Topic 25
Heaps

"You think you know when you can **learn**,
are more sure when you can **write,**
even more when you can **teach**,
but certain when you can **program**."
- Alan Perlis

Priority Queue

- Recall priority queue
  - elements enqueued based on priority
  - dequeue removes the highest priority item
- Options?
  - List? Binary Search Tree? Clicker 1

	Array List enqueue	BST enqueue
A.	O(N)	O(1)
B.	O(N)	O(logN)
C.	O(N)	O(N)
D.	O(logN)	O(logN)
E.	O(1)	O(logN)

Another Option

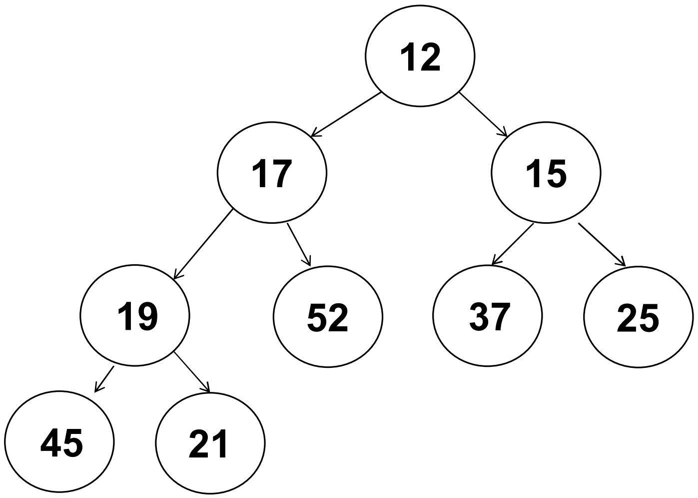
- The *heap* data structure
  - not to be confused with the runtime heap (portion of memory for dynamically allocated variables)
- Typically a complete binary tree (variations with more than 2 children possible)
  - all levels have maximum number of nodes except deepest where nodes are filled in from left to right
- Maintains the *heap order property*
  - in a min heap the value in the root of any subtree is less than or equal to all other values in the subtree

Clicker 2

- In a max heap with no duplicates where is the largest value?

A. the root of the tree
B. in the left-most node
C. in the right-most node
D. a node in the lowest level
E. none of these

Example Min Heap

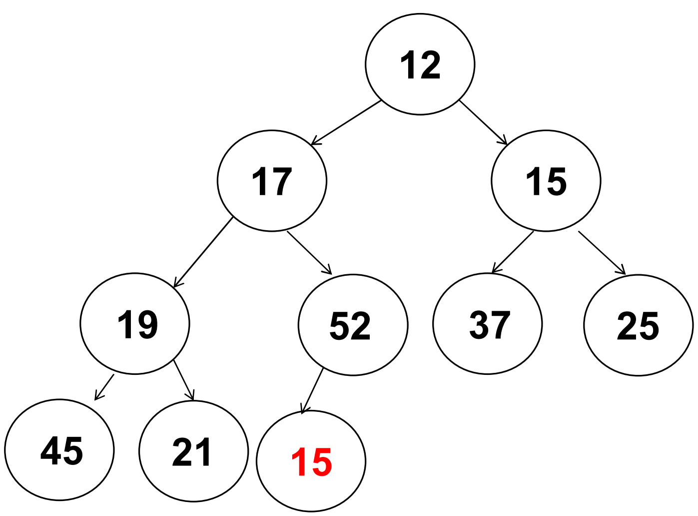


Add Operation

- Add new element to next open spot in array
- Swap with parent if new value is less than parent
- Continue back up the tree as long as the new value is less than new parent node

