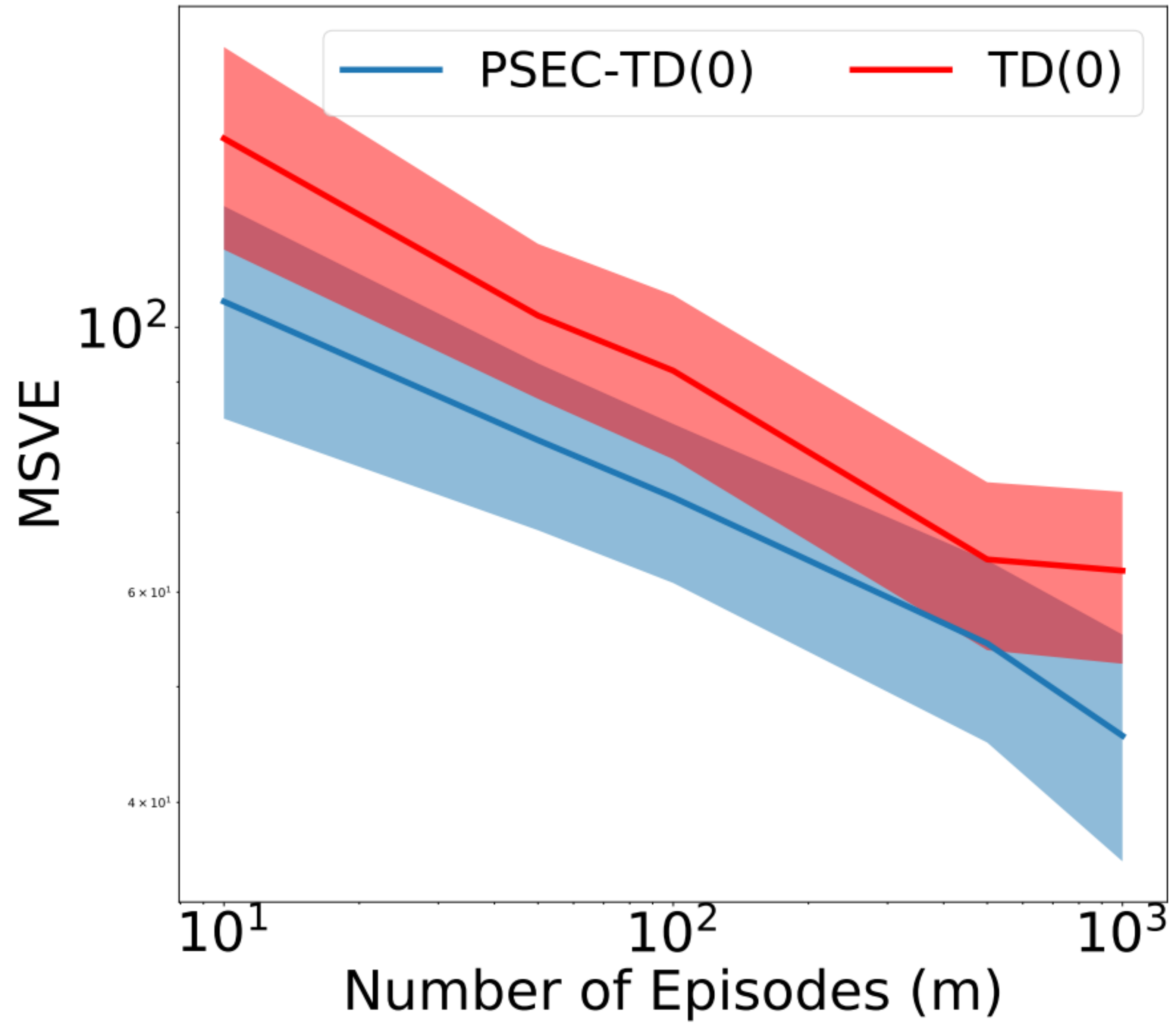

