When variables are admitted, we have expressions of the form

$$\text{For all } x \quad (x \leq 0 \rightarrow x < 1).$$

and write this

$$\forall x \ (x \leq 0 \rightarrow x < 1).$$

Thus we use the symbol "$\forall$" as a shorthand for "for all", and similarly use "$\exists$" for "for some". The following rules for the IMPLY natural deduction prover, are taken from reference (**).

NATURAL SYSTEM

$(H \Rightarrow G)$

H IS A SET OF HYPOTHESES.

G IS A GOAL.

TO FIND A SUBSTITUTION θ FOR WHICH

$$(H\theta \rightarrow G\theta)$$

IS A VALID PROPOSITIONAL FORMULA.

EXAMPLE.

$$P(A) \wedge (P(X) \rightarrow Q(X)) \Rightarrow Q(A)$$

ANSWER: $\theta \equiv A|X$

IMPLY RULES

A PARTIAL SET FROM [12]

I4. $(H \Rightarrow A \wedge B)$ "SPLIT"

IF $(H \Rightarrow A)$ RETURNS θ

AND $(H \Rightarrow B\theta)$ RETURNS λ

THEN RETURN $\theta \circ \lambda$

I3. $(H_1 \vee H_2 \Rightarrow C)$ "CASES"

IF $(H_1 \Rightarrow C)$ RETURNS θ

AND $(H_2\theta \Rightarrow C)$ RETURNS λ

THEN RETURN $\theta \circ \lambda$

I5. $(H \Rightarrow C)$

PUT C':= REDUCE(C); H':= REDUCE(H)

CALL $(H' \Rightarrow C')$

I7. $(H \Rightarrow (A \rightarrow B))$ "PROMOTE"

CALL $(H \wedge A \Rightarrow B)$.

I15. $(H \Rightarrow C)$

PUT C':= DEFINE(C)

CALL $(H \Rightarrow C')$

✓ H2. $(H \Rightarrow C)$ "MATCH"

IF $H\theta \equiv C\theta$, RETURN θ.

✓ H6. $(A \wedge B \Rightarrow C)$ "OR-FORK"

IF $(A \Rightarrow C)$ RETURNS θ (NOT NIL), RETURN θ.

ELSE CALL $(B \Rightarrow C)$

✓ H7. $H \wedge (A \rightarrow D) \Rightarrow C$ "BACK-CHAIN"

IF $(D \Rightarrow C)$ RETURNS θ,

AND $(H \Rightarrow A\theta)$ RETURNS λ,

THEN RETURN $\theta \circ \lambda$

H9. $H \wedge (A = B) \Rightarrow C$ "SUB = "

PUT $A' :=$ CHOOSE (A,B), $B' :=$ OTHER(A,B)

CALL $(H(A'/B') \Rightarrow C(A'/B'))$.

$(\forall x\ P(x) \rightarrow P(a))$

Skolemize (Eliminate Quantifiers, $\forall, \exists$)

$(P(x) \Rightarrow P(a))$

Returns a/x H2

$(Q \wedge P(a) \rightarrow \exists x\, (P(x) \wedge Q))$

$(Q \wedge P(a) \Rightarrow P(x) \wedge Q)$

AND-SPLIT — I4

$(Q \wedge P(a) \Rightarrow P(x))$

$(Q \Rightarrow P(x))$ — H6

FAILS

$(P(a) \Rightarrow P(x))$

Returns a/x — H2

$(Q \wedge P(a) \Rightarrow Q)$

$(Q \Rightarrow Q)$ — H6

Returns "T" — H2

Returns a/x.

~~Example Using IMPLY~~

<u>Theorem.</u> $(P(A) \wedge \forall x(P(x) \rightarrow Q(x)) \rightarrow Q(A))$.

$P(A) \wedge (P(x) \rightarrow Q(x)) \Rightarrow Q(A))$ — Eliminate Quantifier

$(P(A) \Rightarrow (Q(A))$ — H6

FAILS

$(P(x) \rightarrow Q(x)) \Rightarrow Q(A))$

$(Q(x) \Rightarrow Q(A))$ — H7

Return $\theta \equiv A|x$ — H2

$(P(A) \Rightarrow P(x)(A|x))$

Return TRUE.

Returns $A|x$.

Boyer-Moore (UT-Austin)

RECURSIVE FUNCTION PROVER

e.g. Proving Theorems about LISP functions

EX. ORDERED (SORT L)

For Hard theorems, the user suggests a series of lemmas which it proves (like proof-checking)

Ex. Prime Factorization Theorem

Ex. "Verified" a simple compiler for algebraic expressions (McCarthy)

Ex. Halting Problem (unsolvability) 1982

.
.
.