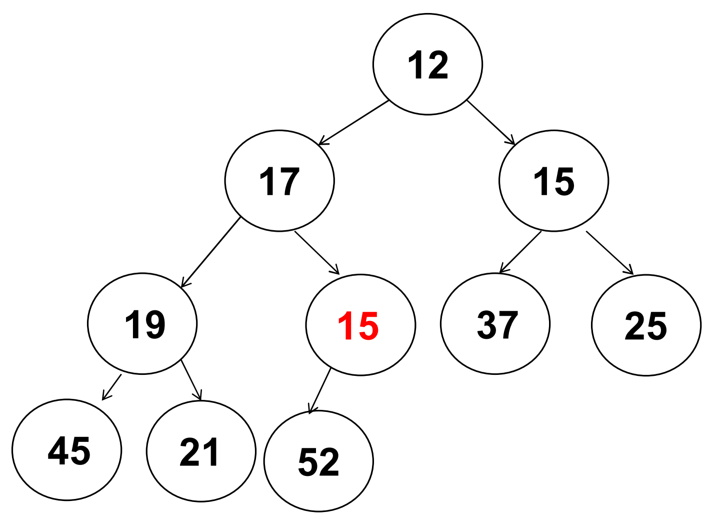
Add Example

- Add 15 to heap (initially next left most node)



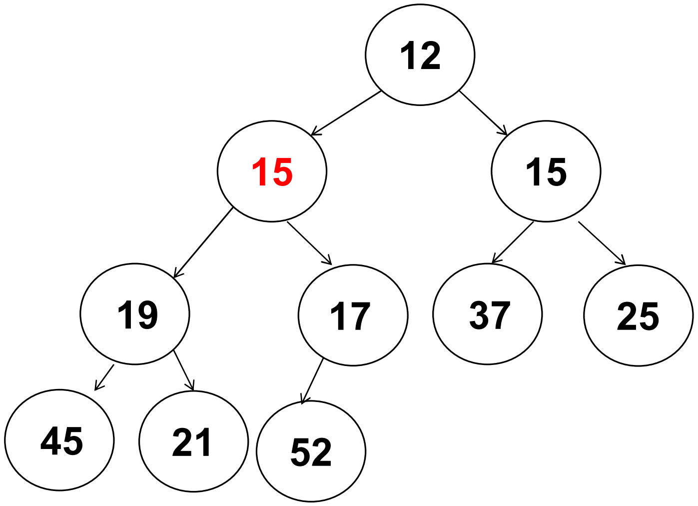
Add Example

- Swap 15 and 52



Enqueue Example

- Swap 15 and 17, then stop

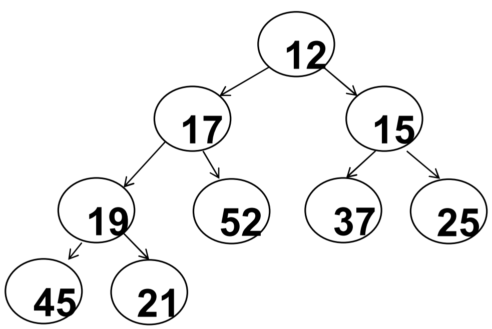


Add Example

- Insert the following values 1 at a time into a min heap:
  16 9 5 8 13 8 8 5 5 19 27 9 3

Internal Storage

- Interestingly heaps are often implemented with an array instead of nodes



for element at index i:
parent index: *i / 2*
left child index: *i * 2*
right child index: *i * 2 + 1*

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**17**	**15**	**19**	**52**	**37**	**25**	**45**	**21**						

