

NSF

## PROGRESS REPORT

June 1, 1974 - May 31, 1975

W. Byrdson

Our work this last year can be divided as follows:

1. Further development of our interactive theorem prover [1].
2. Application of the prover to
  - (a) Program Verification [2,3,4]
  - (b) Limit theorems in Analysis, by the use of Non-standard Analysis [5]
  - (c) Theorems in Topology
3. A new method for proving Presburger formulas (integer valued inequalities [6,7]).
4. Complete Sets of Reductions [8,9,10].
5. Beginning of a book on Automatic Theorem Proving [11].

The interactive prover has been extended and documented [1], and installed as a part of the London-Good program verification system [2]. This program verification work has been in cooperation with Ralph London's group at Information Sciences Institute (USC, Los Angeles) and Don Good's group at UT. The program is running on the DEC 10 at ISI and the CDC 6600 and the DEC 10 at UT. Mabry Tyson has done most of this work, in cooperation with Larry Fagan and Peter Bruell at ISI.

In this application the prover has been exercised on many theorems which have arisen from verifying programs. This experience has been valuable because it has drawn our attention to errors and inadequacies in our program, and has pinpointed

areas where the user has had to supply crucial steps in the proof, but where further extensions of the program can ease this burden on the user.

This work has drawn us more directly into program verification. Mark Moriconi is writing his Ph.D. dissertation on a hierarchical design and verification system, which allows the program to verify his program by steps as it is written, and provides a complete documentation.

In [5] we report another example where the prover (machine alone-- not in interactive mode) has been used to prove substantial theorems in Mathematics. Here we have shown that by transferring these theorems to the setting of non-standard analysis, they then lend themselves to some powerful techniques of our program: Reduction (rewriting), typing, controlled forward chaining, simplification, definition instantiation. Ballantyne is mainly responsible for this.

Our work in topology continues but has been slowed somewhat by our efforts in Program Correctness. Dr. Jared Darlington's visit in the Fall of 1974 helped us further understand the "families" problem (how do we automatically define, or build, a family of sets satisfying certain conditions, given a set of hypothesis?). His work on higher order logic, though it has no means solved this question, has partially opened the door and helped us understand which way we might proceed.

The work on Presburger arithmetic is important because Presburger formulas are the kind that arise from proving program correctness and other practical situations. The algorithms described in [6,7] have been programmed and are used as an integral part of our interactive prover [1,2,3]. At least one other group has reported that they have programmed our algorithms for use in their system.

The work of Dallas Lankford and Michael Richter has recently settled a long outstanding problem in Paramodulation, the functional reflexivity problem [10] (Dan Brand of Waterloo has evidently, independently (and previously) settled this question). More importantly Lankford's work on complete sets of reductions extends some earlier results of Knuth and Bendix, and Slagle. Since many automatic provers are now beginning to make extensive use of the powerful device of rewrite rules (Reductions) it is important that we understand which of the equality units of a theory (or a specific theorem), can be put in a rewrite table (which is much, much, faster to use), and which ones must be retained as equality unit hypothesis. Lankford's work, among other things, gives rules for answering these questions. This too will be presented at the Argonne Workshop.

The book which we are starting will describe the non-resolution side of theorem proving. It will highlight many of the concepts that have emerged (or re-emerged) during the last few years such as: Natural deduction (subgoaling), Reduction, Procedures, Simplification, Types, Forward Chaining, Induction, and Interaction. [11].

We have had much encouragement on the book from others in the field.

## PLANS FOR THE YEAR 1975-76

### Program Verification

- (a) Further exercise the program on a number of examples.
- (b) Extend the program to handle automatically some of the steps in the proof now being done by the human user.
- (c) Further improve the man-machine interface.
- (d) Completion of Moriconi's Ph.D. Dissertation on a design-verification system, and implementation of it.

### Analysis

We will push the non-standard analysis approach to see if it will yield the automatic proof of really substantial theorems in analysis, and to discover what its limitations are and why. We also wish to develop algorithms which will automatically convert formulas (including definitions) in standard analysis to the corresponding formulas in non-standard analysis, and vice versa.

### Topology

Here we will continue with some "generating of families" methods we are devising. Also we hope to exercise the interactive prover on many example theorems in topology to help pinpoint areas that need more work. Our objective remains to prove "substantial" theorems automatically.

### Book

Complete it (essentially) this year.

### Complete Sets of Reductions

Work here will concentrate on proving completeness of systems using sets of reductions, in computer implementation, publication of results, and in installing these concepts into our programs.

