Locking and Equality Atom Term Locking for Resolution and Paramodulation

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Abstract

Two restrictions of resolution and paramodualtion, named locking and equality atom term locking, are defined and shown to be complete. Locking restricts the number of new resolvents and paramodulants produced at each round by assigning a positive integer, called the index, to each literal of each clause and requiring that resolution and paramodulation be done only with literals which are of minimal index among the indices of their respective clauses. Resolvents and paramodulants are locked hereditarily in the sense that if two literals merge as the result of a resolution or paramodulation then the merged literal is assigned the minimal index of

its parents. Equality atom term locking restricts the number of paramodulants produced at each round in a manner analogous to locking. In addition, a new proof of the ground completeness of paramodulation is established without using the maximal model theorem.

1 Introduction

Several strategies for resolution and apramodulation, such as set of support [1], linearity [2], and renamable paramodulation [3], have been investigated with the hope that these or other restrictions would ultimately lead to an efficient theorem prover for the first order logic with equality. In this paper we present two strategies for resolution and paramodulation which are compatible and so may be used as a combination strategy. We assume the reader is familiar with the basic terminology of resolution and paramodulation found in [4,5], as well as the papers mentioned above.

2 Ground Resolution and Paramodulation

When R. Anderson and W. W. Bledsoe [6] established the ground completeness of resolution and paramodulation with the excess literal parameter method, they remarked that the method reduced the need for the maximal model theorem to the ground unit case [46]. By constructing a maximal model for a suitable set of ground unit clauses we show that the dependence upon the maximal model theorem can be eliminated entirely.

<u>Theorem 1</u> If S is a finite, R-unsatisfiable set of ground clauses then there exists a deduction of $\square$ or $\{t_1 \neq t_1, \ldots, t_m \neq t_m\}$ from S, where $t_1, \ldots, t_m$ are in the Herbrand universe of S.

<u>Proof</u> The only case of the excess literal parameter method which is not straightforward is the unit case, so let us assume S consists entirely of unit clauses. We argue by contradiction, assuming that for each non-negative integer n (1) $\square \notin RP_n(S)$ and (2) $\{t \neq t\} \notin RP_n(S)$ for each term t in the Herbrand universe of S (throughout, $RP_n(S)$ denotes S together with the set of all clauses generated from S after n rounds of resolution and paramodulation). Let $S' = \bigcup_{i=1}^{\infty} RP_i(S)$, let $S'' = S' \cup \{\{t = t\} \mid t \text{ is in the Herbrand universe of } S\}$, let $S''' = \bigcup_{i=1}^{\infty} RP_i(S'')$, and let $M = (\bigcup_{X \in S'''} X) \cup (N - C)$ where N is the negative interpretation of S''' and where C is the set of all complements of positive members of $\bigcup_{X \in S'''} X$. We will show that M is an R-model of S''',

hence an R-model of S, violating the R-unsatisfiability of S.

We now establish that M is a model of S''' by demonstrating that (3) M does not contain a complementary pair, and (4) $C \cap M$ is not empty for each clause C of S'''. If M did contain a complementary pair then, because of the definition of M, the pair would be in $\bigcup_{X \in S'''} X$. It can be shown that each of the pair has a linear deduction from S'' such that all the side clauses are members of S''. If either deduction has side clauses which are members of $\{\{t = t\} \mid t \text{ is in the Herbrand universe of } S\}$ then new linear deductions can be constructed by deleting those side clauses. If both top clauses are members of S' then the two new deductions are deductions from S, violating (1). If the top clause of one of the deductions is $\{t = t\}$, where t is in the Herbrand universe of S, then it can be shown that $\{t \neq t\}$ can be deduced from S, violating (2). This completes the proof of (3). For (4), notice that because of the definition of M, $C \cap M = C$ for any clause C of S'''. If C were empty then $\square$ would have been a member of S''', which is impossible because of (3).

