CS 380S

0x1A Great Papers in Computer Security

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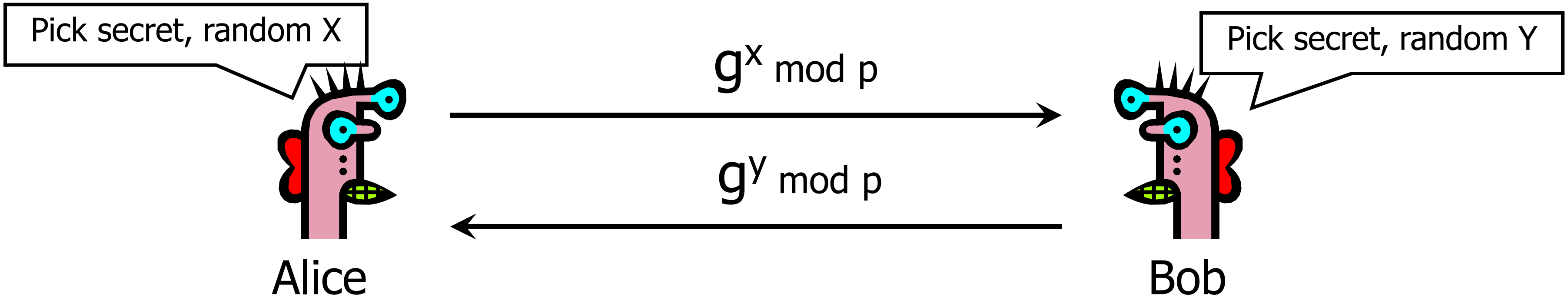
W. Diffie and M. Hellman

New Directions in Cryptography

(ToIT 1976)

Diffie-Hellman Key Establishment

- Alice and Bob never met and share no secrets
- Public information: p and g, where p is a large prime number, g is a generator of Z^*_p
  - $Z^*_p=\{1, 2 \ldots p-1\}$; $\forall a \in Z^*_p\ \exists i$ such that $a=g^i \bmod p$



Pick secret, random X

g^x mod p

g^y mod p

Pick secret, random Y

Alice

Bob

Compute $k=(g^y)^x=g^{xy} \bmod p$

Compute $k=(g^x)^y=g^{xy} \bmod p$

Why Is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem: given g^x mod p, it's hard to extract x
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem: given g^x and g^y, it's hard to compute g^{xy} mod p
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:

  given g^x and g^y, it's hard to tell the difference between g^{xy} mod p and g^r mod p where r is random

Security of Diffie-Hellman Protocol

◆ Assuming the DDH problem is hard, Diffie-Hellman protocol is a secure key establishment protocol against <u>passive</u> attackers

- Eavesdropper can't tell the difference between the established key and a random value
- Can use the established key for symmetric cryptography
  - Approx. 1000 times faster than modular exponentiation

◆ Basic Diffie-Hellman protocol is not secure against an active, man-in-the-middle attacker

Public-Key Encryption

- **Key generation:** computationally easy to generate a pair (public key PK, private key SK)
  - Computationally infeasible to determine private key SK given only public key PK
- **Encryption:** given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- **Decryption:** given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to compute M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

ElGamal Encryption

Key generation

- Pick a large prime p, generator g of Z^*_p
- Private key: random x such that $1 \le x \le p-2$
- Public key: $(p, g, y = g^x \bmod p)$

Encryption

- Pick random k, $1 \le k \le p-2$
- $E(m) = (g^k \bmod p, m \cdot y^k \bmod p) = (\gamma, \delta)$

Decryption

- Given ciphertext (γ,δ), compute $\gamma^{-x} \bmod p$
- Recover $m = \delta \cdot (\gamma^{-x}) \bmod p$

When Is Encryption “Secure”?

- Hard to recover the key?
  - What if attacker can learn plaintext without learning the key?
- Hard to recover plaintext from ciphertext?
  - What if attacker learns some bits or some property of the plaintext?
- (Informal) goal: ciphertext should hide all “useful” information about the plaintext
  - … except its length

Attack Models

Assume that the attacker knows the encryption algorithm and wants to decrypt some ciphertext

- Ciphertext-only attack
- Known-plaintext attack (stronger)
  - Knows some plaintext-ciphertext pairs
- Chosen-plaintext attack (even stronger)
  - Can obtain ciphertext for any plaintext of his choice
- Chosen-ciphertext attack (very strong)
  - Can decrypt any ciphertext <u>except</u> the target

The Chosen-Plaintext (CPA) Game

Idea: attacker should not be able to learn
any property of the encrypted plaintext

- Attacker chooses as many plaintexts as he wants and learns the corresponding ciphertexts
- When ready, he picks two plaintexts M_0 and M_1
  - He is even allowed to pick plaintexts for which he previously learned ciphertexts!
- He receives either a ciphertext of M_0, or a ciphertext of M_1
- He wins if he guesses correctly which one it is