In Honor of Elijah, The Meme King, Spring 2020

PriorityQueue Class

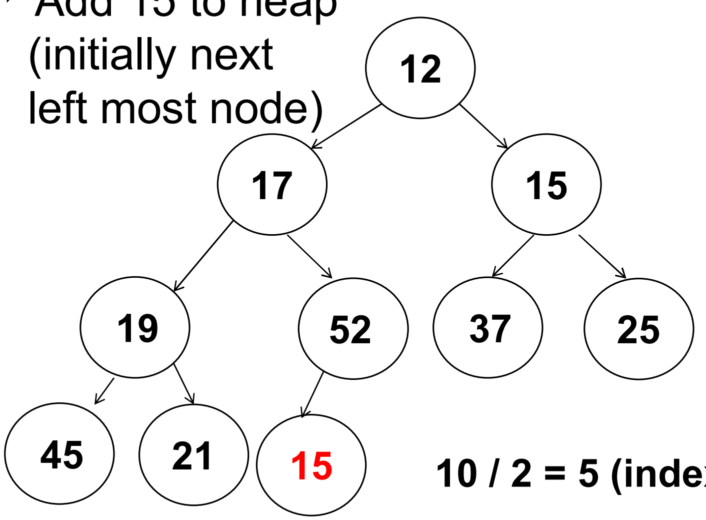
```java
public class PriorityQueue<E extends Comparable<? super E>>
{

    private int size;
    private E[] con;

    public PriorityQueue() {
        con = getArray(2);
    }

    private E[] getArray(int size) {
        return (E[]) (new Comparable[size]);
    }
```


PriorityQueue enqueue / add

```java
public void enqueue(E val) {
    if ( size >= con.length - 1 )
        enlargeArray(con.length * 2);

    size++;
    int indexToPlace = size;
    while ( indexToPlace > 1
            && val.compareTo( con[indexToPlace / 2] ) < 0 ) {

        con[indexToPlace] = con[indexToPlace / 2]; // swap
        indexToPlace /= 2; // change indexToPlace to parent
    }
    con[indexToPlace] = val;
}

private void enlargeArray(int newSize) {
    E[] temp = getArray(newSize);
    System.arraycopy(con, 1, temp, 1, size);
    con = temp;
}
```


Enqueue / add Example With Array Shown

- Add 15 to heap (initially next left most node)

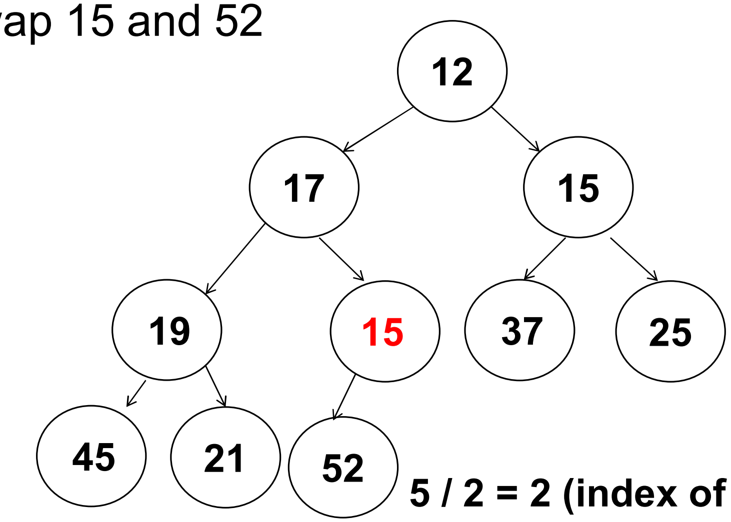


10 / 2 = 5 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**17**	**15**	**19**	**52**	**37**	**25**	**45**	**21**	**15**					

Enqueue Example With Array Shown

- Swap 15 and 52

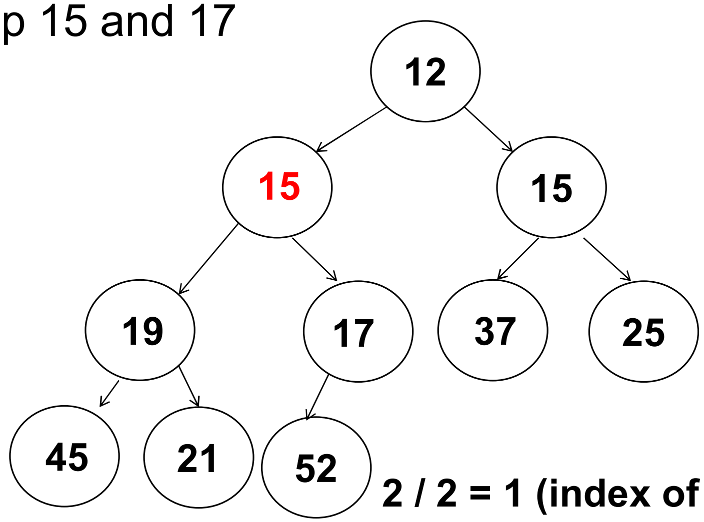


5 / 2 = 2 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**17**	**15**	**19**	**15**	**37**	**25**	**45**	**21**	**52**					