To complete the proof that M is an R-model of S''' it suffices to show that if x and $t_1 = t_2$ are members of M and x' is the result of replacing one occurrence of t_1 in x by t_2 then x' is a member of M. Let $x \in M$ and observe that either $x \in \bigcup_{X \in S'''} X$ or $x \in N - C$. In the first case, because S is closed under paramodulation, if $t_1 = t_2$ is in M then x' is in $\bigcup_{X \in S'''} X$, hence in M. In the second case x and hence x' are negations of atoms. Let x be the negation of the atom a and assume $x' \in M$. Since M is a model, $a \in M$, hence $a \in \bigcup_{X \in S'''} X$. Let a' be the result of replacing by t_1 in a the corresponding occurrence of t_2 which replaced an occurrence of t_1 in x (when passing from x to x' earlier). Since S''' is closed under paramodulation, if $t_1 = t_2$ is in M then it follows that $a' \in \bigcup_{X \in S'''} X$. But now both a' and x are in M, and since a' and x are complements, (3) is contradicted. This completes the proof of Theorem 1.

It is intriguing to notice that in view of Theorem 1 the unit reflexive axiom, $\{x = x\}$, is the only reflexive axiom that must be added to an R-unsatisfiable set of ground clauses in order to insure refutation completeness. However, all known methods of lifting paramodulation (for example, see [1]) involve the addition of the functional reflexive axioms.

3 Locking

A locked literal is an ordered pair (x,i) where x is a literal and i is a positive integer (i is called the index of the literal 1), and a locked clause is a finite set of locked literals. Let MERGE be the function defined by $\mathrm{MERGE}(C) = \{(c,i) \mid i = \min\{k \mid (c,k) \in C\}$ and $(c,j) \in C$ for some $j\}$. (MERGE is the function whose domain is the set of all locked clauses, and which merges all locked literals $(c,k_1), \ldots, (c,k_n)$ of a given clause which have the same literal into a single locked literal (c,i) where i is the minimum of $k_1, \ldots k_n$.) R is a ground locked resolvent of C and D means $R = \mathrm{MERGE}(\,(C - \{(c,i)\}) \cup (D - \{(d,j)\})\,)$ where $(c,i) \in C$, $(d,j) \in D$, c and d are complements, i is a minimal index of the indices of C, j is a minimal index of the indices of D. P is a ground locked paramodulant of C by E means $P = \mathrm{MERGE}(\,(C - \{(c,i)\}) \cup \{(c',i)\} \cup (E - \{(t_1 = t_2, j)\})\,)$ where $(c,i) \in C$, $(t_1 = t_2, j) \in E$, c' is the result of replacing one occurrence of t_1 in c by t_2 or of replacing one occurrence of t_2 in c by t_1, i is a minimal index of the indices of C, and j is a minimal index of the indices of E. A locked R-refutation from S is a locked deduction of $\square$ from S where $\{x = x\}$ is allowed only to resolve. The following locked R-refutations may be compared with the P-hyper-refutations of [3].

<u>Example 1</u> $\{(\neg Qa,1),\ (\neg Sa,2),\ (\neg Ta,3),\ (a = b,4)\}$, $\{(Qa,n)\}$, $\{(Sa,m)\}$, $\{(Ta,p)\}$, $\{(fa \neq fb,q\}$, $\{(x = x,r)\}$ (the locking of unit clauses is irrelevant, so we have used arbitrary instead of specific integers to lock the unit clauses of these examples).

L_1: (1) $\{(\neg Sa,2),\ (\neg Ta,3),\ (a = b,4)\}$

L_2: (2) $\{(\neg Ta,3),\ (a = b,4)\}$

L_3: (3) $\{(a = b,4)\}$

L_4: (4) $\{(\neg Qb,1),\ (\neg Sa,2),\ (\neg Ta,3),\ (a = b,4)\}$

(5) $\{(Qb,n)\}$

(6) $\{(Sb,m)\}$

(7) $\{(Tb,p)\}$

(8) $\{(fa \neq fa,q)\}$

L_5: (9) $\square$

<u>Example 2</u> $\{(fa \neq fb,1),\ (Qc,7)\}$, $\{(fc \neq fd,2),\ (\neg Qc,3)\}$, $\{(\neg Qc,4),\ (c = d,5)\}$, $\{(a = b,6),\ (Qc,8)\}$, $\{(x = x,m)\}$

[illegible]: in this example, we [illegible] none in this example)