Applications: PV, Proof-checking, related to programming

Uses: Induction, Generalization, etc., etc.

ATP is a part of AI, but more than that.

Earliest Provers had AI features

(1) knowledge base

(2) reasoning rules

Later provers tended toward (2) alone.

Why is there still a problem?

Why not use EMYCIN and TEIRESIAS?

Ans.: These (EMYCIN and TEIRESMS) are best for applications needing

(1) much expert knowledge, and

(2) shallow reasoning.

This is fine for many of life's problems, but ATP's needs are more severe:

(1) much expert knowledge,

(2) deeper reasoning.

- Expert knowledge is hard to encode for advanced mathematics. It is
  - easy to prove <u>all</u> geometry theorem of a certain type.
  - hard to <u>discover</u> the proof of a new theorem.
  - hard to discover a new theorem.

- Ongoing research in ATP is exciting. We will not have time to even mention much of it here.

Theorems which do not contain variables to be instantiated (bound) are called <u>ground</u> theorems.

State of the art remark:

> All ground theorems (that arise naturally) are easy to prove by modern ATP programs. But much needs to be done to handle theorems with variables.

Assertion: Much of the difficulty in ATP will be eliminated if we have programs that can

- successfully fetch the appropriate lemmas (and not useless ones)
- properly bind these lemmas' variables.

Assertion: Many of the concepts used successfully by human provers have yet to be property exploited by ATP programs:

- use of examples
  - as counterexamples (some done)
  - as guides to proof discovery (a little has been done)
- conjecturing (Lenat's work, little else)
- analogy (very little)
- Agenda Mechanism - to control the search (two Ph.D.'s theses)
- Special-purpose subprovers
  - equality packages (lots has been done) (see slide)
  - inequality packages (lots has been done)
- Domain specific heuristics

Many other ideas that we are considering are not mentioned here. These have much in common with AI research.

EQUALITY

$(A = B \wedge H \longrightarrow C)$

1. CHOOSE either A or B to replace the other.

   (Replacing both ways is disastrous)

2. REDUCTIONS

   A---> B   A is always replaced by B

   Ex. $A \cap \emptyset \dashrightarrow \emptyset$
   $P \wedge T \dashrightarrow P$
   (Car (Cons x y)) ---> x

   Rewrite Rules

3. Complete Sets of Reductions

   Convert all equality units to Reductions

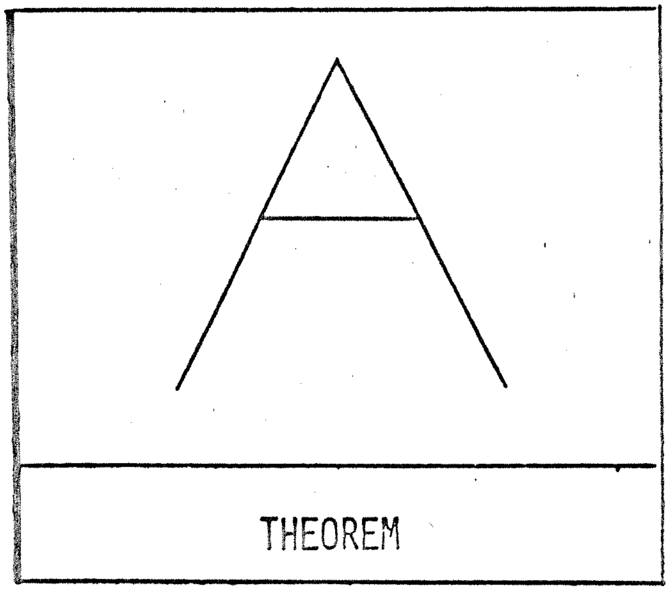
   If possible the saving is enormous.

4. EQUALITY PACKAGE

Man-machine interactive prover and proof checkers are expected to play an important role in future ATP. Examples of these are

