CS 380S

0x1A Great Papers in Computer Security

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http://www.cs.utexas.edu/~shmat/courses/cs380s/

Attacking Cryptographic Schemes

- Cryptanalysis
  - Find mathematical weaknesses in constructions
  - Statistical analysis of plaintext / ciphertext pairs
- Side-channel attacks
  - Exploit characteristics of implementations
  - Power analysis
  - Electromagnetic radiation analysis
  - Acoustic analysis
  - Timing analysis

Timing Attack

- Basic idea: learn the system's secret by observing how long it takes to perform various computations
- Typical goal: extract private key
- Extremely powerful because isolation doesn't help
  - Victim could be remote
  - Victim could be inside its own virtual machine
  - Keys could be in tamper-proof storage or smartcard
- Attacker wins simply by measuring response times

RSA Cryptosystem

- **Key generation:**
  - Generate large (say, 512-bit) primes p, q
  - Compute $n=pq$ and $\varphi(n)=(p-1)(q-1)$
  - Choose small e, relatively prime to $\varphi(n)$
    - Typically, $e=3$ (may be vulnerable) or $e=2^{16}+1=65537$ (why?)
  - Compute unique d such that $ed = 1 \bmod \varphi(n)$
  - Public key = (e,n); private key = d
    - Security relies on the assumption that it is difficult to compute roots modulo n without knowing p and q
- **Encryption** of m (simplified!): $c = m^e \bmod n$
- **Decryption** of c: $c^d \bmod n = (m^e)^d \bmod n = m$

How RSA Decryption Works

- RSA decryption: compute y^x mod n
  - This is a modular exponentiation operation
- Naïve algorithm: square and multiply

$$
\begin{array}{l}
\texttt{Let } s_0 = 1\texttt{.} \\
\texttt{For } k = 0 \texttt{ upto } w-1\texttt{:} \\
\quad \texttt{If } (\text{bit } k \text{ of } x) \texttt{ is } 1 \texttt{ then} \\
\quad\quad \texttt{Let } R_k = (s_k \cdot y) \bmod n\texttt{.} \\
\quad \texttt{Else} \\
\quad\quad \texttt{Let } R_k = s_k\texttt{.} \\
\quad \texttt{Let } s_{k+1} = R_k^2 \bmod n\texttt{.} \\
\texttt{EndFor.} \\
\texttt{Return } (R_{w-1})\texttt{.}
\end{array}
$$

Kocher's Observation

Whether iteration takes a long time depends on the k^{th} bit of secret exponent

```
Let s_0 = 1.
For k = 0 upto w − 1:
    If (bit k of x) is 1 then
        Let R_k = (s_k · y) mod n.
    Else
        Let R_k = s_k.
    Let s_{k+1} = R_k^2 mod n.
EndFor.
Return (R_{w−1}).
```

This takes a while to compute

This is instantaneous

Exploiting Timing Information

- Different timing given operands
- Assumption / heuristic: timings of subsequent multiplications are independent
  - Given that we know the first k-1 bits of x … Exact timing
    Given a guess for the k^{th} bit of x … Exact guess
    … Time for remaining bits independent
- Given measurement of total time can see whether there is correlation between "time for k^{th} bit is long" and "total time is long"

Outline of Kocher's Attack

- Idea: guess some bits of the exponent and predict how long decryption will take
- If guess is correct, will observe correlation; if incorrect, then prediction will look random
  - This is a signal detection problem, where signal is timing variation due to guessed exponent bits
  - The more bits you already know, the stronger the signal, thus easier to detect (error-correction property)
- Start by guessing a few top bits, look at correlations for each guess, pick the most promising candidate and continue

D. Brumley and D. Boneh

Remote Timing Attacks are Practical

(USENIX Security 2003)

RSA in OpenSSL

- OpenSSL is a popular open-source toolkit
  - mod_SSL (in Apache = 28% of HTTPS market)
  - stunnel (secure TCP/IP servers)
  - sNFS (secure NFS)
  - Many more applications
- Kocher's attack doesn't work against OpenSSL
  - Instead of square-and-multiply, OpenSSL uses CRT, sliding windows and two different multiplication algorithms for modular exponentiation
    - CRT = Chinese Remainder Theorem
    - Secret exponent is processed in chunks, not bit-by-bit

Chinese Remainder Theorem

- $n = n_1 n_2 \ldots n_k$
  where $\gcd(n_i, n_j) = 1$ when $i \neq j$
- The system of congruences
  $x = x_1 \bmod n_1 = \ldots = x_k \bmod n_k$
  - Has a simultaneous solution x to all congruences
  - There exists exactly one solution x between 0 and n-1
- For RSA modulus n=pq, to compute x mod n it's enough to know x mod p and x mod q

