CS 380S

0x1A Great Papers in Computer Security

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Secure Multi-Party Computation

- General framework for describing computation between parties who do not trust each other
- Example: elections
  - N parties, each one has a "Yes" or "No" vote
  - Goal: determine whether the majority voted "Yes", but no voter should learn how other people voted
- Example: auctions
  - Each bidder makes an offer
  - Goal: determine whose offer won without revealing losing offers

More Examples

- Example: distributed data mining
  - Two companies want to compare their datasets without revealing them
    - For example, compute the intersection of two customer lists
- Example: database privacy
  - Evaluate a query on the database without revealing the query to the database owner
  - Evaluate a statistical query without revealing the values of individual entries

A Couple of Observations

- We are dealing with distributed multi-party protocols
  - "Protocol" describes how parties are supposed to exchange messages on the network
- All of these tasks can be easily computed by a trusted third party
  - Secure multi-party computation aims to achieve the same result without involving a trusted third party

How to Define Security?

- Must be mathematically rigorous
- Must capture all realistic attacks that a malicious participant may try to stage
- Should be “abstract”
  - Based on the desired “functionality” of the protocol, not a specific protocol
  - Goal: define security for an entire class of protocols

Functionality

- K mutually distrustful parties want to jointly carry out some task
- Model this task as a "functionality"

$f: (\{0,1\}^*)^K \rightarrow (\{0,1\}^*)^K$

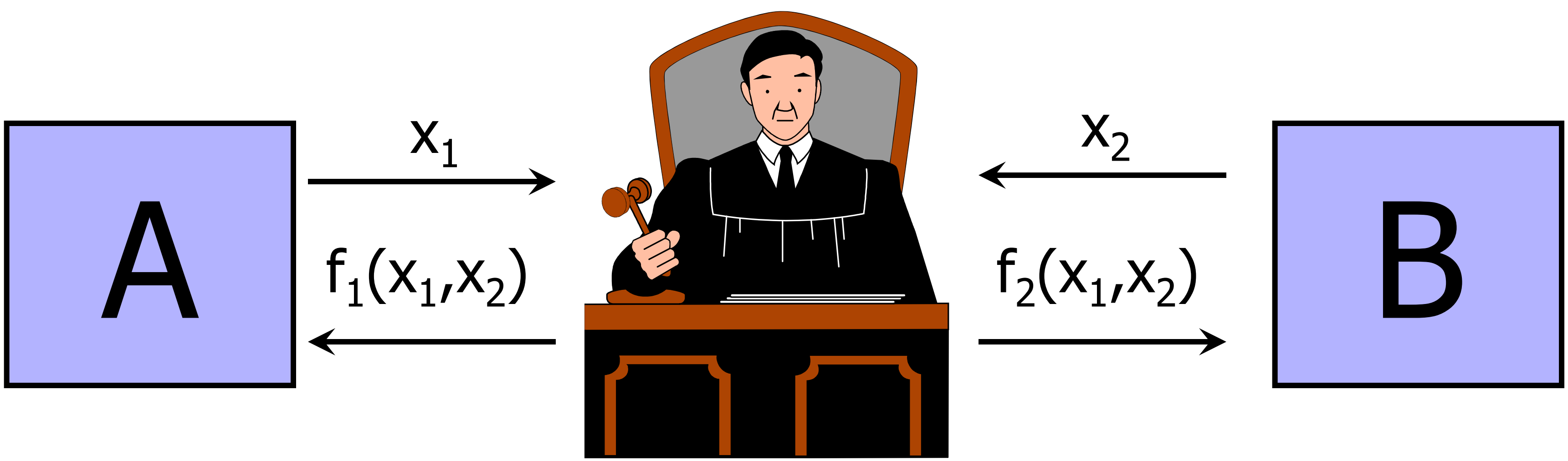
K inputs (one per party); each input is a bitstring

K outputs

- Assume that this functionality is computable in probabilistic polynomial time

Ideal Model

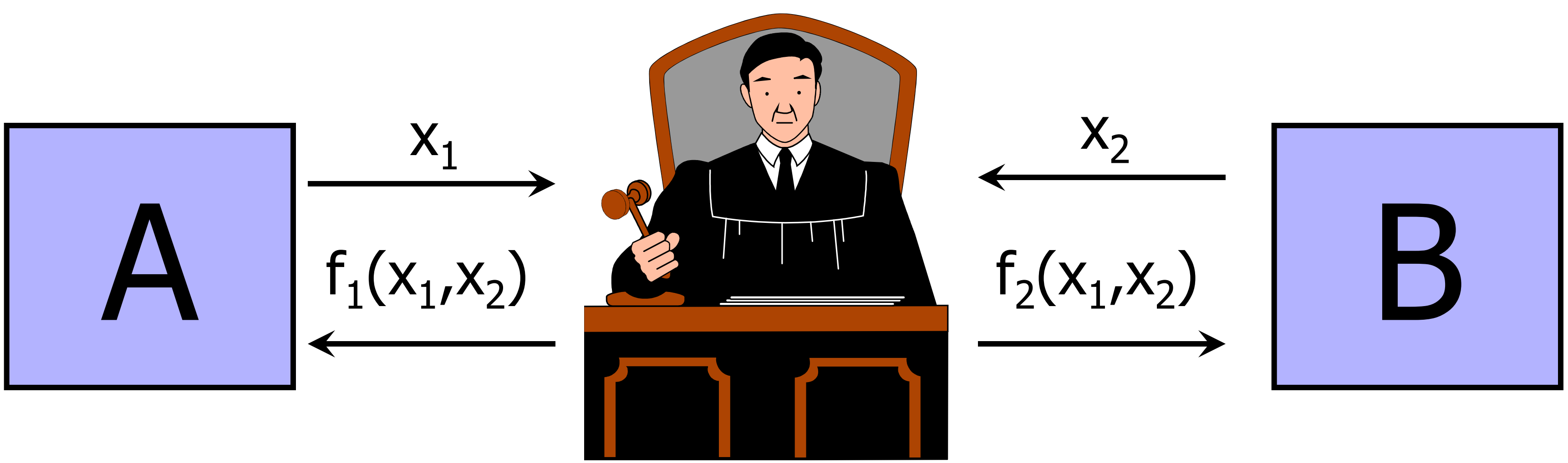
- Intuitively, we want the protocol to behave "as if" a trusted third party collected the parties' inputs and computed the desired functionality
  - Computation in the ideal model is secure by definition!