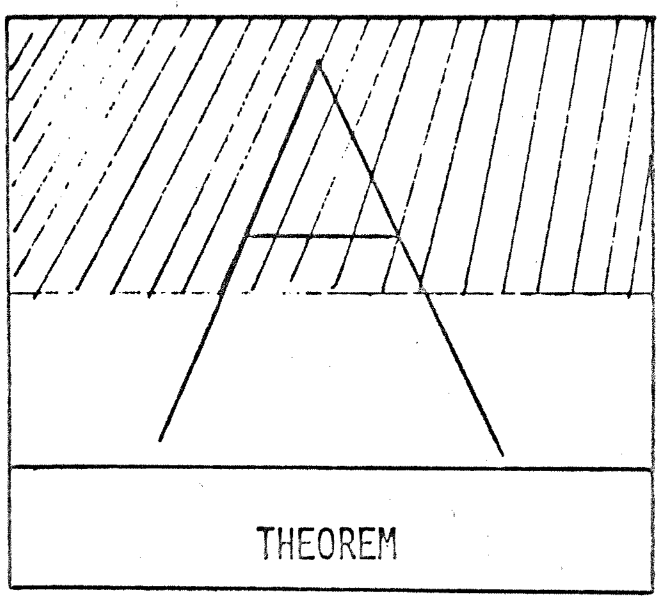
- Program Verification (mentioned earlier)
- Wos and Winker's Prover at Argonne National Laboratory
- The Boyer and Moore prover for recursive function theory
- The proof of the four-color problem in topology
- Our current attempts to prove the Poincare conjecture.

A man-machine prover must allow the user easy access. The user must not be asked to prove easy things, the machine must be able to detect when it needs help from the user and to communicate with him on what is needed without excessive work on the user's part. Such an interface is used in the Don Good PV program at UT-Austin but needs much improvement.



AXIOMS AND SUPPORTING THEOREMS NEEDED IN THE PROOF.

THEOREM BEING PROVED



BUILT IN PROCEDURES AND REDUCTION TABLES

GIVEN ONLY WHEN NEEDED

If the axioms and supporting theorem ("lemmas") shown in the slide are to be supplied ahead of time by the user, then the user would have to prove the theorem before he asks the computer to do so. Ridiculous! Whereas a really good man-machine prover will have many such lemmas "built-in" and will elicit others from the user as needed in the proof.

Several proof checkers have been built but most suffer from the fact that the user cannot submit his proofs in natural form. Work is underway to partially remedy that.

WOODLANDS AI LABORATORY
WAIL

3 August 1982

Mike Ballantyne and Woody Bledsoe are conducting a study on the feasibility of estaolishing an AI Laboratory at the Woodlands.

The Woodlands is a new city located about 40 miles northwest of Houston on Interstate 45, which has been under planning for twelve Years, is now partially built, and promises to be one of the loveliest communities in the world. It is being built on a plan that provides for the environmental, social, and employment needs of its citizens: extensive wooded parks which permeate all of the housing areas, golf courses (the Houston Open is played yearly on one of the Woodlands' golf courses), tennis courts, swimming polls, ice skating, etc, housing areas (in all price ranges), schools, churches, community centers, businesses, high technology industry (including energy and medical), research and development laboratories, etc. It is designed to provide all levels of housing needs and jobs for every adult who lives there. We feel that this will be one of the choicest places to live and work.

The Woodlands AI Laboratory (WAIL) will be part of HARC (Houston Area Research Center - See attached brochure) which is associated with the University of Houston, Rice University, and Texas A&M, and which is attempting to bring research and development laboratories to The Woodlands. The Woodlands Corporation, which is principally owned by The Mitchell Energy Corporation, has donated a 150 acre site to HARC and provided several million dollars in start-up funding for the next few years. It is envisioned that other Energy and Medical related industries in the Houston are would sustain the funding for the long run. (A 150 acre site has also been donated to the Texas Medical Center).

The Woodlands AI Laboratory would initially concentrate on applied AI, such as expert systems, industrial robotry, etc, which will be useful to businesses and industries in the Woodlands and Houston areas, especially those related to energy, medical, and computing research, development and applications, and later expand to others such as Natural Language interfaces, program verification, Vision, problem solving and search, knowledge represention and acquisition, theorem proving, program synthesis and understanding, etc.

Initial housing and funding for WAIL will come from those provided to HARC. We feel that the existing and projected funding is very secure, and that WAIL will be able to survive the incubation stage and become a strong, well known, laboratory.

As part of our feasibility study we will talk with a number of individuals throughout the country and abroad. These include prominent AI researchers from Universities, research laboratories, and industrial AI groups, and others in research and development laboratories throughout industry:

Stanford, MIT, CMU, U. Md, U. Texas, Rutgers, Rochester, U.Penn., U. Illinois, Yale, etc.

SRI, ISI, SUMEX, BBN, etc

Schlumberger, TI, Fairchild, Hewlett-Packard, Machine Intelligence Corporation, etc

Texas Medical Center, MD Anderson Hospital, etc

We are seeking advise on the following points:

- Possible Projects for WAIL
    - Applications oriented
    - Long range research projects
- Existing AI Projects (in other Laboratories)
- Prospects for heading and staffing WAIL
- General Advice