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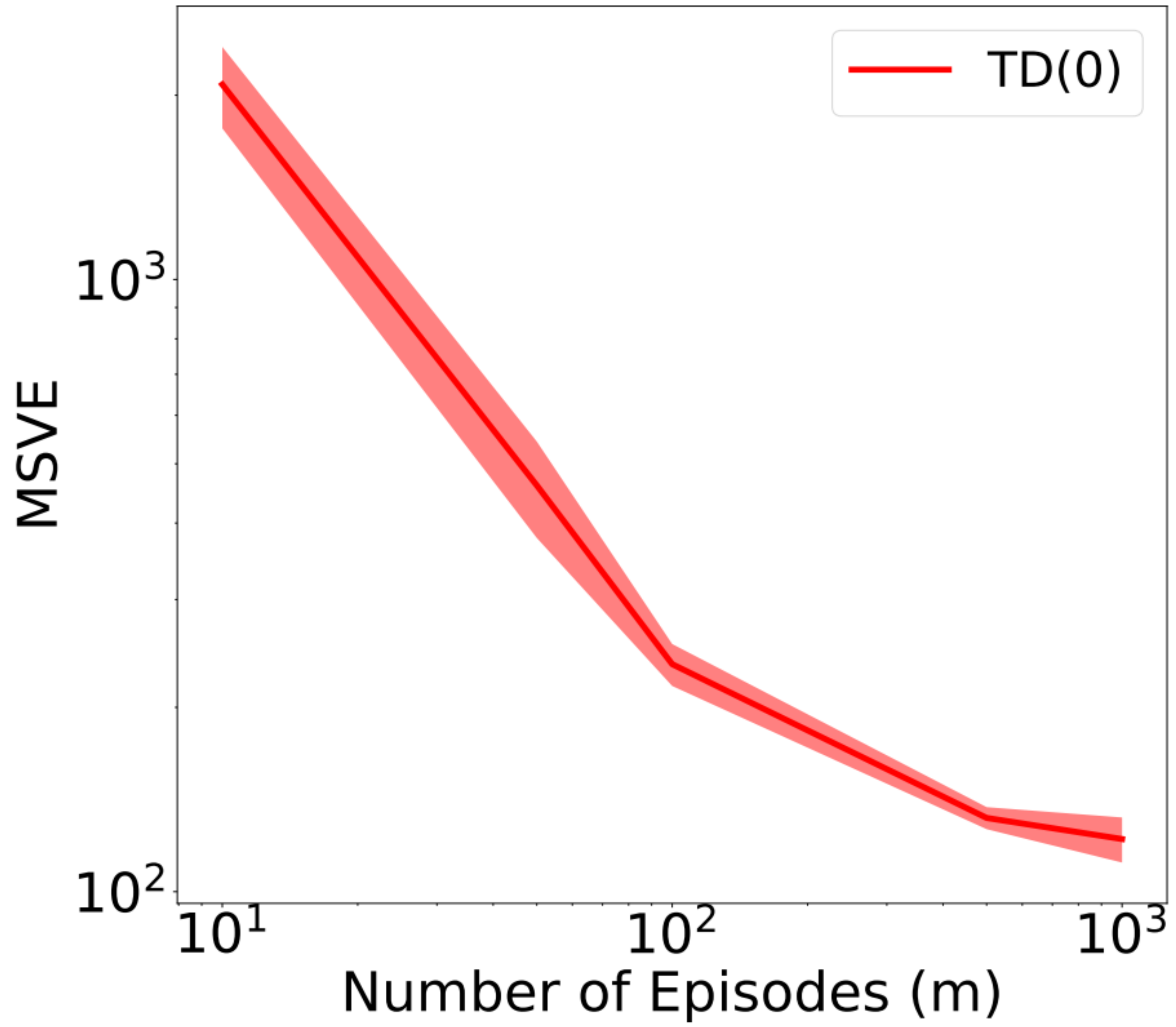