RSA Decryption With CRT

- To decrypt c, need to compute $m=c^d \bmod n$
- Use Chinese Remainder Theorem (why?)
  - $d_1 = d \bmod (p-1)$
  - $d_2 = d \bmod (q-1)$
  - $qinv = q^{-1} \bmod p$

  these are precomputed

  - Compute $m_1 = c^{d1} \bmod p$; $m_2 = c^{d2} \bmod q$
  - Compute $m = m_2+(qinv*(m_1-m_2) \bmod p)*q$

Attack this computation in order to learn q.
This is enough to learn private key **(why?)**

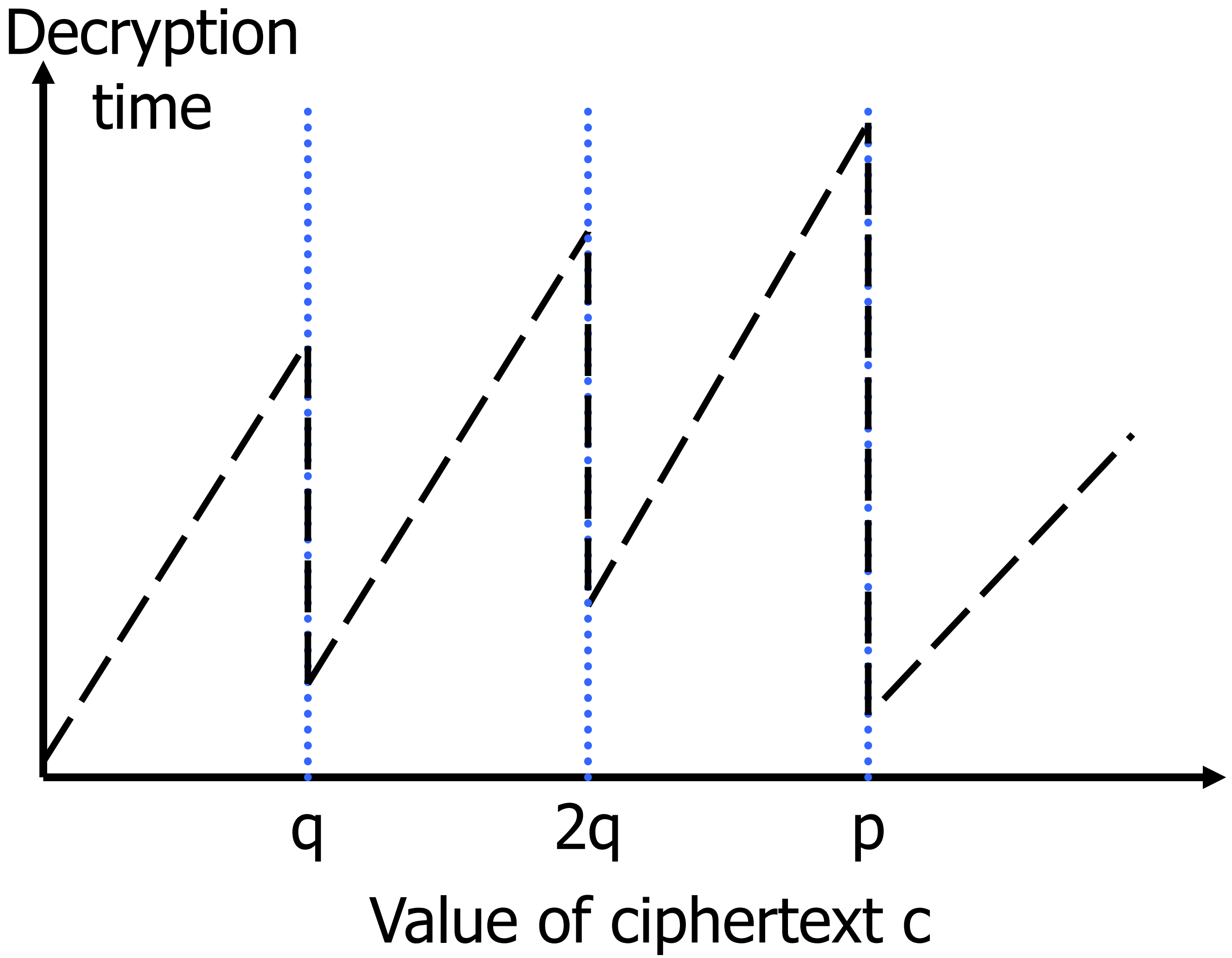
Montgomery Reduction

- Decryption requires computing $m_2 = c^{d2} \bmod q$
- This is done by repeated multiplication
  - Simple: square and multiply (process d_2 1 bit at a time)
  - More clever: sliding windows (process d_2 in 5-bit blocks)
- In either case, many multiplications modulo q
- Multiplications use Montgomery reduction
  - Pick some $R = 2^k$
  - To compute $x*y \bmod q$, convert x and y into their Montgomery form $xR \bmod q$ and $yR \bmod q$
  - Compute $(xR * yR) * R^{-1} = zR \bmod q$
    - Multiplication by R^{-1} can be done very efficiently