Slighty More Formally

- A protocol is secure if it emulates an ideal setting where the parties hand their inputs to a "trusted party," who locally computes the desired outputs and hands them back to the parties

[Goldreich-Micali-Wigderson 1987]



Adversary Models

- Some participants may be dishonest (corrupt)
  - If all were honest, we would not need secure multi-party computation
- Semi-honest (aka passive; honest-but-curious)
  - Follows protocol, but tries to learn more from received messages than he would learn in the ideal model
- Malicious
  - Deviates from the protocol in arbitrary ways, lies about his inputs, may quit at any point
- For now, focus on semi-honest adversaries and two-party protocols

Correctness and Security

- How do we argue that the real protocol "emulates" the ideal protocol?
- Correctness
  - All honest participants should receive the correct result of evaluating functionality f
    - Because a trusted third party would compute f correctly
- Security
  - All corrupt participants should learn no more from the protocol than what they would learn in the ideal model
  - What does a corrupt participant learn in ideal model?
    - His own input and the result of evaluating f

Simulation

- Corrupt participant's view of the protocol = record of messages sent and received
  - In the ideal world, this view consists simply of his input and the result of evaluating f
- How to argue that real protocol does not leak more useful information than ideal-world view?
- Key idea: simulation
  - If real-world view (i.e., messages received in the real protocol) can be simulated with access only to the ideal-world view, then real-world protocol is secure
  - Simulation must be <u>indistinguishable</u> from real view

Technicalities

- Distance between probability distributions A and B over a common set X is

  $\frac{1}{2} * \text{sum}_X(|\Pr(A=x) - \Pr(B=x)|)$

- Probability ensemble A_i is a set of discrete probability distributions
  - Index i ranges over some set I
- Function f(n) is negligible if it is asymptotically smaller than the inverse of any polynomial

  $\forall$ constant c $\exists$m such that $|f(n)| < 1/n^c \;\forall n>m$

Indistinguishability Notions

- Distribution ensembles A_i and B_i are equal
- Distribution ensembles A_i and B_i are statistically close if $dist(A_i,B_i)$ is a negligible function of i
- Distribution ensembles A_i and B_i are computationally indistinguishable ($A_i \approx B_i$) if, for any probabilistic polynomial-time algorithm D, $|Pr(D(A_i)=1) - Pr(D(B_i)=1)|$ is a negligible function of i
  - No efficient algorithm can tell the difference between A_i and B_i except with a negligible probability

SMC Definition (First Attempt)

- Protocol for computing $f(X_A,X_B)$ betw. A and B is secure if there exist efficient simulator algorithms S_A and S_B such that for all input pairs (x_A,x_B) …
- Correctness: $(y_A,y_B) \approx f(x_A,x_B)$
  - Intuition: outputs received by honest parties are indistinguishable from the correct result of evaluating f
- Security: $view_A(\text{real protocol}) \approx S_A(x_A,y_A)$

  $view_B(\text{real protocol}) \approx S_B(x_B,y_B)$
  - Intuition: a corrupt party's view of the protocol can be simulated from its input and output
- This definition does not work! Why?

Randomized Ideal Functionality

- Consider a coin flipping functionality

  f()=(b,-) where b is random bit

  - f() flips a coin and tells A the result; B learns nothing
- The following protocol "implements" f()
  1. A chooses bit b randomly
  2. A sends b to B
  3. A outputs b
- It is obviously insecure (why?)
- Yet it is correct and simulatable according to our attempted definition (why?)

