THE REFUTATION COMPLETENESS OF PERMUTATIVE NARROWING AND BLOCKED RESOLUTION

by

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INTRODUCTION

The concept of complete sets of permutative reductions has been defined by Lankford and Ballantyne (2) and will not be discussed in detail here. In this note, we consider the problem of constructing refutation complete algorithms incorporating complete sets of permutative reductions. The paradigm we follow has been developed by Lankford (1) and Slagle (4) for ordinary reductions. Familiarity with the articles mentioned above is assumed. Below, we define permutative narrowing and establish the refutation completeness of blocked resolution and permutative narrowing in the presence of complete sets of permutative reductions.

Let $\mathcal{P}$ be a finite set of equations and let $\approx_{\mathcal{P}}$ be the congruence relation defined by $t \approx_{\mathcal{P}} u$ iff $t = u$ is an equality-consequence of $\mathcal{P}$. A finite set $\mathcal{P}$ of equations is called a set of permuters in case for each term t, the $\approx_{\mathcal{P}}$ -equivalence class of t is finite. A complete set of permutative reductions is a pair of sets $\mathcal{R}$, $\mathcal{P}$ where $\mathcal{P}$ is a set of permuters,

$\mathcal{R}$ is a set of permutative rewrite rules of the form $\approx_{\mathcal{P}}(L) \longrightarrow \approx_{\mathcal{P}}(R)$, and the system $\mathcal{R}, \mathcal{P}$ have the finite and unique termination properties, cf. Lankford and Ballantyne (2). When $\mathcal{P}$ is understood, we omit reference to it.

PERMUTATIVE NARROWING

An immediate $\mathcal{P}$-inference of a clause C is any paramodulant of C by a member of $\mathcal{P}$ on a subterm of C which is not a variable. A $\mathcal{P}$-inference of a clause C is the end result of a finite sequence of immediate $\mathcal{P}$-inferences starting with C. An immediate $\mathcal{P}$-narrowing of a clause C is the result C'' of fully permutatively reducing C', where C' is an immediate narrowing by $L \longrightarrow R$ of a $\mathcal{P}$-inference of C, for some $\approx(L) \longrightarrow \approx(R) \in \mathcal{R}$. Here it is assumed that L and R are the least members of $\approx(L)$ and $\approx(R)$ for some fixed well-ordering. A $\mathcal{P}$-narrowing of a clause is the end result of a finite sequence of immediate $\mathcal{P}$-narrowings, followed by a $\mathcal{P}$-inference.

Theorem If $\mathcal{R}, \mathcal{P}$ are a complete set of permutative reductions, $\mathcal{E}(\mathcal{R}) = \{L = R \mid \approx(L) \longrightarrow \approx(R) \in \mathcal{R}\}$, $\mathcal{C}$ is a set of clauses which do not contain any equality predicates, $\mathcal{E}(\mathcal{R}) \cup \mathcal{P} \cup \mathcal{C}$ is equality unsatisfiable, and $\mathcal{C}^*$ is the result of fully permutatively reducing $\mathcal{C}$ by $\mathcal{R}, \mathcal{P}$, then there is a finite set $\mathcal{C}^N$ of $\mathcal{P}$-narrowings of $\mathcal{C}^*$ such that a blocked resolution deduction of the empty clause may be constructed from $\mathcal{C}^N$.

Proof We first consider the case when all clauses of $\mathcal{C}$ are ground. In this case, methods of Lankford (1) can be used to show that $\mathcal{C}^*$ is unsatisfiable. Moreover, any refutation from $\mathcal{C}^*$ is blocked in the ground case. Another demonstration of the ground case can be obtained by observing that a complete set of permutative reductions determines a normal form for $\mathcal{E}(\mathcal{R}) \cup \mathcal{P}$ and applying the results of Plotkin (3).

For the general case, the above ground refutation completeness result can be lifted similar to the lifting of ordinary narrowing as done by Lankford (1) and Slagle (4). Let C be a clause, λ be a substitution, and $(C\lambda)^*$ be the result of permutatively reducing $C\lambda$ as far as possible. It can be shown that there is a permutative narrowing C^N of C which has $(C\lambda)^*$ as a substitution instance.

To illustrate permutative narrowing and blocked resolution, let

$\mathcal{R}$: $f(a) \longrightarrow b$,

$\mathcal{P}$: $f(g(x)) = g(f(x))$, and

$\mathcal{C}$: $P(f(x))$,

$\neg P(g(b))$.

The permutative narrowings are:

N1. P(b) ,

N2. P(g(b)) ,

N3. P(g(g(b)) ,

... .

In practice, permutative narrowings would be generated by rounds. In the above example, a blocked refutation would be obtained in the second round.

CONCLUDING REMARKS

The main result of this note deals with complete sets of permutative reductions and sets of clauses which do not contain the equality predicate. The results of Lankford (1) for ordinary reduction suggest that the results of this note can be extended to the cases when the permutative reductions are incomplete and when the equality predicate occurs in clauses.

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