Enqueue Example With Array Shown

- Swap 15 and 17

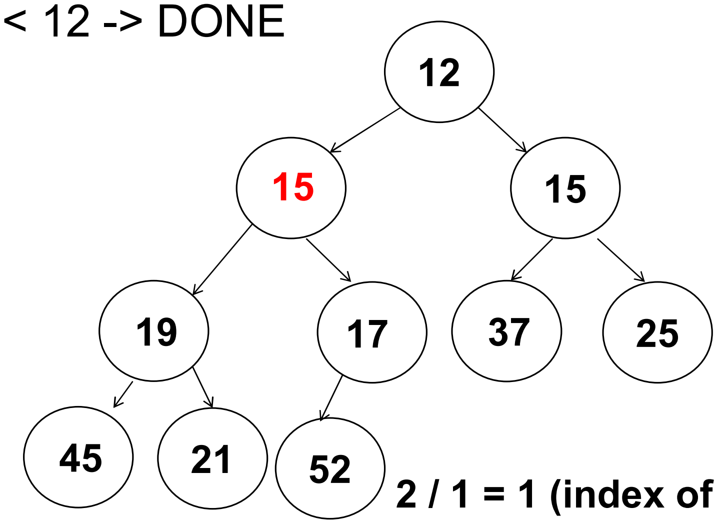


2 / 2 = 1 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**15**	**15**	**19**	**17**	**37**	**25**	**45**	**21**	**52**					

Enqueue Example With Array Shown

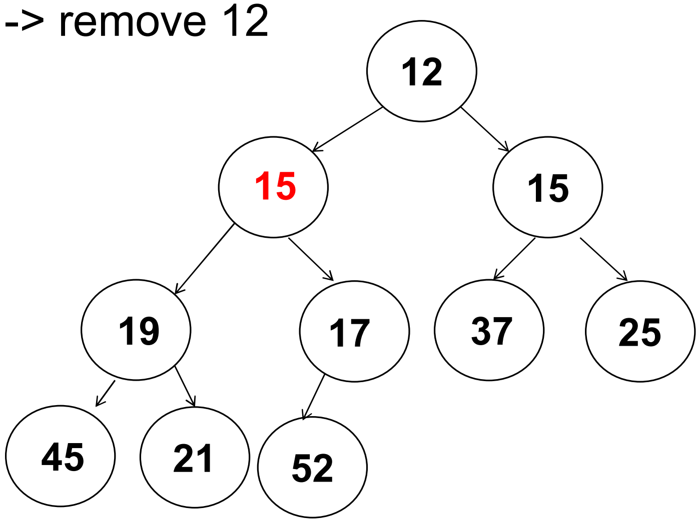
- 15 !< 12 -> DONE



2 / 1 = 1 (index of parent)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	**15**	**16**	**19**	**17**	**37**	**25**	**45**	**21**	**52**					