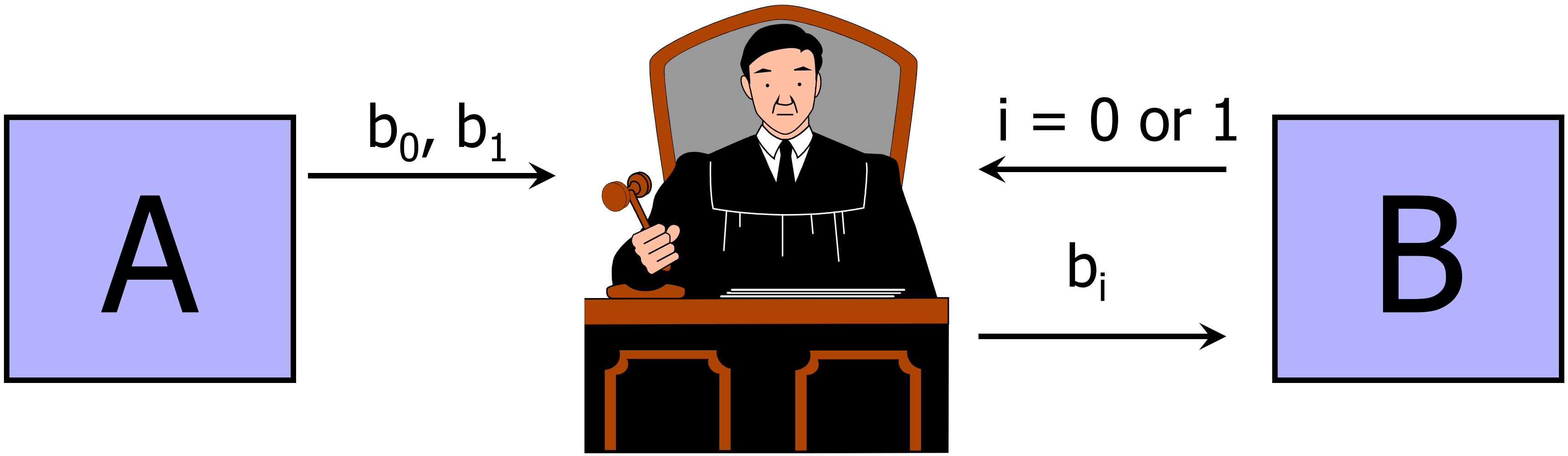
SMC Definition

- Protocol for computing $f(X_A,X_B)$ betw. A and B is secure if there exist efficient simulator algorithms S_A and S_B such that for all input pairs (x_A,x_B) …
- Correctness: $(y_A,y_B) \approx f(x_A,x_B)$
- Security: $(\text{view}_A(\text{real protocol}), y_B) \approx (S_A(x_A,y_A), y_B)$
  $(\text{view}_B(\text{real protocol}), y_A) \approx (S_B(x_B,y_B), y_A)$
  - Intuition: if a corrupt party's view of the protocol is correlated with the honest party's output, the simulator must be able to capture this correlation
- Does this fix the problem with coin-flipping f?

Oblivious Transfer (OT)

[Rabin 1981]

- Fundamental SMC primitive



- A inputs two bits, B inputs the index of one of A's bits
- B learns his chosen bit, A learns nothing
  - A does not learn which bit B has chosen; B does not learn the value of the bit that he did not choose
- Generalizes to bitstrings, M instead of 2, etc.

One-Way Trapdoor Functions

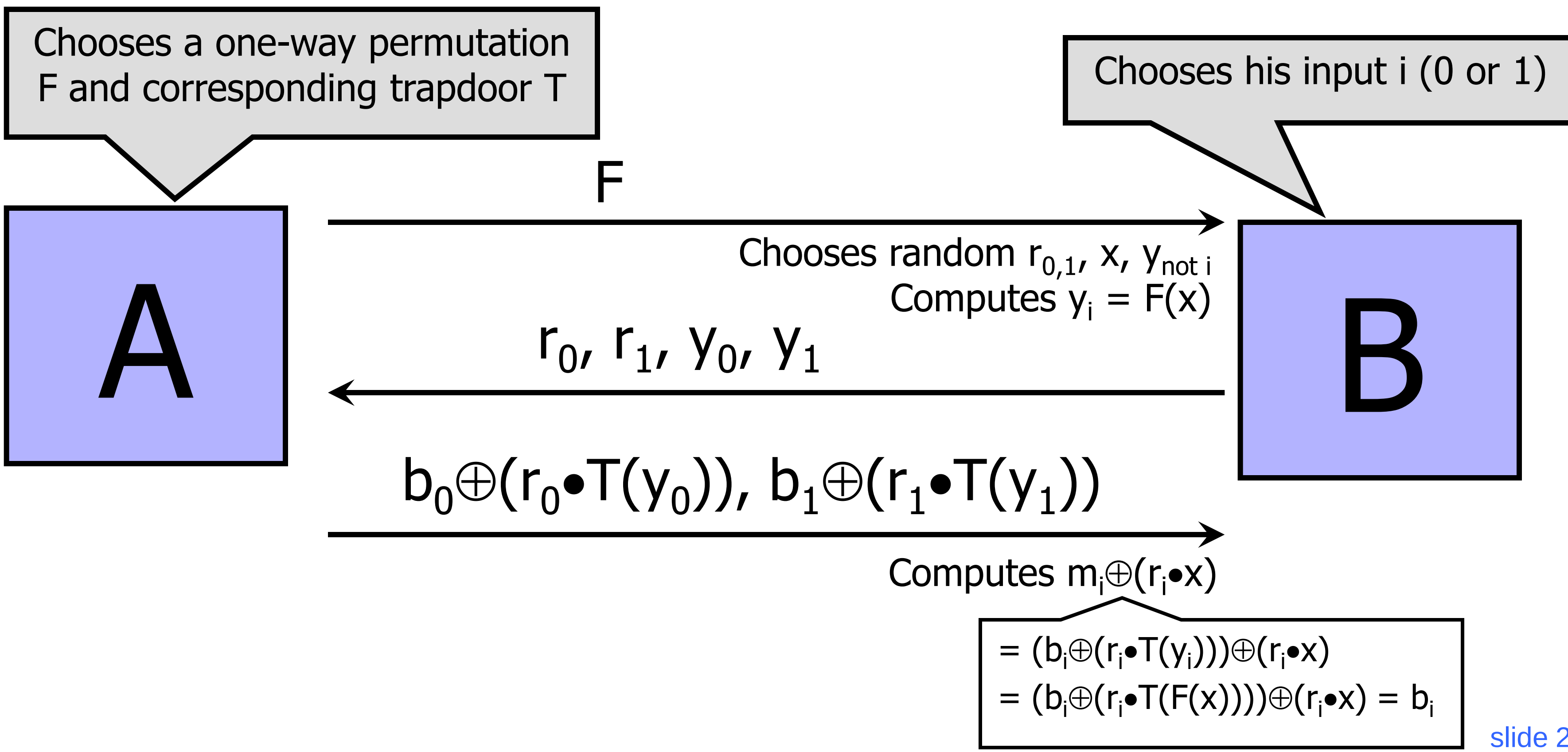
- Intuition: one-way functions are easy to compute, but hard to invert (skip formal definition)
  - We will be interested in one-way permutations
- Intution: one-way trapdoor functions are one-way functions that are easy to invert given some extra information called the trapdoor
  - Example: if $n=pq$ where p and q are large primes and e is relatively prime to $\varphi(n)$, $f_{e,n}(m) = m^e \bmod n$ is easy to compute, but it is believed to be hard to invert
  - Given the trapdoor d s.t. $de=1 \bmod \varphi(n)$, $f_{e,n}(m)$ is easy to invert because $f_{e,n}(m)^d = (m^e)^d = m \bmod n$

