ELEMENTARY THEORY OF NUMBERS – W. J. Leveque

THEOREM 1–1. If a is positive and b is arbitrary, there is exactly one pair of integers q, r such that the conditions

$$b = qa + r, \quad 0 \leq r < a, \qquad (6)$$

hold.

Proof: First, we show that (6) has at least one solution.

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To show the uniqueness of q and r, assume that q' and r' also are integers such that

$$b = q'a + r', \quad 0 \leq r' < a.$$

Then if $q' < q$, we have

$$b - q'a = r' \geq b - (q - 1)a = r + a \geq a,$$

and this contradicts the inequality $r' < a$. Hence $q' \geq q$.

Similarly, we show that $q \geq q'$. Therefore $q = q'$, and consequently $r = r'$. ▲

Figure 28

```
|-TO
|-SHOW
|-UNIQUENESS
|-IMPLICITLY-SUPPOSE (B = Q*A+R & 0 <= R & R < A)
|-SUPPOSE (B = Q1*A+R1 & 0 <= R1 & R1 < A)
| |-ASSUME
| |-THAT
| |-(FORMULA (B = Q1*A+R1 & 0 <= R1 & R1 < A))
| |-BREAK
|-PROVE (Q = Q1 & R = R1)
  |-PROVE Q = Q1
  | |-PROVE (Q1 >= Q & Q >= Q1)
| | |-PROVE Q1 >= Q
  | | | |-SUPPOSE Q1 < Q
  | | | | |-THEN
  | | | | |-IF
  | | | | |-(FORMULA (Q1 < Q))
  | | | |-CONTRADICTION
  | | | | |-PROVE R1 >= A
  | | | | | |-WE
  | | | | | |-HAVE
  | | | | | |-(FORMULA (B-Q1*A = R1 & R1 >= B-(Q-1)*A
  | | | | | |     & B-(Q-1)*A = R+A & R+A >= A))
  | | | | | |-DEDUCE B-Q1*A = R1
  | | | | | |-DEDUCE B-Q1*A >= B-(Q-1)*A
```

Figure 29

```
. . . . . .
| | | | | |-DEDUCE R+A >= A
| | | | |-AND
| | | | |-THIS
| | | | |-CONTRADICTS
| | | | |-(FORMULA (R1 < A))
| | | | |-DEDUCE R1 < A
| | | | |-DEDUCE R1 >= A <=> NOT(R1 < A)
| | | | |-BREAK
| | | |-HENCE
| | | |-(FORMULA (Q1 >= Q))
| | | |-DEDUCE Q1 < Q <=> NOT(Q1 >= Q)
| | |-BREAK
| | |-PROVE Q >= Q1
| |   |-SIMILARLY
| |   |-WE
| |   |-SHOW
| |   |-(FORMULA (Q >= Q1))
| |-BREAK
| |-THEREFORE
| |-(FORMULA (Q = Q1))
| |-DEDUCE (Q1 >= Q & Q >= Q1) => Q = Q1
|-AND
|-CONSEQUENTLY
|-(FORMULA (R = R1))
|-BREAK
|-DEDUCE Q = Q1 => R = R1
```

Figure 30

are used, stored in a concept hierarchy graph, to facilitate the proper acquisition and use. (Plummer, a visitor at the University of Texas, is finishing his PhD Thesis under Bundy at Edinburgh).

7 CONCLUDING REMARKS

Logic is emerging as a foundation for AI and all of Computer Science. The consequence of this is that some form of *automatic reasoning* is a *requirement* for most AI programs. Much of the research in ATP over the last thirty years is applicable to this need.

As these programs grow more complex, the corresponding inference problems will become more difficult, comparable in difficulty to the proof substantial theorems in mathematics.

We have reviewed the current research on automated reasoning and given a proposed classification of that work.

We note that some research areas, such as *clause-compiling* and *parallel processing*, are very exciting, and this is rightly so. But we wonder whether these efforts on *fast implementation*, which are very important in their own right, might divert us from the even more important areas (in the long run) of *tactics* and *strategy*.

Under tactics, we are especially hopeful about the work on *larger-inference-steps*, and the work on special purpose systems such as those for the use of rewrite rules.

We believe that more *large scale experiments* are needed, wherein researchers exercise their provers on worthwhile examples, rather than play with toy problems and/or a couple of harder problems (such as the Steamroller problem or the Intermediate Value Theorem).

What about Strategy? Are we to soon attain "over all" strategies for our provers? There has been some promising work on Analogy and Machine Learning; a little on Conjecturing, Abstractions, and using Examples to guide proof discovery, but not much else.

We feel that fundamental progress will require advances in representing and accessing the knowledge used by human mathematicians. This knowledge includes examples, rules, heuristics, and motivations, in addition to the more commonly recognized declarative facts represented by axioms and lemmas. The experiments we have reported on demonstrate simplified approaches to representing one or more forms of mathematical knowledge, but the realization of an integrated truly powerful system remains for the future.

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