Schindler's Observation

- At the end of Montgomery reduction, if $zR > q$, then need to subtract q
  - Probability of this extra step is proportional to c mod q
- If c is close to q, a lot of subtractions will be done
- If c mod q = 0, very few subtractions
  - Decryption will take longer as c gets closer to q, then become fast as c passes a multiple of q
- By playing with different values of c and observing how long decryption takes, attacker can guess q!
  - Doesn't work directly against OpenSSL because of sliding windows and two multiplication algorithms

Reduction Timing Dependency

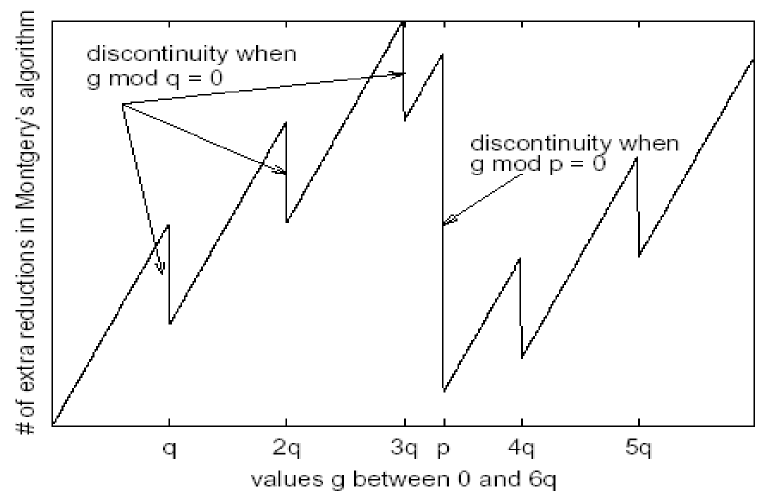


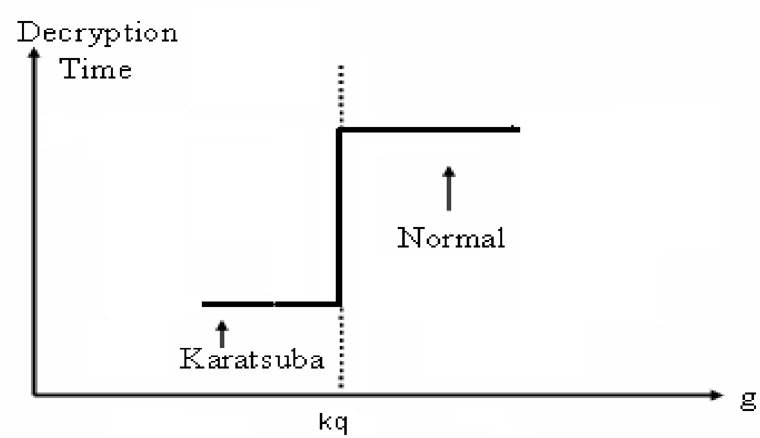
Integer Multiplication Routines

- 30-40% of OpenSSL running time is spent on integer multiplication
- If integers have the same number of words n, OpenSSL uses Karatsuba multiplication
  - Takes $O(n^{\log_2 3})$
- If integers have unequal number of words n and m, OpenSSL uses normal multiplication
  - Takes $O(nm)$