L_1: (1) $\{(fa \neq fa,1),\ (Qc,7)\}$

(2) $\{(fb \neq fb,1),\ (Qc,7)\}$

L_2: (3) $\{(Qc,7)\}$

(4) $\{(fb \neq fa,1),\ (Qc,7)\}$

L_3: (5) $\{(c = d,6)\}$

L_4: (6) $\{(fc \neq fc,2),\ (\neg Qc,3)\}$

(7) $\{(fd \neq fd,2),\ (\neg Qc,3)\}$

(8) $\{(\neg Qd,4),\ (c = d,5)\}$

(9) $\{(Qd,7)\}$

L_5: ~~(10) $\{(\neg Qc, 7)\}$~~

(10) $\{(\neg Qc, 3)\}$

(11) $\{(fd \neq fc, 2), (\neg Qc, 3)\}$

L_6: (12) $\square$

Although the locked R-refutations of these examples are more efficient than the corresponding P-hyper-refutations (9 clauses vs. 13 clauses, and 12 clauses vs. 29 clauses), it is not our intention to claim an overall increase in efficiency. We have no guarantee that the search is not forced so deep in order to find a locked R-refutation that the total number of clauses produced is greater than the number produced in the ordinary way.

<u>Theorem 2</u> If S is a finite, R-unsatisfiable set of ground clauses and S' is the set of clauses of S which have been locked in some way, together with $\{(x = x, n)\}$, then there is a locked R-refutation from S'.

<u>Proof</u> For the unit case of the excess literal parameter method, by Theorem 1, there is a deduction of $\square$ or $\{t \neq t\}$ from S. Since all the clauses of each deduction are unit clauses, the corresponding deductions from S' are locked. In the case of the deduction of $\{(t \neq t, m)\}$, it lock resolves with $\{(x = x, n)\}$ to produce $\square$.

For the induction step, it can be shown that there is a locked clause C with at least two members such that the index of one of the members (c,i) satisfies $i = \max\{j \mid$ there exist a clause $D \in S'$ and a literal d such that $(d,j) \in D\}$. Split S on the literal c and let S_1, S_1', S_2, and S_2' denote the respective splittings with $\{c\}$ in S_2. It is clear that S_1 and S_1' satisfy the induction hypothesis, so there is a locked R-refutation from S_1'. When $\{(c,i)\}$ is adjoined to the R-refutation the resulting deduction of $\square$ or $\{(c,i)\}$ remains locked. Since S_2 and S_2' also satisfy the induction hypothesis, there is a locked R-refutation from S_2'. If $\{(c,i)\}$ is not involved in the locked R-refutation from S_2' then that deduction is a locked R-refutation from S'. If $\{(c,i)\}$ is involved in the locked R-refutation from S_2' then then the combination of the deduction from S_2' with the deduction of $\{(c,i)\}$ from S_1' is a locked R-refutation from S'. This completes the proof of Theorem 2.

The lifting of locking is straightforward and so it will not be described here.

Locking is based on the simple idea that if one were to resolve or paramodulate only on one literal in each clause than the number of clauses deduced at each round would certainly be reduced. So it seems that the best use of locking is realized when each literal of each clause of the input set is assigned a distinct positive integer. We believe that a further increase in efficiency, reminiscent of hyper-deduction, is achieved by locking positive literals high and negative literals low (see Example 2).

4 Equality Atom Term Locking

Equality atom term locking restricts the number of paramodulants produced at each round by assigning an arbitrary symbol * to one or both terms of each equality atom of each clause and requiring that substitution be done only with those terms assigned the symbol *. Resolvents and paramodulants are term locked hereditarily in the sense that if two equality atoms merge as the result of a resolution ar paramodualtion then the merged literal has the symbol * assigned to a term iff at least one of the parents had the symbol * assigned to its corresponding term.