InvertedPendulum

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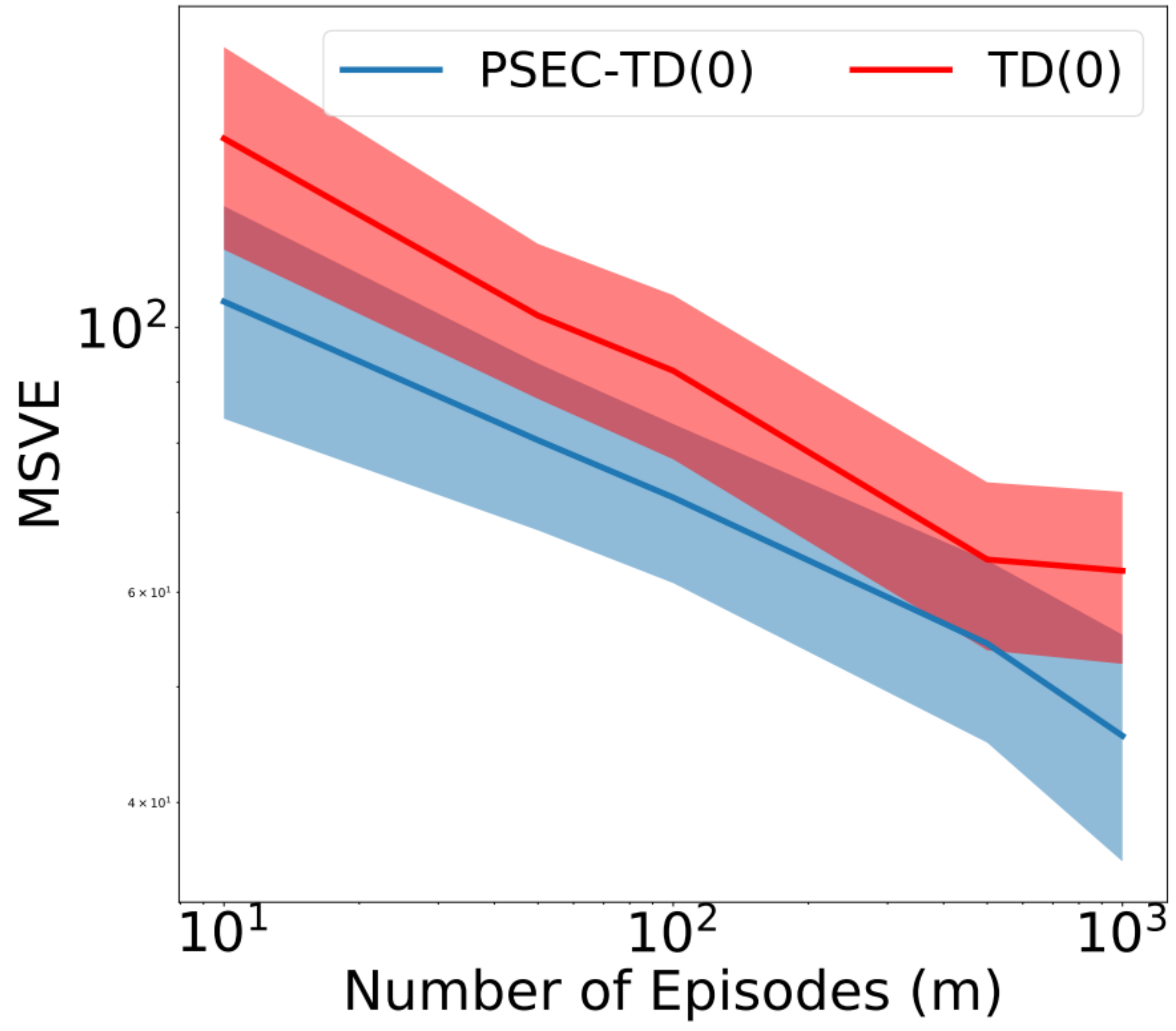