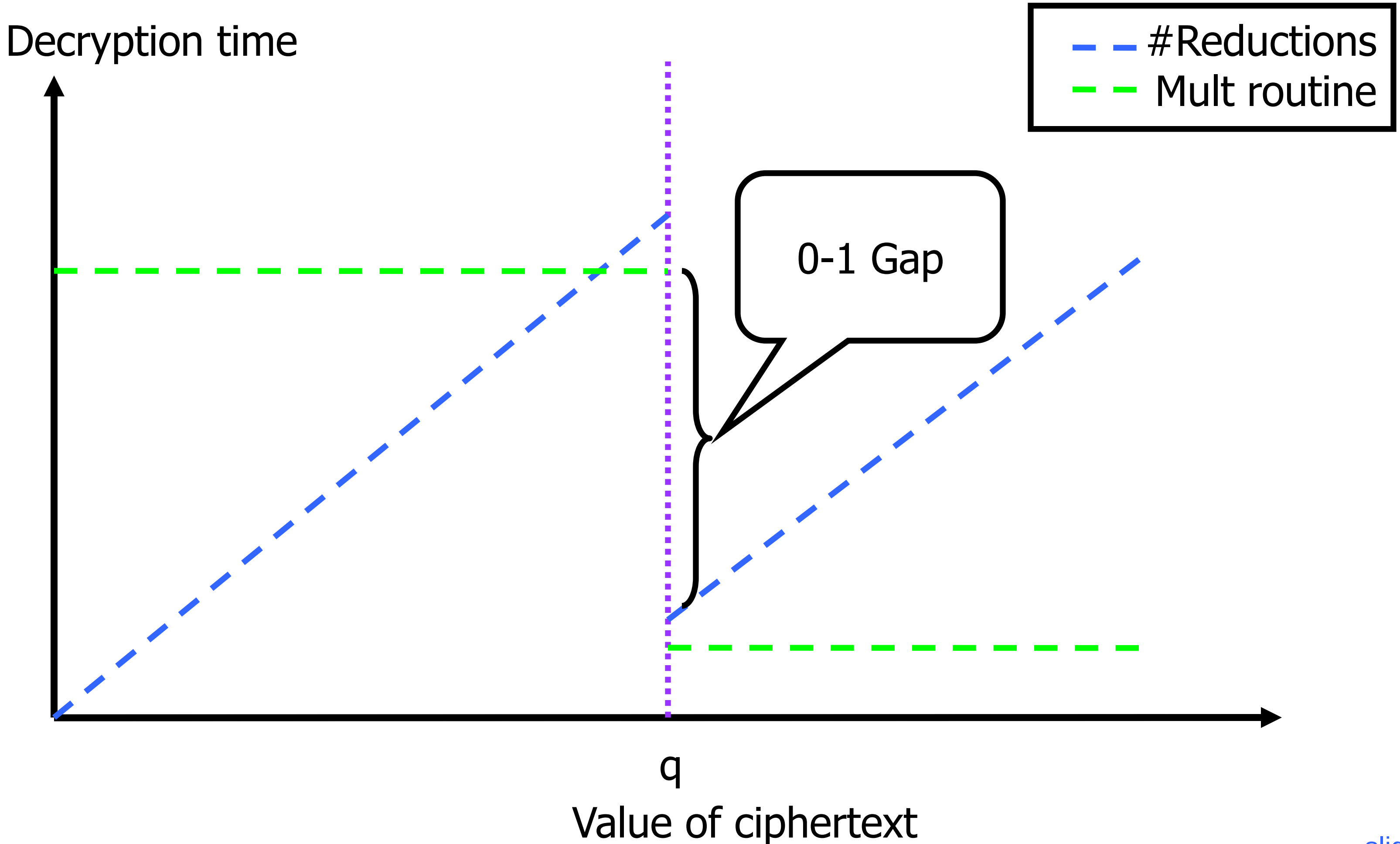
Summary of Time Dependencies

	g<q	g>q
Montgomery effect	Longer	Shorter
Multiplication effect	Shorter	Longer
g is the decryption value (same as c) Different effects… but one will always dominate!		