A <u>term locked equality atom</u> is a string $t_1 *= t_2$, $t_1 =* t_2$, or $t_1 *=* t_2$ where t_1 and t_2 are terms. A <u>term locked literal</u> is an ordinary literal (other than an ordinary equality atom) or a term locked equality atom, and a <u>term locked clause</u> is a finite set of term locked literals. Let MRG be the function defined by $MRG(C) = (C - A) \cup A_1 \cup A_2 \cup A_3$ where C is a term locked clause, A is the set of all term locked equality atoms of C, $A_1 = \{t_1 \ {*}{=}\ t_2 \mid t_1 \ {*}{=}\ t_2$ is a member of A and neither $t_1 \ {=}{*}\ t_2$ or $t_1 \ {*}{=}{*}\ t_2$ is a member of $A\}$, $A_2 = \{t_1 \ {=}{*}\ t_2 \mid t_1 \ {=}{*}\ t_2$ is a member of A and neither $t_1 \ {*}{=}\ t_2$ or $t_1 \ {*}{=}{*}\ t_2$ is a member of $A\}$, and $A_3 = \{t_1 \ {*}{=}{*}\ t_2 \mid t_1 \ {*}{=}{*}\ t_2$ is a member of A or $t_1 \ {*}{=}\ t_2$ and $t_1 \ {=}{*}\ t_2$ are members of $A\}$. R is a ground term locked resolvent of C and D means $R = MRG(\ (C - \{c\}) \cup (D - \{d\})\ )$ where $c \in C$, $d \in D$, and c and d are complements. P is a ground term locked paramodulant of C by E means $P = MRG(\ (C - \{c\}) \cup \{c'\} \cup (E - \{e\})$ where $c \in C$, $e \in E$, and (1) e is $t_1 \ {*}{=}\ t_2$ or $t_1 \ {*}{=}{*}\ t_2$ and c' is the result of replacing one occurrence of t_1 in c by t_2, or (2) e is $t_1 \ {=}{*}\ t_2$ or $t_1 \ {*}{=}{*}\ t_2$ and c' is the result of replacing one occurrence of t_2 in c by t_1. A term locked R-refutation from $S \cup \{\{x=x\}\}$ is a term locked deduction of $\square$ from S where $\{x = x\}$ is allowed only to resolve. Since it will be shown that locking and equality atom term locking are compatible, the following example is Example 2 which has in addition been term locked.

Example 3 $\{(fa \neq fb,1), (Qc,7)\}$, $\{(fc \neq fd,2), (Qc,3)\}$, $\{(\neg Qc,4), (c \ {*}{=}\ d,5)\}$, $\{(a \ {*}{=}\ b,6), (Qc,8)\}$, $\{(x = x,m)\}$.

TL_1: (1) $\{(fb \neq fb,1), (Qc,7)\}$

TL_2: (2) $\{(Qc,7)\}$

TL_3: (3) $\{(c *= d,5)\}$

TL_4: (4) $\{(fd \neq fd,2), (\neg Qc,3)\}$

(5) $\{(Qd,4), (c *= d,5)\}$

(6) $\{(Qd,7)\}$

TL_5: (7) $\{(\neg Qc,3)\}$

TL_6: (8) $\Box$

<u>Theorem 3</u> If S is a finite, R-unsatisfiable set of ground clauses, and S' is the set of clauses of S which have been term locked in some way then there exists a term locked R-refutation from S' or there exists a term locked deduction of $\{t_1 \neq t_1, \ldots, t_m \neq t_m\}$ where $t_1, \ldots, t_m$ are in the Herbrand universe of S.

<u>Proof</u> Our procedure for establishing the unit case of the excess literal parameter method may briefly be described as follows. It can be shown that (1) there is a linear deduction of a clause C from S such that C is a complement of some clause of S, all the side clauses of the linear deduction are members of S, and each inference of the linear deduction is a paramodulation, or (2) there is a linear deduction of $\{t \neq t\}$ from S such that t is in the Herbrand universe of S, all the side clauses are members of S, and each inference is a paramodulation. In case of (1), the deduction from S may be transformed (by appropriate insertion of *) into a linear deduction from