### References

1. W. W. Bledsoe and Mabry Tyson. The UT interactive theorem prover. University of Texas, Mathematics Department Memo ATP 17, May 1975
2. D.I. Good, R.L. London, and W. W. Bledsoe. An interactive verification system. Proc. of the 1975 International Conference on Reliable Software, Los Angeles, April 1975. and IEEE Trans on Software Engineering 1 (1975) pp. 59-67
3. W. W. Bledsoe and Mabry Tyson. Typing and proof by cases in program verification. University of Texas, Mathematics Department Memo ATP-15, May 1975. To be presented at the Automatic Theorem Proving Workshop, Argonne National Laboratory, Illinois - June 3-5, 1975
4. Mark Moriconi. A hierarchical design - Verification Methodology. (This is an overview of Moriconi's forthcoming Ph.D. dissertation).
5. Michael Ballantyne and W. W. Bledsoe. Automatic Proof of Limit Theorems in Analysis. University of Texas Mathematics Department Memo ATP 23, June 1975. To be presented at the Automatic Theorem Proving Workshop, Argonne National Laboratory, Illinois, June 3-5, 1975.
6. W. W. Bledsoe. The Sup-Inf Method in Presburger Arithmetic. University of Texas Mathematics Department Memo ATP 18, December 1974.
7. W. W. Bledsoe. A New Method for Proving Certain Presburger Formulas. To be presented at the Fourth International Joint Conference on Artificial Intelligence., Tblisi, USSR, September 3-8, 1975. (This is a reduced version of [6]).
8. Dallas Lankford. Complete Sets of Reductions. University of Texas Mathematics Department Memo ATP21, January 1975.

9. Dallas Lankford. Blocked resolution and canonical paramodulation. University of Texas Mathematics Department Memo ATP 25, Forthcoming.
10. Dallas Lankford. The functional reflexive problem. Submitted to Journal of Association for Computing Machinery, January 1975.
11. Letter about a proposed book.
12. Mark Moriconi. Semi-automatic synthesis of induction predicates. University of Texas Mathematics Department Memo ATP 16, June 1974
13. Robert Anderson and Vesko Marinov. Canonical form of a set of models. University of Texas Mathematics Department Memo ATP 19, September 1974.
14. R. H. Bing, W. W. Bledsoe, and R. D. Mauldin. Sets generated by rectangles. Pacific Journal of Math, 51 (1975).
15. Computer output from a run at ISI

#### OTHER SUPPORT

I have no other support for the work in this proposal. I am provided one month per year support on NIH grant #GM-20028-02 (Professor Richard Richardson-Principal Investigator). On that project a man-machine system is being used with pattern recognition techniques to construct chromosome maps.

Mark Moriconi has been partially supported on a related project, Contract #DAAB03-75-C-0136.

5-7-75

Type theory provides a natural framework within which to analyze certain very important aspects of human problem solving and theorem proving. In particular, type theory and type reflection can be used to see clearly how the non-finitary concepts of mathematics are manipulated in both human and machine theorem proving. In this work, I plan to give 1) a brief theoretical description of type theory and type reflection, 2) a wide selection of detailed examples illustrating the roll of type distinctions and type reflections and finally 3) an informal discussion of the implications of type reflection to automatic theorem proving. In order to maximize the impact of these ideas, the examples will be selected from axiomatic set theory and topology, classical analysis, applied mathematics, recursion theory and automata theory and especially from established automatic theorem proving programs.

Richard Stark



W. W. Bledsoe

Estimated Budget for Second Twelve Months  
(September 1, 1975 through August 31, 1976)

A. Salaries and Wages

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Salaries paid from grant funds at The University of Texas at Austin conform to the rates approved by the Board of Regents for salaries paid from regular University funds.

1. Senior Personnel

a. 1-Principal Investigator - 2 summer months (W. W. Bledsoe) \$6,800

2. Other Personnel (non-faculty)

a. 1-Post-doctoral research associate - .55 T.-2 summer months  
F.T. rate \$918 (Dallas Lankford) (\$505) 1,010

b. 4-Research Assistants

(1) 1/2 T.-9 academic months, F.T.-2 summer months at F.T.  
rate \$739 per month (Michael Ballantyne) 4,804

(2) 1/2 T.-4 academic months, at F.T. rate \$859 (Mark  
Moriconi) 1,718

(3) 1/2 T.-9 academic months, F.T. rate- summer month  
at F.T. rate \$804 per month (Mabry Tyson) 4,422

(4) 3 summer months at F.T. rate \$595 per month (Peter Bruell) 1,785

TOTAL SALARIES AND WAGES 20,539

B. Fringe Benefits

Federal Social Security (5.85% of the first \$14,100 of each salary  
Unemployment Compensation Insurance (1.0% of the first \$4200 of each  
salary); Workmen's Compensation Insurance (0.3% of total salaries);  
Insurance Premium Sharing (\$15.00 per month for full-time employees  
or an equivalent amount for those working half-time or more who are  
enrolled in insurance benefit plans). Under University accounting  
procedures these items are direct costs.

1,364

TOTAL A & B

\$21,923

C. Expendable Supplies and Equipment

521

D. <u>Travel</u>	
1. Domestic - For travel to scientific meetings	1,000
E. <u>Publication Costs</u>	400
F. <u>Computer Charges</u> - CDC - 6600 - 23 hours @ 230 per hour; ( 1/2 to be borne by The University of Texas); main- tenance of two computer terminals - \$240	2,885
G. <u>Total Direct Costs</u>	\$26,709
H. <u>Indirect Costs</u>	
On-Campus - 54% of Salaries and Wages Predetermined 9/1/75 - 8/31/76 and provisional 9/1/76 until amended	11,091
Total Estimated Budget for Second Twelve Months	\$37,800

W. W. Bledsoe DCR 74-12886  
September 1, 1975 - August 31, 1976

Estimated Residuals

Salaries	0
Other Expenses	0
Travel	0
Publication Costs	300
Computer Costs	200
TOTAL	<u>\$500</u>