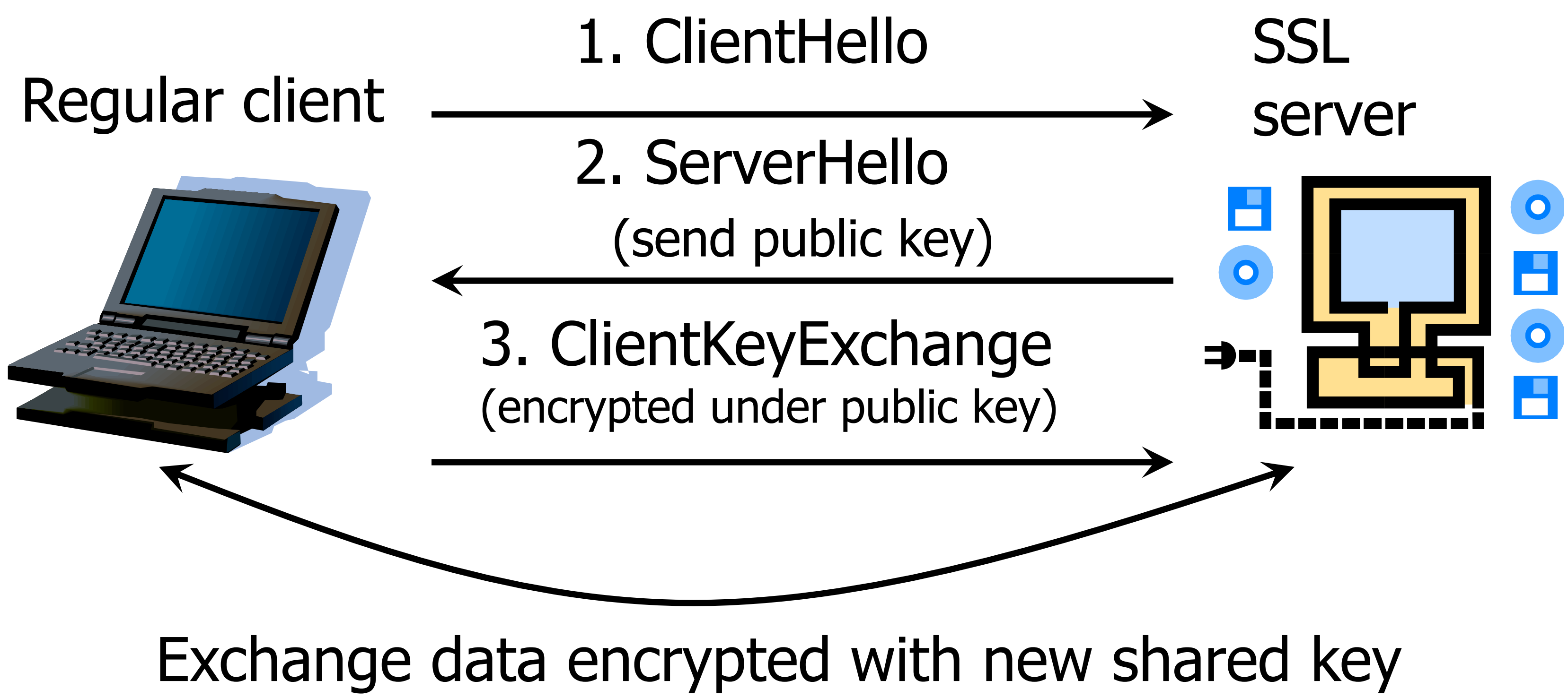




Discontinuity in Decryption Time



Normal SSL Handshake

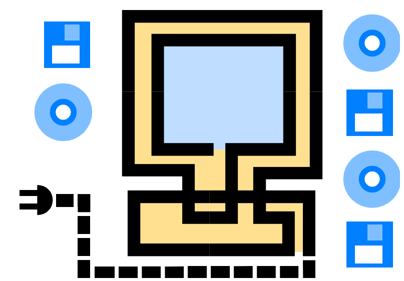


Attacking SSL Handshake

Attacker

1. ClientHello

SSL
server



2. ServerHello
(send public key)

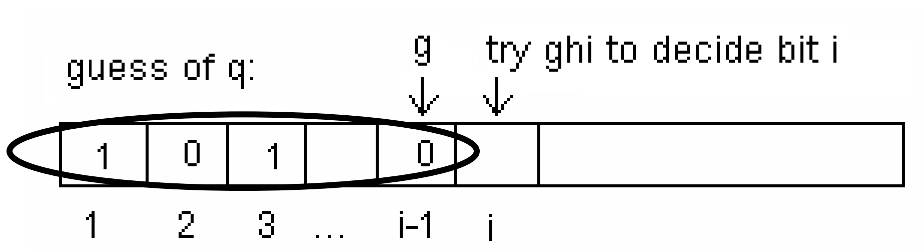
3. Record time t_1
Send guess g or g_{hi}