CPA Game: Formalization

- Define Enc(M_0, M_1, b) to be a function that returns encrypted M_b

  0 or 1

  - Think of Enc as a magic box that computes ciphertexts on attacker's demand… he can obtain a ciphertext of any plaintext M by submitting $M_0=M_1=M$, or he can submit $M_0 \neq M_1$
- Attacker's goal is to learn just one bit b

Chosen-Plaintext Security

◆ Consider two experiments (A is the attacker)

Experiment 0	Experiment 1
A interacts with Enc(-,-,0) and outputs bit d	A interacts with Enc(-,-,1) and outputs bit d

- Identical except for the value of the secret bit
- d is attacker's guess of the secret bit

◆ Attacker's advantage is defined as

If A "knows" secret bit, he should be able to make his output depend on it

| Prob(A outputs 1 in Exp0) - Prob(A outputs 1 in Exp1)) |

◆ Encryption scheme is chosen-plaintext secure if this advantage is negligible for any efficient A

Simple Example

- Any deterministic, stateless encryption scheme is insecure against chosen-plaintext attack
  - Attacker can easily distinguish encryptions of different plaintexts from encryptions of identical plaintexts

Attacker A interacts with Enc(-,-,b)

```
Let X,Y be any two different plaintexts
$C_1 \leftarrow$ Enc(X,Y,b);
$C_2 \leftarrow$ Enc(Y,Y,b);
If $C_1=C_2$ then output 1 else output 0
```

- The advantage of this attacker A is 1

Prob(A outputs 1 if b=0)=0   Prob(A outputs 1 if b=1)=1

Semantic Security

[Goldwasser and Micali 1982]

- Ciphertext hides even partial information about the plaintext
  - No matter what prior knowledge attacker has about the plaintext, it does not increase after observing ciphertext
- Equivalent to ciphertext indistinguishability under the chosen-plaintext attack
  - It is infeasible to find two messages whose encryptions can be distinguished

Semantic Security of ElGamal

Semantic security of ElGamal encryption is equivalent to DDH

- Given an oracle for breaking DDH, show that we can find two messages whose ElGamal ciphertexts can be distinguished
- Given an oracle for distinguishing ElGamal ciphertexts, show that we can break DDH
  - Break DDH = given a triplet $<g^a, g^b, Z>$, we can decide whether $Z=g^{ab} \bmod p$ or Z is random

DDH $\Rightarrow$ ElGamal

- Pick any two messages m_0, m_1
- Receive $E(m) = g^k, m \cdot y^k$
  - $y = g^x$ is the ElGamal public key
  - To break ElGamal, must determine if $m=m_0$ or $m=m_1$
- Run the DDH oracle on this triplet:

$$<g^k, y \cdot g^v, (m \cdot y^k) \cdot g^{kv}/m_0> \ = \ <g^k, g^{x+v}, m \cdot g^{(x+v)k}/m_0>$$

  - v is random
- If this is a DH triplet, then $m=m_0$, else $m=m_1$
- This breaks semantic security of ElGamal (why?)

(1) ElGamal $\Rightarrow$ DDH

- Suppose some algorithm A breaks ElGamal
  - Given any public key, A produces plaintexts m_0 and m_1 whose encryptions it can distinguish with advantage Adv

  We will use A to break DDH
  - Decide, given (g^a, g^b, Z), whether $Z=g^{ab} \bmod p$ or not
- Give $y=g^a \bmod p$ to A as the public key
- A produces m_0 and m_1
- Toss a coin for bit x and give A the ciphertext $(g^b, m_x \cdot Z) \bmod p$
  - This is a valid ElGamal encryption of m_x **iff** $Z=g^{ab} \bmod p$

(2) ElGamal $\Rightarrow$ DDH

◆ A receives $(g^b, m_x \cdot Z)$ mod p
  - This is a valid ElGamal encryption of m_x **iff** $Z=g^{ab}$ mod p

◆ A outputs his guess of bit x (why?)

◆ If A guessed x correctly, we say that $Z=g^{ab}$ mod p, otherwise we say that Z is random

◆ What is our advantage in breaking DDH?
  - If $Z=g^{ab}$ mod p, we are correct with probability Adv(A)
  - If Z is random, we are correct with probability ½
  - Our advantage in breaking DDH is Adv(A)/2

Beyond Semantic Security

- Chosen-ciphertext security
  - "Lunch-time" attack [Naor and Yung 1990]
  - Adaptive chosen-ciphertext security [Rackoff and Simon 1991]
- Non-malleability [Dolev, Dwork, Naor 1991]
  - Infeasible to create a "related" ciphertext
  - Implies that an encrypted message cannot be modified without decrypting it
  - Equivalent to adaptive chosen-ciphertext security