- Remove -> remove 12

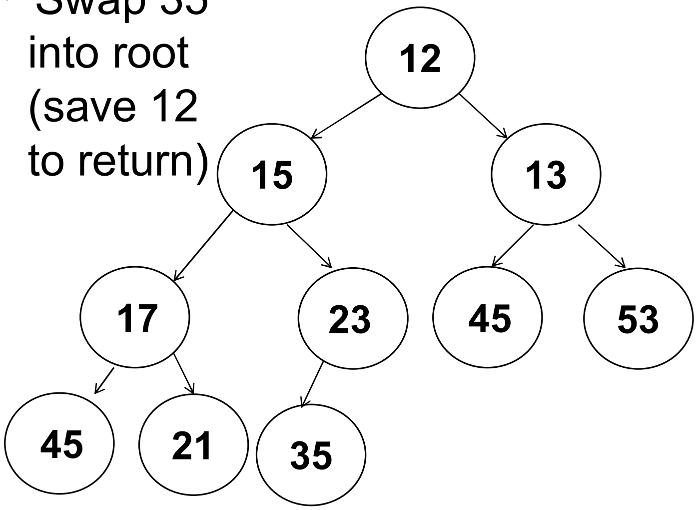


Remove / Dequeue

- min value / front of queue is in root of tree
- swap value from last node to root and move down swapping with smaller child unless values is smaller than both children

Dequeue Example

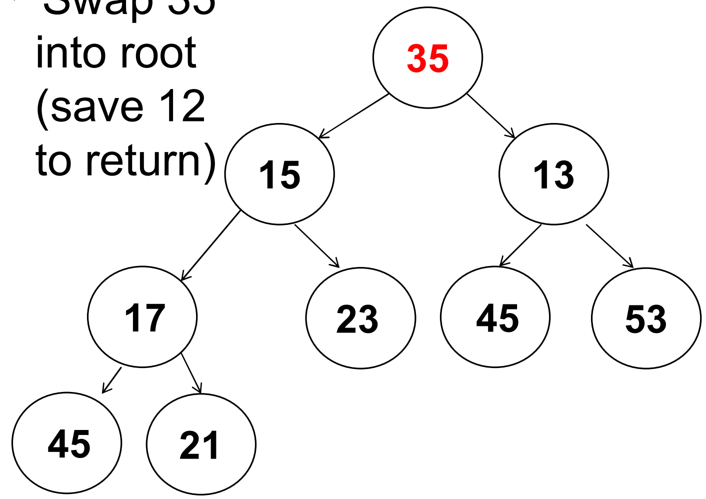
- Swap 35 into root (save 12 to return)



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	12	15	13	17	23	45	53	45	21	35					

Dequeue Example

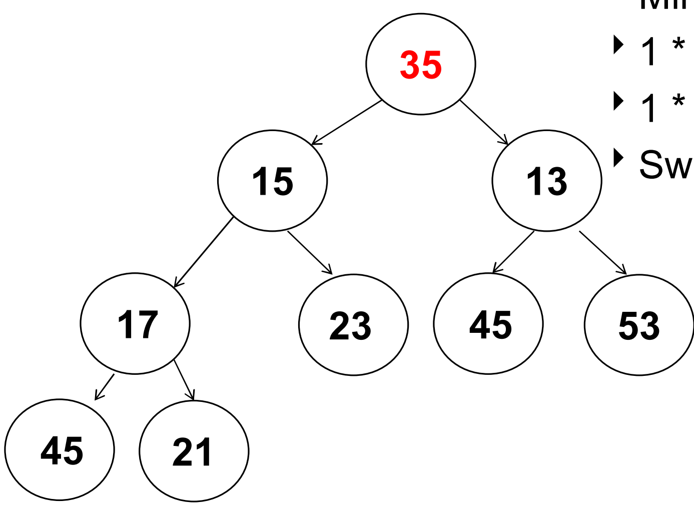
- Swap 35 into root (save 12 to return)