4. Alert

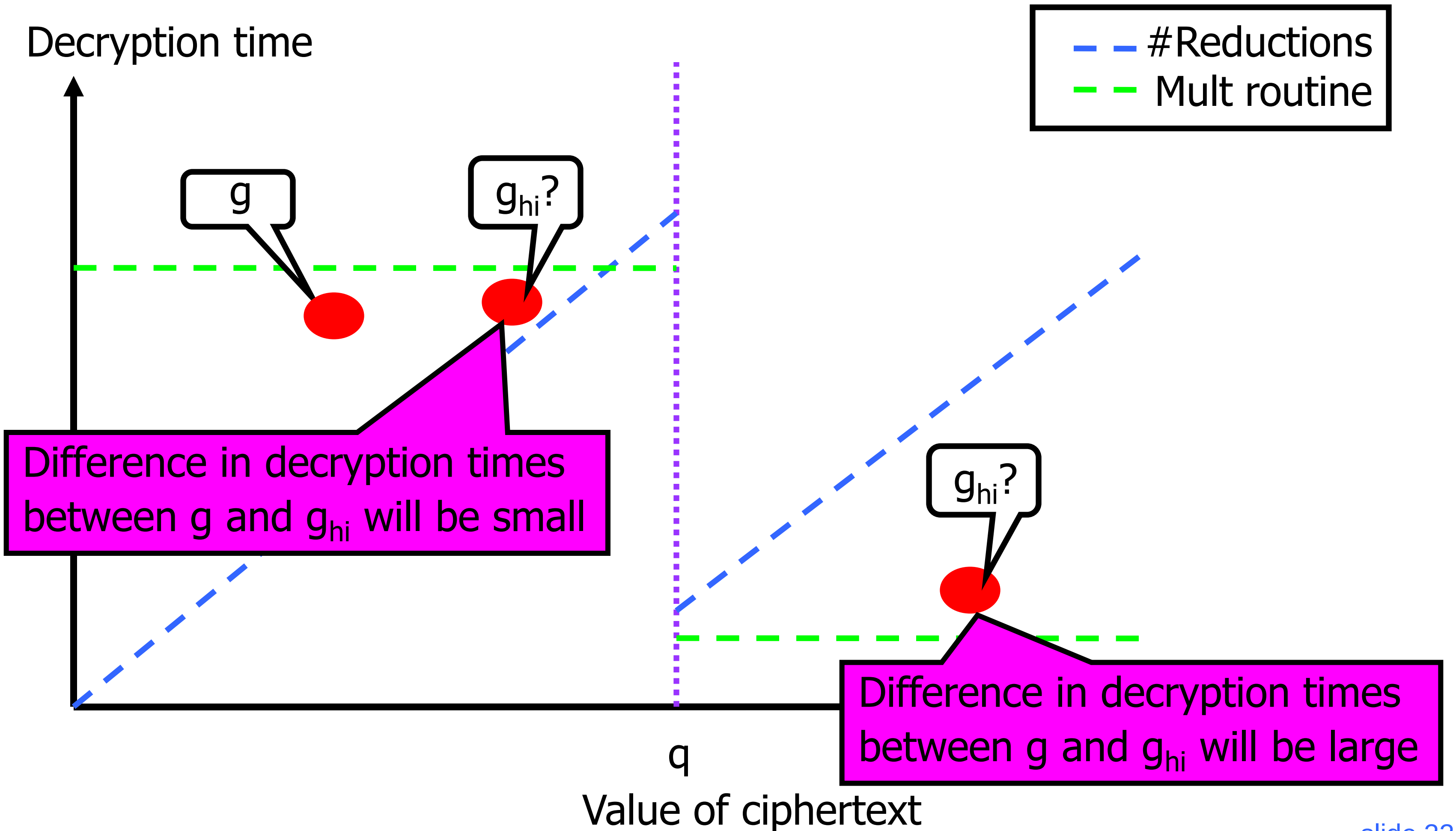
5. Record time t_2
Compute $t_2 – t_1$

Attack Overview

- Initial guess g for q between 2^{511} and 2^{512} (why?)
- Try all possible guesses for the top few bits
- Suppose we know i-1 top bits of q. Goal: i^{th} bit.
  - Set g =<known i-1 bits of q>000000
  - Set g_{hi}=<known i-1 bits of q>100000 - note: $g<g_{hi}$
    - If $g<q<g_{hi}$ then the i^{th} bit of q is 0
    - If $g<g_{hi}<q$ then the i^{th} bit of q is 1
- Goal: decide whether $g<q<g_{hi}$ or $g<g_{hi}<q$