Hard-Core Predicates

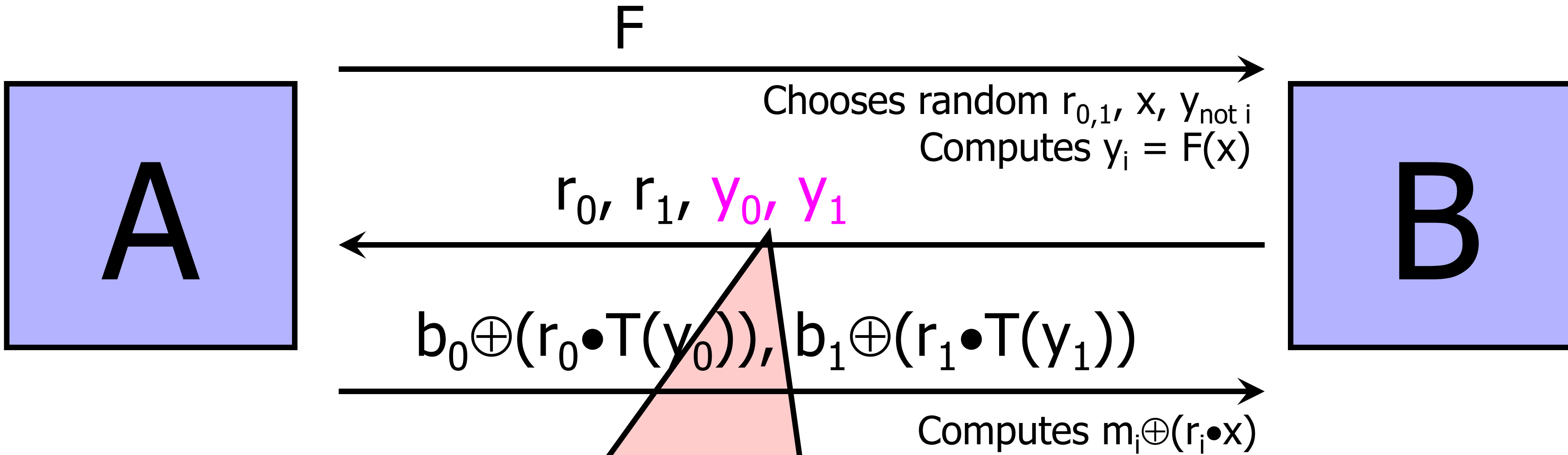
- Let f: S→S be a one-way function on some set S
- B: S→{0,1} is a hard-core predicate for f if
  - Intuition: there is a bit of information about x such that learning this bit from f(x) is as hard as inverting f
  - B(x) is easy to compute given $x \in S$
  - If an algorithm, given only f(x), computes B(x) correctly with prob > ½+ε, it can be used to invert f(x) easily
    - Consequence: B(x) is hard to compute given only f(x)
- Goldreich-Levin theorem
  - B(x,r)=r•x is a hard-core predicate for g(x,r) = (f(x),r)
    - f(x) is any one-way function, $r \bullet x = (r_1 x_1) \oplus \ldots \oplus (r_n x_n)$

Oblivious Transfer Protocol

- Assume the existence of some family of one-way trapdoor permutations



Proof of Security for B

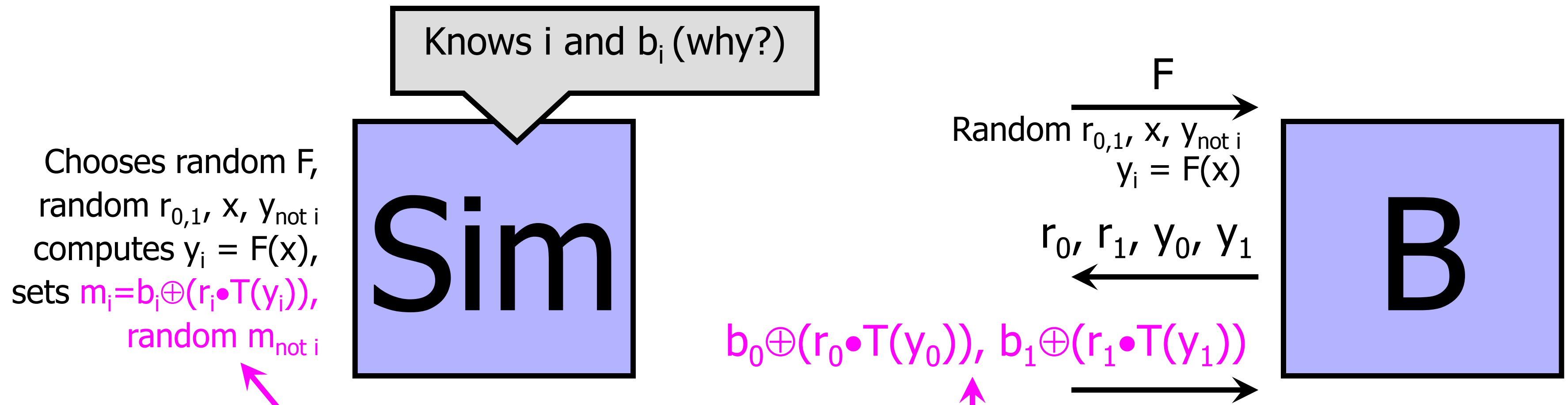


y_0 and y_1 are uniformly random regardless of A's choice of permutation F (why?)