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	35	15	13	17	23	45	53	45	21						

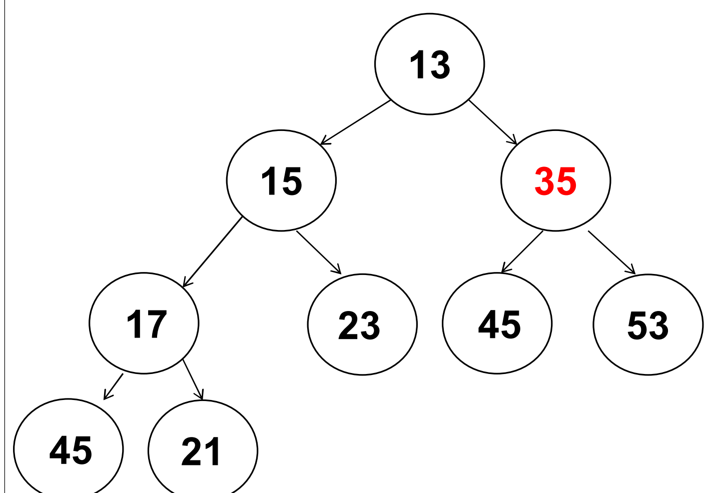
Dequeue Example

- Min child?
- 1 * 2 = 2 -> 15
- 1 * 2 + 1 = 3 -> 13
- Swap with 13



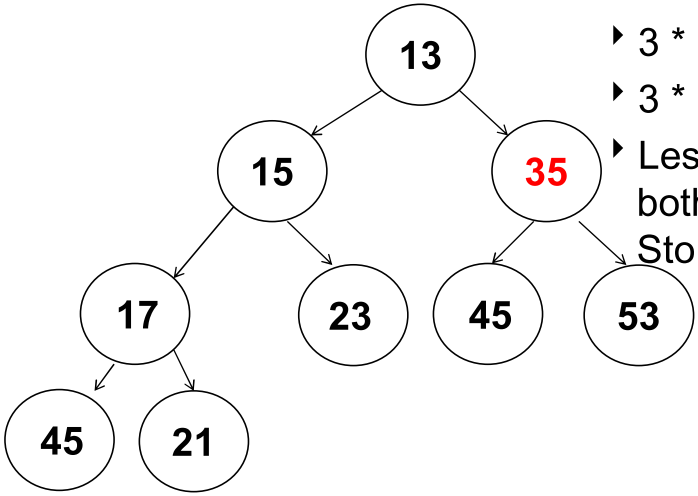
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	35	15	13	17	23	45	53	45	21						

Dequeue Example



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	13	15	35	17	23	45	53	45	21						

Dequeue Example



- Min child?
- 3 * 2 = 6 -> 45
- 3 * 2 + 1 = 7 -> 53
- Less than or equal to both of my children! Stop!

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	13	**15**	**35**	**17**	**23**	**45**	**53**	**45**	**21**						

Dequeue Code

```
public E dequeue( ) {
    E top = con[1];
    int hole = 1;
    boolean done = false;
    while ( hole * 2 < size && ! done ) {
        int child = hole * 2;
        // see which child is smaller
        if ( con[child].compareTo( con[child + 1] ) > 0 )
            child++;    // child now points to smaller

        // is replacement value bigger than child?
        if (con[size].compareTo( con[child] ) > 0 ) {
            con[hole] = con[child];
            hole = child;
        }
        else
            done = true;
    }
    con[hole] = con[size];
    size--;
    return top;
}
```


Clicker 3 - PriorityQueue Comparison

- Run a Stress test of PQ implemented with Heap and PQ implemented with BinarySearchTree
- What will result be?