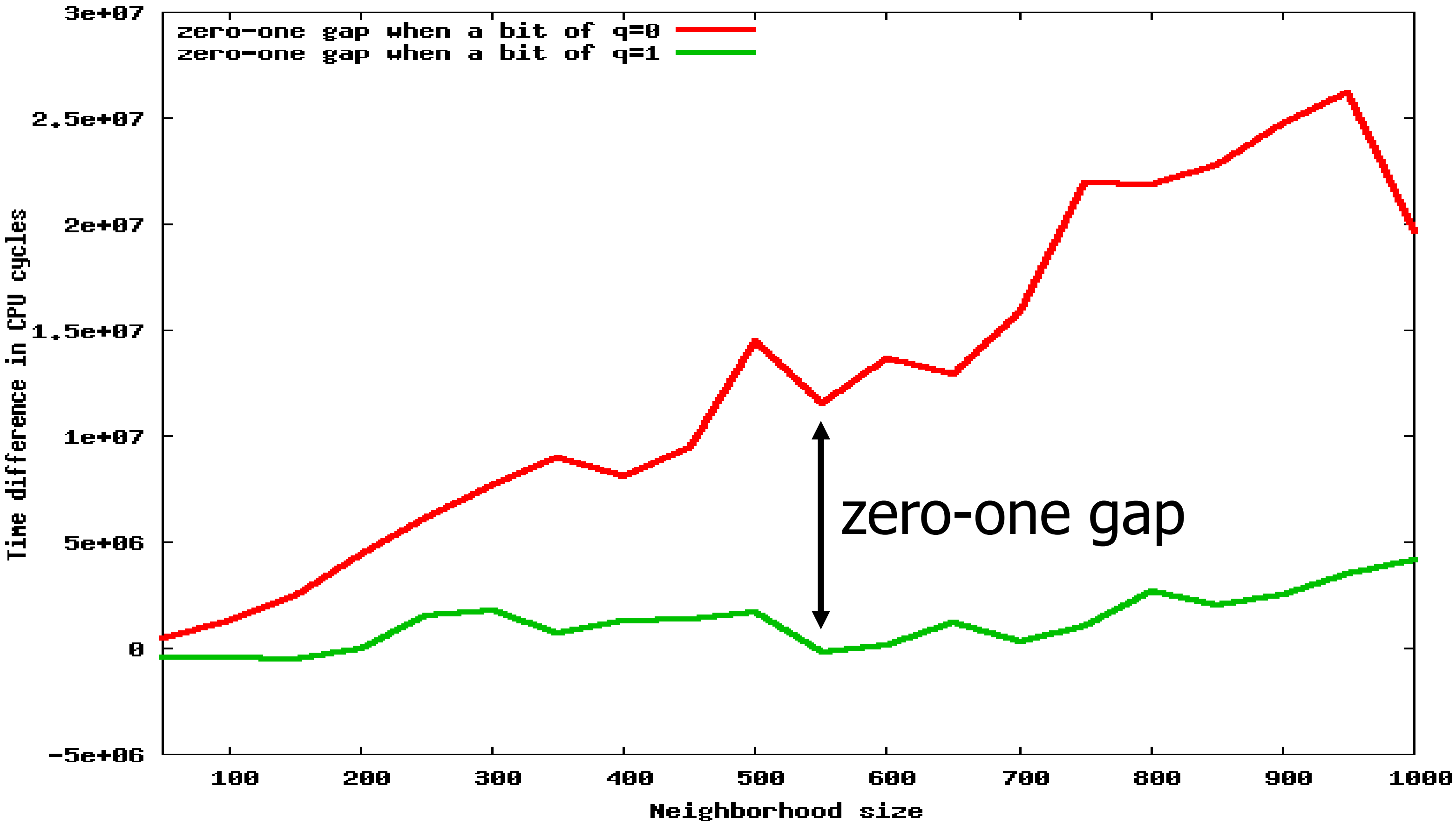
Two Possibilities for g_{hi}



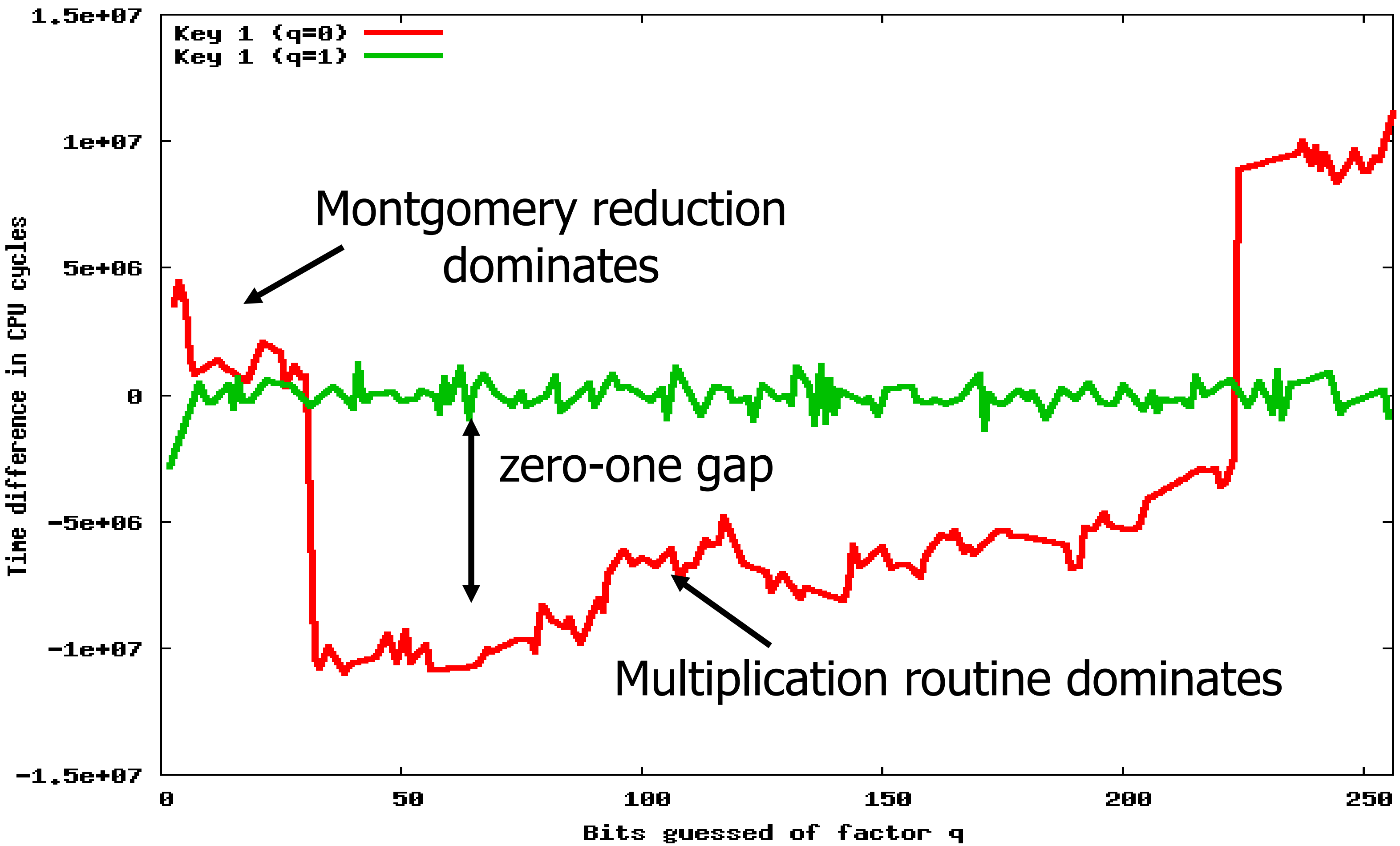
Timing Attack Details

- What is "large" and "small"?
  - Know from attacking previous bits
- Decrypting just g does not work because of sliding windows
  - Decrypt a neighborhood of values near g
  - Will increase difference between large and small values, resulting in larger 0-1 gap
- Attack requires only 2 hours, about 1.4 million queries to recover the private key
  - Only need to recover most significant half bits of q