Therefore, A's view is independent of B's input i.

Proof of Security for A (Sketch)

- Need to build a simulator whose output is indistinguishable from B's view of the protocol



The only difference between simulation and real protocol:

In simulation, $m_{not\ i}$ is random (why?)

In real protocol, $m_{not\ i}=b_{not\ i}\oplus(r_{not\ i}\bullet T(y_{not\ i}))$

Proof of Security for A (Cont'd)

- Why is it computationally infeasible to distinguish random m and $m'=b\oplus(r\bullet T(y))$?
  - b is some bit, r and y are random, T is the trapdoor of a one-way trapdoor permutation
- $(r\bullet x)$ is a hard-core bit for $g(x,r)=(F(x),r)$
  - This means that $(r\bullet x)$ is hard to compute given $F(x)$
- If B can distinguish m and $m'=b\oplus(r\bullet x')$ given only $y=F(x')$, we obtain a contradiction with the fact that $(r\bullet x')$ is a hard-core bit
  - Proof omitted

Naor-Pinkas Oblivious Transfer

Setting: order-q subgroup of Z*p, p is prime, q divides p-1
g is a generator group for which CDH assumption holds

Messages m_0 and m_1

Choice: bit σ

S

C

Chooser does not know discrete log of C

C

Chooses random k

Sets $PK_\sigma = g^k$, $PK_{1-\sigma} = C/PK_\sigma$

PK_0

Chooses random r, computes PK_1

g^r, $m_0 \oplus Hash((PK_0)^r, 0)$, $m_1 \oplus Hash((PK_1)^r, 1)$

Chooser knows discrete log either for PK_0, or for PK_1, but not both

Computes $(g^r)^k = (PK_\sigma)^r$ and decrypts m_σ

Chooser does not know the discrete log of $PK_{1-\sigma}$, thus cannot distinguish between a random value g_z and $(PK_{1-\sigma})^r$ - **why?**

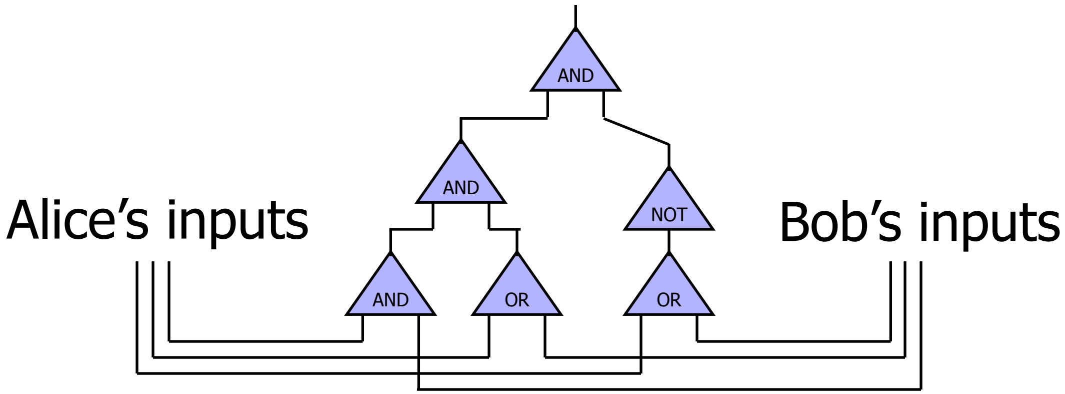
A. Yao

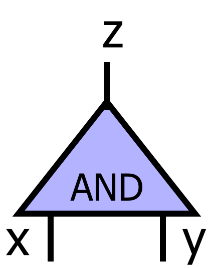
Protocols for Secure Computations

(FOCS 1982)

Yao's Protocol

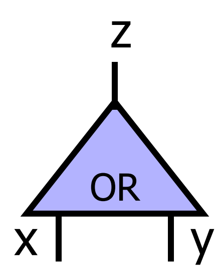
- Compute any function securely
  - … in the semi-honest model; can be extended to malicious
- First, convert the function into a boolean circuit





Truth table:

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

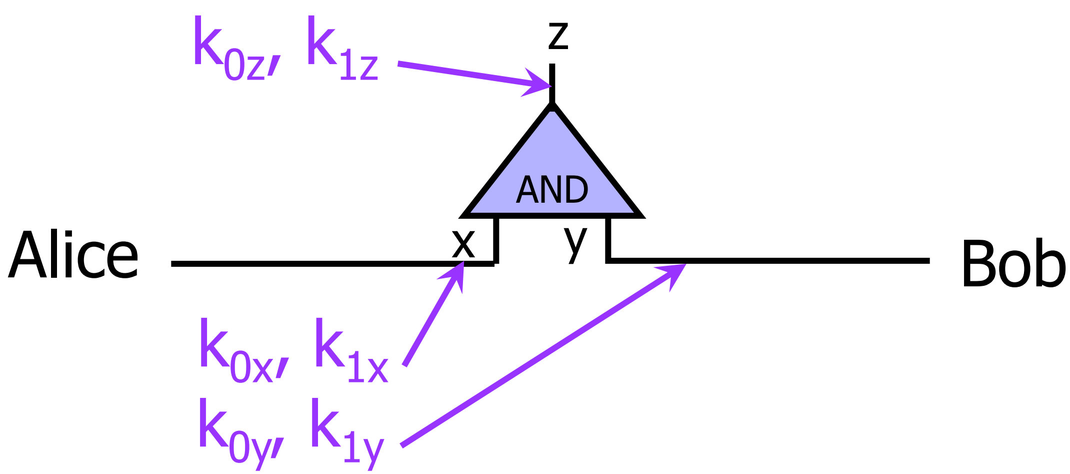


Truth table:

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

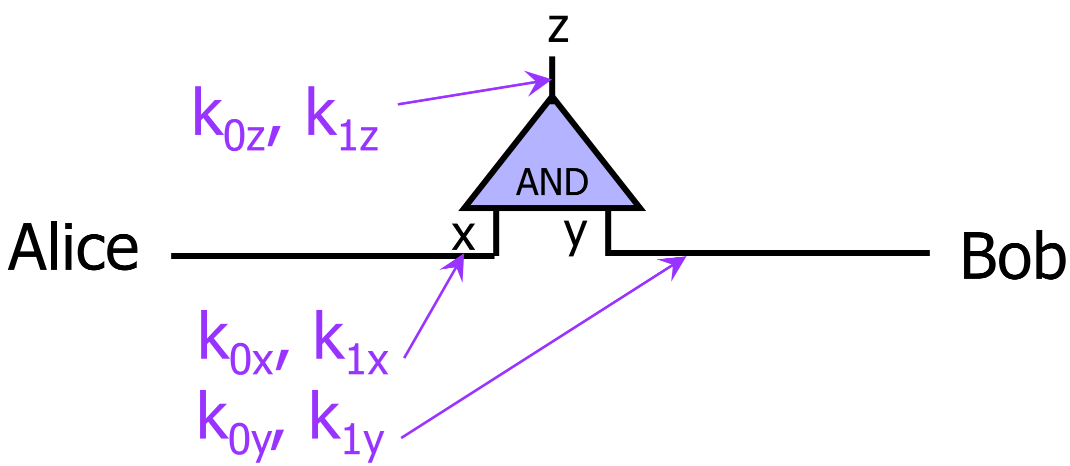
1: Pick Random Keys For Each Wire

- Evaluate <u>one gate</u> securely
  - Later generalize to the entire circuit
- Alice picks two random keys for each wire
  - One key corresponds to "0", the other to "1"
  - 6 keys in total for a gate with 2 input wires



2: Encrypt Truth Table

- Alice encrypts each row of the truth table by encrypting the output-wire key with the corresponding pair of input-wire keys



Original truth table:

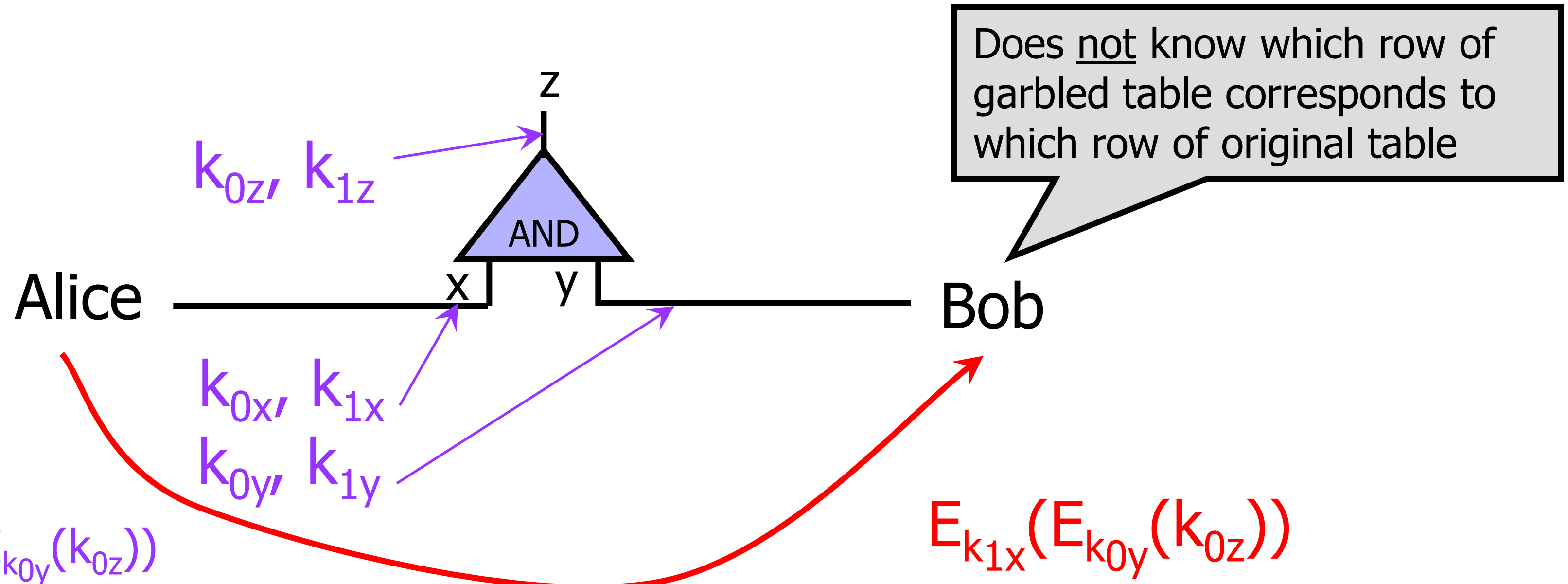
x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