A. Heap takes half the time or less of BST

B. Heap faster, but not twice as fast

C. About the same

D. BST faster, but not twice as fast

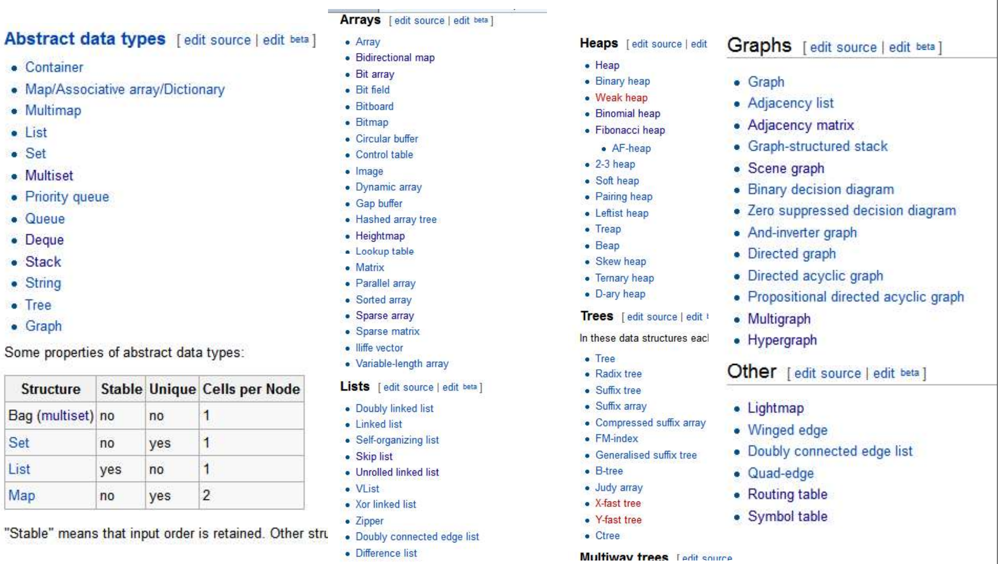
E. BST takes half the time or less of Heap

Data Structures

- Data structures we have studied
  - arrays, array based lists, linked lists, maps, sets, stacks, queues, trees, binary search trees, graphs, hash tables, red-black trees, priority queues, heaps, tries
- Most program languages have some built in data structures, native or library
- Must be familiar with performance of data structures
  - best learned by implementing them yourself

Data Structures

- We have ***not*** covered every data structure



Abstract data types [edit source | edit beta]

- Container
- Map/Associative array/Dictionary
- Multimap
- List
- Set
- Multiset
- Priority queue
- Queue
- Deque
- Stack
- String
- Tree
- Graph

Some properties of abstract data types:

Structure	Stable	Unique	Cells per Node
Bag (multiset)	no	no	1
Set	no	yes	1
List	yes	no	1
Map	no	yes	2

"Stable" means that input order is retained. Other stru

Arrays [edit source | edit beta]

- Array
- Bidirectional map
- Bit array
- Bit field
- Bitboard
- Bitmap
- Circular buffer
- Control table
- Image
- Dynamic array
- Gap buffer
- Hashed array tree
- Heightmap
- Lookup table
- Matrix
- Parallel array
- Sorted array
- Sparse array
- Sparse matrix
- Iliffe vector
- Variable-length array

Lists [edit source | edit beta]

- Doubly linked list
- Linked list
- Self-organizing list
- Skip list
- Unrolled linked list
- VList
- Xor linked list
- Zipper
- Doubly connected edge list
- Difference list

Heaps [edit source | edit

- Heap
- Binary heap
- Weak heap
- Binomial heap
- Fibonacci heap
  - AF-heap
- 2-3 heap
- Soft heap
- Pairing heap
- Leftist heap
- Treap
- Beap
- Skew heap
- Ternary heap
- D-ary heap

Trees [edit source | edit

In these data structures eacl

- Tree
- Radix tree
- Suffix tree
- Suffix array
- Compressed suffix array
- FM-index
- Generalised suffix tree
- B-tree
- Judy array
- X-fast tree
- Y-fast tree
- Ctree

Multiway trees [edit source

Graphs [edit source | edit beta]

- Graph
- Adjacency list
- Adjacency matrix
- Graph-structured stack
- Scene graph
- Binary decision diagram
- Zero suppressed decision diagram
- And-inverter graph
- Directed graph
- Directed acyclic graph
- Propositional directed acyclic graph
- Multigraph
- Hypergraph

Other [edit source | edit beta]

- Lightmap
- Winged edge
- Doubly connected edge list
- Quad-edge
- Routing table
- Symbol table

http://en.wikipedia.org/wiki/List_of_data_structures

Data Structures

- deque, b-trees, quad-trees, binary space partition trees, skip list, sparse list, sparse matrix, union-find data structure, Bloom filters, AVL trees, 2-3-4 trees, and more!
- Must be able to learn new and apply new data structures