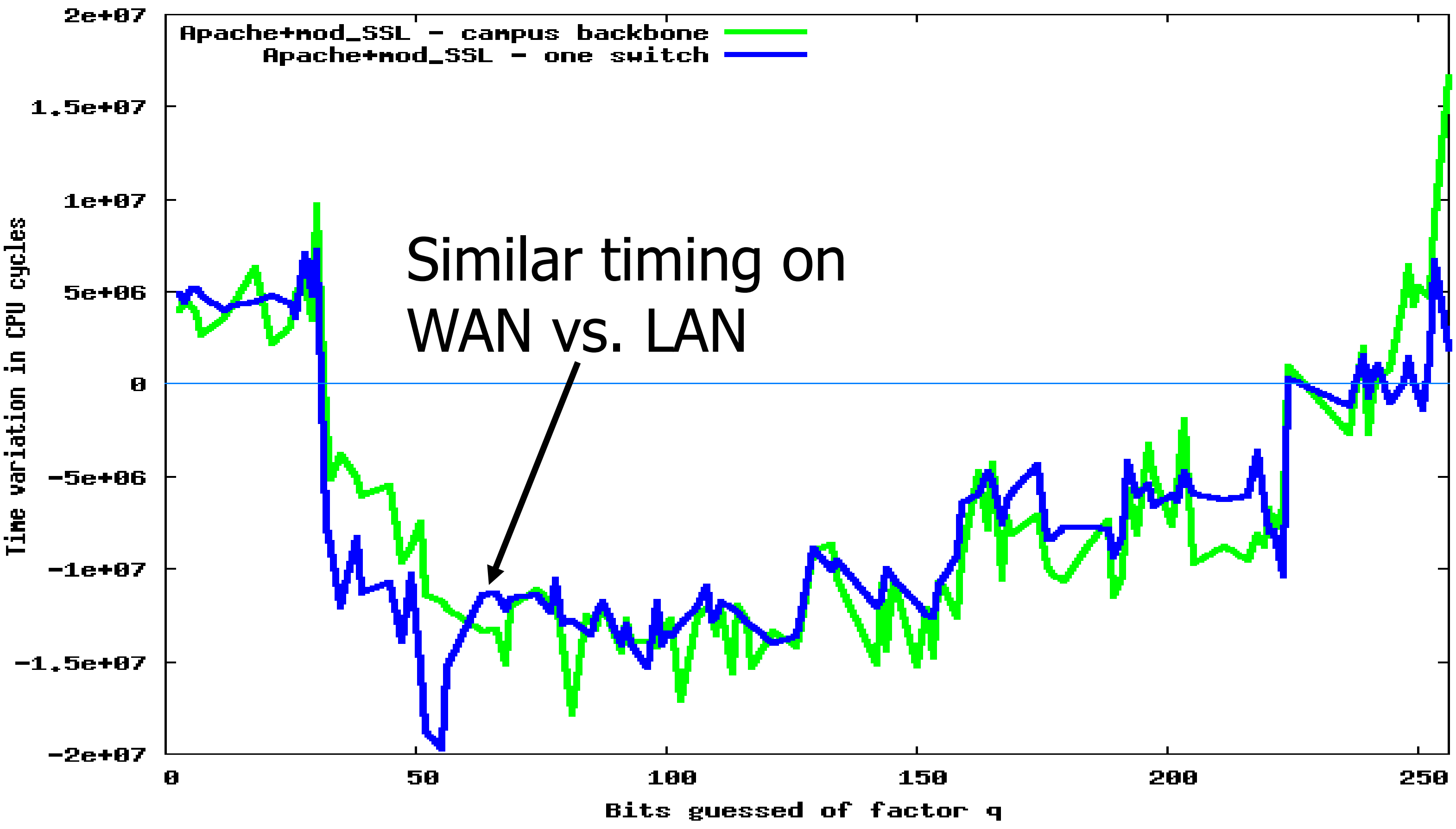
Impact of Neighborhood Size



Extracting RSA Private Key



Works On The Internet



Defenses

- Bad: require statically that all decryptions take the same time
  - For example, always do the extra “dummy” reduction
  - … but what if compiler optimizes it away?
- Bad: dynamically make all decryptions the same or multiples of the same time “quantum”
  - Now all decryptions have to be as slow as the slowest decryption
- Good: Use RSA blinding

RSA Blinding

- Instead of decrypting ciphertext c, decrypt a random ciphertext related to c
  - Compute $x' = c*r^e \bmod n$, r is random
  - Decrypt x' to obtain m'
  - Calculate original plaintext $m = m'/r \bmod n$
- Since r is random, decryption time is random
- 2-10% performance penalty

Blinding Works