Encrypted truth table:

$$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$$
$$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$$
$$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$$
$$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$$

3: Send Garbled Truth Table

- Alice randomly permutes (“garbles”) encrypted truth table and sends it to Bob



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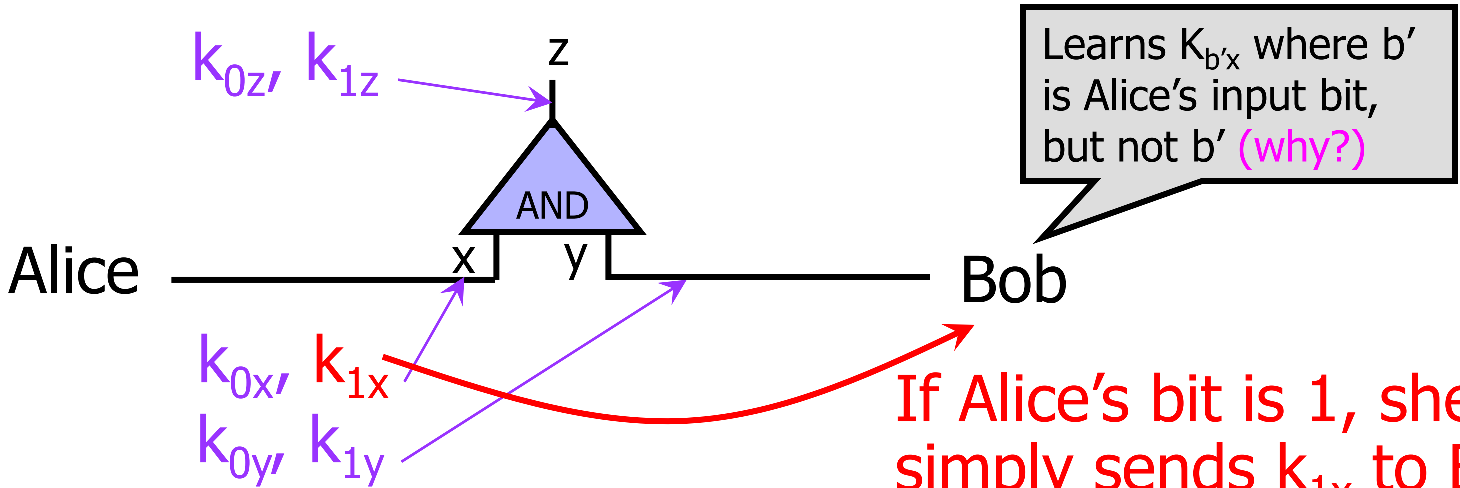
Garbled truth table:

$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$
$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$
$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$
$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$

4: Send Keys For Alice's Inputs

- Alice sends the key corresponding to her input bit
  - Keys are random, so Bob does not learn what this bit is

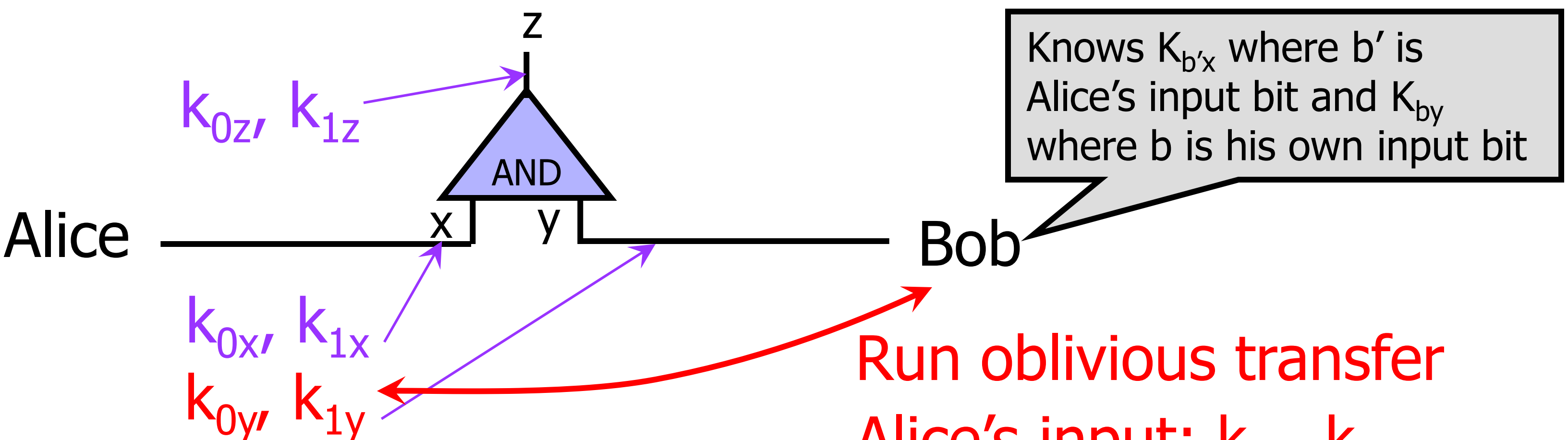


If Alice's bit is 1, she simply sends k_{1x} to Bob; if 0, she sends k_{0x}

Garbled truth table:
$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$
$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$
$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$
$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$

5: Use OT on Keys for Bob's Input

- Alice and Bob run oblivious transfer protocol
  - Alice's input is the two keys corresponding to Bob's wire
  - Bob's input into OT is simply his 1-bit input on that wire