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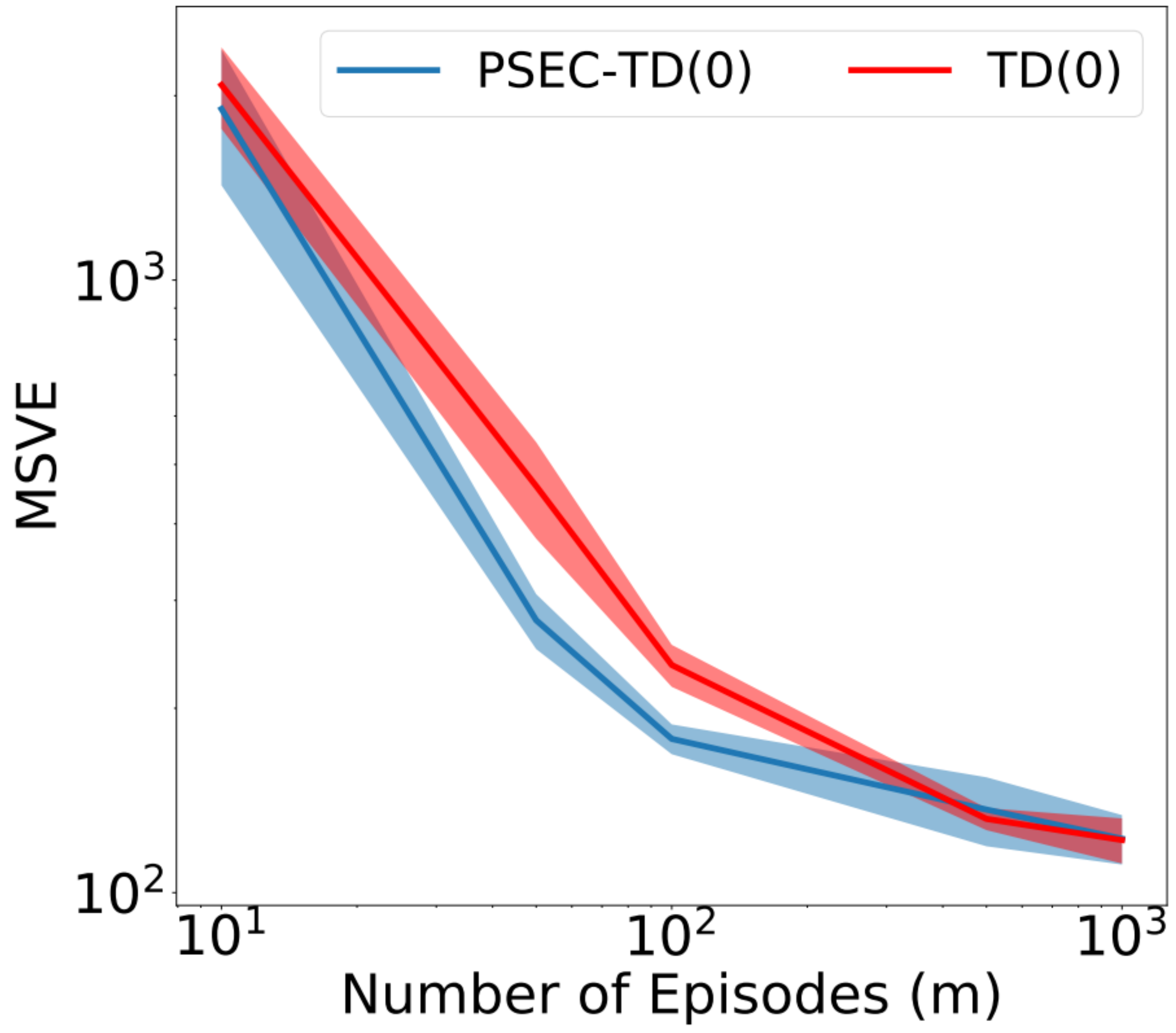

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  - Can PSEC be applied to off-policy TD(0)?

Related Work

- Estimating the behavior policy from data [Li et al., 2015, Narita et al., 2018, Hirano et al., 2003].
- Reducing sampling error in policy evaluation [Hanna et al., 2019] and policy gradient learning [Hanna and Stone, 2019].
- Reducing sampling error in action-values [van Seijen et al., 2009, Precup et al., 2000]

Open Questions

- Reduce sampling error in n-step and TD(\lambda).
- Evaluate actor-critic algorithms with an improved value function estimate.
- Extend *batch* PSEC to *online* TD(0).

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- PSEC-TD(0) brings benefit to **discrete/continuous state/action spaces**.
- While primarily shown for **on-policy** TD(0), PSEC is also applicable in **off-policy** TD(0).

Thank You!

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Josiah Hanna

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Peter Stone

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Recent extension: On Sampling Error in Batch Action-Value Prediction Algorithms