S' whose side clauses are members of S' (if C is $\{e\}$ where e is an equality atom then each clause of the linear deduction which is not a side clause inherits the term locking of the clause above it). A contra term locked paramodulant is a paramodulant of term locked clauses such that the term opposite the term of substitution is locked (we emphasize that contra term locked paramodulation is not necessarily paramodulation such that the term of substitution is not locked). It is clear that the following transformation, which we refer to as the transformation, may be applied (perhaps none, and in any case a finite number of times) to the above transformed linear deduction to result in a linear deduction of the transformed C whose top clause is a member of S', whose side clauses may be deduced from S' by term locked paramodulation, and such that the sequence of inferences consists of zero or more term locked inferences followed by zero or more contra term locked inferences (all of the term locked inferences form an initial segment of the sequence, while all of the contra term locked inferences form a final segment). The transformation is best stated in theorem form: if P_0, P_1, P_2, E_1, and E_2 are term locked clauses, P_1 is a contra term locked paramodulant of P_0 by E_1, and P_2 is a term locked paramodulant of P_1 by E_2 then there exists a term locked clause P such that P is a term locked paramodulant of P_0 by E_2 and P_2 is a contra term locked paramodulant of P by E_1, or there exists a clause E such that either E is a term locked paramodulant of E_1 by E_2 and P_2 is a contra term locked paramodulant of P_0 by E or E is a term locked paramodulant of E_2 by E_1 and P_2 is a term locked paramodulant of P_0 by E. The truth of the transformation follows by considering the cases determined by whether the term of substitution of E_1 is contained in, is contained by, or is indipendent of the term of substitution of E_2. Now let P_0, E_1, P_1, ... , E_k, P_k, E_{k+1}, P_{k+1}, ... ,E_n, P_n denote the deduction (here, P_i is a paramodulant of P_{i-1} by E_i) mentioned above, where the first k inferences are term locked and the last n - (k + 1) inferences are contra term locked. Since P_n is the transformed C, since the transformed C is a complement of a member C' of S', and since the final n - (k + 1) inferences are contra term locked it follows that E_n, ... , E_{k+1} may be used to construct a term locked deduction of the complement of P_k from S'. Thus there is a term locked deduction of $\square$ from S'. The case of (2) is done similarly.

The induction step is almost obvious, and so we omit it. This completes the proof of Theorem 2.

The lifting of term locking possesses no quirks and so is omitted.

Equality atom term locking was conceived of as an analogy to locking. The direct analogy, locking with positive integers instead of the symbol *, has not been shown to be complete, but little, if any, efficiency would seem to be lost. If the input set has all of its equality atoms locked on the left (or right) then all inferred equality atoms are locked on only one side, and that is about the best that could be hoped for. By locking a term of maximal length in each equality atom, term locking can be viewed as a complete restriction which closely approximates the demodulation of ~~G. Robinson and~~ L. Wos et al. [7]. The heuristics of one sided paramodulation and demodulation can be combined if we transform the input set by switching terms of the equality atoms so that a term of maximal length is on the left and then term locking the transformed set on the left. Heuristics other than the two just mentioned might suggest other input schemes. As we will show next in Theorem 4, one especially nice feature of equality atom term locking is that it is compatible with locking (also, notice the increase in efficiency of Example 3 over Example 2).

<u>Theorem 4</u> If S is a finite, R-unsatisfiable set of ground clauses and S' is the set of all members of S which have been locked and term locked in some way, together with $\{(x = x,m)\}$, then there is a locked and term locked refutation from S'.

<u>Proof</u> In the unit case, since every deduction is a locked deduction, by Theorem 3 there is a locked and term locked deduction of $\square$ or $\{(t \neq t,i)\}$. Resolving $\{(t \neq t,i)\}$ and $\{(x = x,m)\}$ produces a locked and term locked deduction of $\square$.

For the induction step, splitting is done on a literal of maximal index, as in Theorem 2. When the split singleton is adjoined to the locked and term locked refutation obtained from the induction hypothesis, the resulting deduction is clearly locked and term locked. This completes the proof of Theorem 4.

Again, the lifting is routine and so will not be discussed here. In all cases, the functional reflexive units must be added to insure refutation completeness at the general leval.

5 Conclusions

We have established the ground completeness of resolution and paramodulation without using the maximal model theorem and in the process learned that unit reflexivity is sufficient for ground completeness. We have shown that locking, equality atom term locking, and the combination of these two strategies are refutation complete for R-unsatisfiable, functionally-reflexive sets. We believe that a good implementation of the combination of the two is by the following outline: rearrange each equality atom so that a term of maximal length is on the left then term lock on the left; lock each literal with a different positive integer; and lock positive literals high and negative literals low.

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