Garbled truth table: $E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$

$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$

$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$

$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$

Run oblivious transfer

Alice's input: k_{0y}, k_{1y}

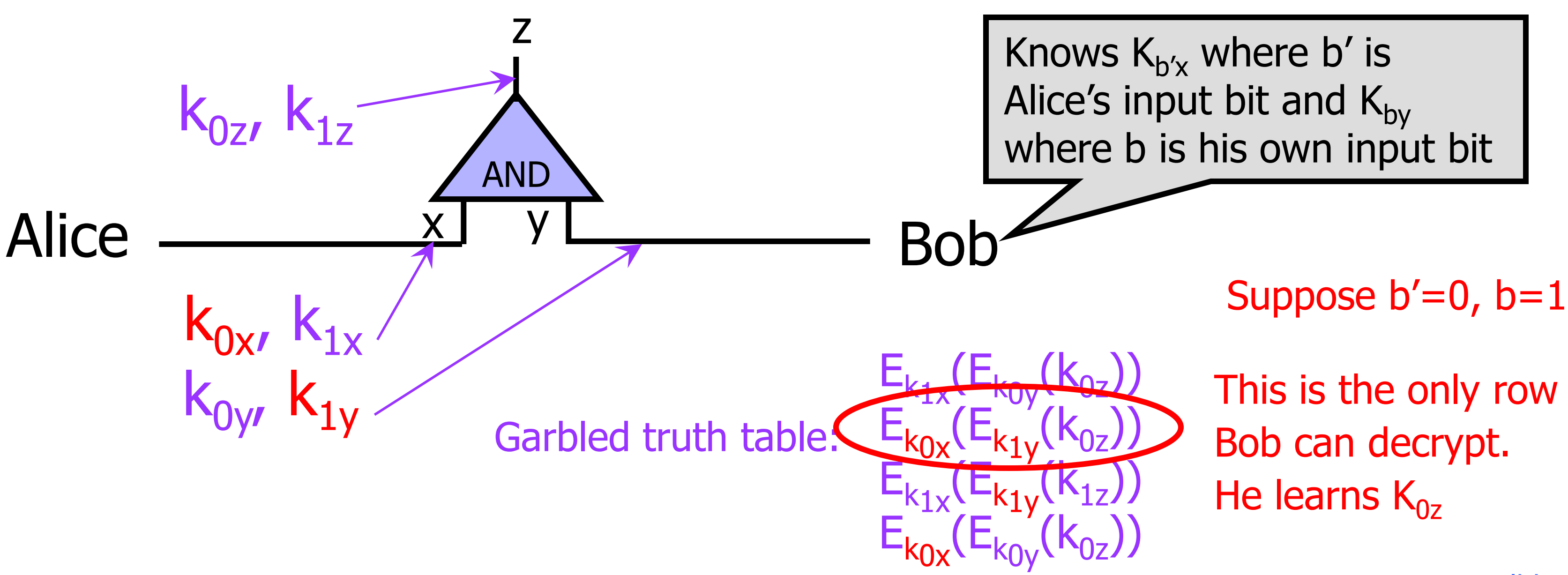
Bob's input: his bit b

Bob learns k_{by}

What does Alice learn?

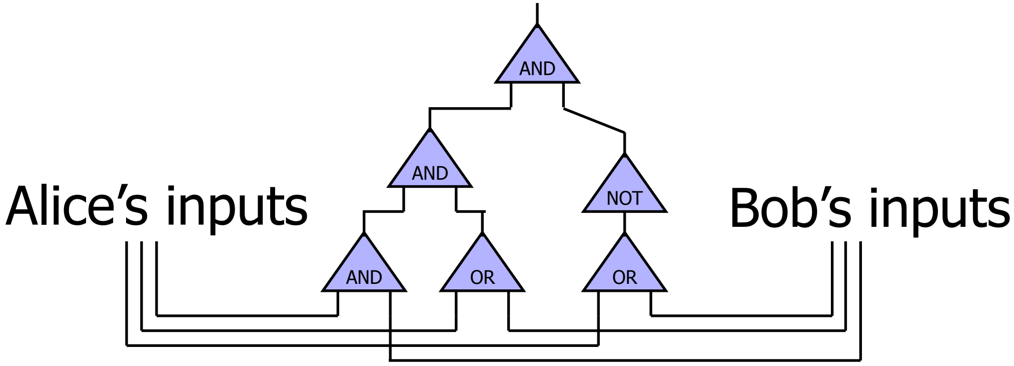
6: Evaluate One Garbled Gate

- Using the two keys that he learned, Bob decrypts exactly one of the output-wire keys
  - Bob does not learn if this key corresponds to 0 or 1
    - Why is this important?



7: Evaluate Entire Circuit

- In this way, Bob evaluates entire garbled circuit
  - For each wire in the circuit, Bob learns only one key
  - It corresponds to 0 or 1 (Bob does not know which)
    - Therefore, Bob does not learn intermediate values (why?)



- Bob tells Alice the key for the final output wire and she tells him if it corresponds to 0 or 1
  - Bob does not tell her intermediate wire keys (why?)

Brief Discussion of Yao's Protocol

- Function must be converted into a circuit
  - For many functions, circuit will be huge (can use BDD)
- If m gates in the circuit and n inputs, then need 4m encryptions and n oblivious transfers
  - Oblivious transfers for all inputs can be done in parallel
- Yao's construction gives a constant-round protocol for secure computation of any function in the semi-honest model
  - Number of rounds does not depend on the number of inputs or the size of the circuit!