AN ANALYTIC MODEL OF A MULTIPROGRAMMING SYSTEM INCLUDING A JOB MIX

by

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ABSTRACT

This paper presents an analytic model of a multiprogramming system where several processors share a common input/output system and where jobs are distinguishable from one another. There are m classes of jobs and each class is associated with a unique processor. Jobs arrive to the system in a random manner. Several scheduling disciplines are included in the study.

INTRODUCTION

Most queueing models of computer systems developed so far, restrict attention to systems where all jobs are indistinguishable from one another. The model described here, considers a computer utility in which on arrival a job is identified with a particular class and assigned to a processor accordingly; however, all processors in the system share a common I/O system with L different channels.

Processes arrive at the system in a Poisson manner with average arrival rate W; a new arrival is assigned to the i^{th} processor (i^{th} class-job) with probability P_{oi}. After being serviced in the i^{th} processor it joins the j^{th} I/O channel with probability P_{iI} P_{ij} or it departs from the system with probability $1 - P_{iI}$. Once served in the j^{th} I/O channel it joins the queue of the i^{th} processor and the cycle starts over until it finally departs from the system. This job's service demand in its associated CPU (i^{th} processor) follows a probability distribution with mean $1/U_i$. Three different types of distributions and processor scheduling algorithms are included in the analysis:

a) exponential where the scheduling discipline is any discipline that makes no use of known running times or of the future behavior of the jobs.

b) any distribution with a rational Laplace-Stieltjes transform (1,2) with the scheduling discipline either LAST COME FIRST SERVED Preemptive Resume (LCFSPR) or Processor-Sharing.

Chandy (3) proved that b) behaves as an exponential distribution with the same mean.

The service demand of any job in the j^{th} I/O channel is exponential with mean $1/V_j$ (see figure 1).

MATHEMATICAL ANALYSIS

Let $(N_1, N_2, \ldots N_m, \bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_L)$ denote the generic state in the Markov process, where:

$\underline{m}$ is the number of processor (job classes) and

$\underline{L}$ the number of I/O channels.

N_i is the number of jobs waiting or being served in the i^{th} processor's queue.

$\bar{Y}_i$ is a vector (list) which contains all jobs that are waiting or being serviced in the i^{th} I/O channel. This vector (list) indicates the order of arrival of all jobs to this queue together with the class of each job. In addition, let Y_{ir} denote the total number of r-class jobs in the i^{th} I/O channel queue. The author derived the solution by solving a subset of the regular balance equations (Feller, 5) called the local balance equations (LBE). This concept was introduced by Chandy (3). The LBE greatly simplify the process of solving the balance equations for <u>some</u> Markovian systems. The concept underlying the LBE, is that for some general Markov processes, the rate at which the system leaves a state due to the movement of a job out of any queue is equal to the rate at which the system enters that state due to the movement of a job into that queue. It is obvious that if the local balance equations (LBE) are satisfied, the regular balance equations are also automatically satisfied.

LOCAL BALANCE EQUATIONS

For the $\underline{i}^{th}$ processor the LBE are as follows:

$$(1)\quad U_i P[N_1, N_2, \ldots, N_i, \ldots N_m, \bar{Y}_1, \bar{Y}_2, \ldots \bar{Y}_L] = \sum_{j=1}^{L} V_j \; P[N_1, \ldots, N_i-1, \ldots, N_m, \bar{Y}_1, \ldots \bar{Y}_j^{(+i)}, \ldots \bar{Y}_L) + W \; P_{oi} \; P[N_1, \ldots, N_i-1, \ldots, N_m, \bar{Y}_1, \ldots \bar{Y}_L)$$

Where $\bar{Y}_j^{(+i)}$ denotes a vector identical to $\bar{Y}_j$ except that it has an additional i-class job at the head of the vector (queue).

For the K^{th} I/O channel the LBE are:

(2) $V_k P[N_1, \ldots, N_m, \bar{Y}_1, \ldots, \bar{Y}_k, \ldots \bar{Y}_L)$

$$= P_{iI} P_{iK} U_i P[N_1, \ldots, N_i+1, \ldots, N_m, \bar{Y}_1, \ldots, \bar{Y}_k^{(-i)}, \ldots \bar{Y}_L]$$

for $i = 1, \ldots, m$

where:

a) the job at the rear of $\bar{Y}_K$ is an i-class job

b) $\bar{Y}_K^{(-i)}$ is identical to $\bar{Y}_K$ except for the absence of an i-class job at its rear.

It can be trivially checked by direct substitution that the unique steady state solution to this Markovian system is:

(3) $$P(N_1, N_2, \ldots N_m, \bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_L) = \text{NORN} \prod_{i=1}^{m} \left[\frac{W P_{oi}}{U_i (1-P_{iI})}\right]^{N_i} \prod_{j=1}^{m} \left[\frac{W P_{oj} P_{jI} P_{jK}}{V_K (1-P_{jI})}\right]^{Y_{Kj}}$$

where

(3) $$\text{NORN} = \frac{1}{\sum\limits_{\text{all feasible states}} P(N_1, \ldots, N_m, Y_1, \ldots, Y_L)}$$

APPLICATIONS AND EXTENSIONS

This model was inspired on a dual-processor system at the Computation Center of the University of Texas (CDC 6400-6600) to which it is applied as a means for system improvement and design. In this system jobs are characterized in a number of different ways including origin, equipment required, etc., and sent to one or the other processor for service according to their classification. Notice that a job is completely identified with unique class throughout its existence in the system.

Great flexibility is introduced in the model in the way of parameters and other features such as distributions and scheduling disciplines.

Several extensions and/or reductions of this model are possible. For example, the model can be modified to encompass systems where input and compute bound jobs share a unique set of hardware resources. Optimal distribution of load in a configuration like that at the University of Texas Computation Center may be obtained via the model in a straightforward manner. Extension of the model to closed cyclic networks (1,2) can easily be carried out. Also arbitrarily complex open networks satisfy the LBE. The solution to these systems is presented in the appendix.

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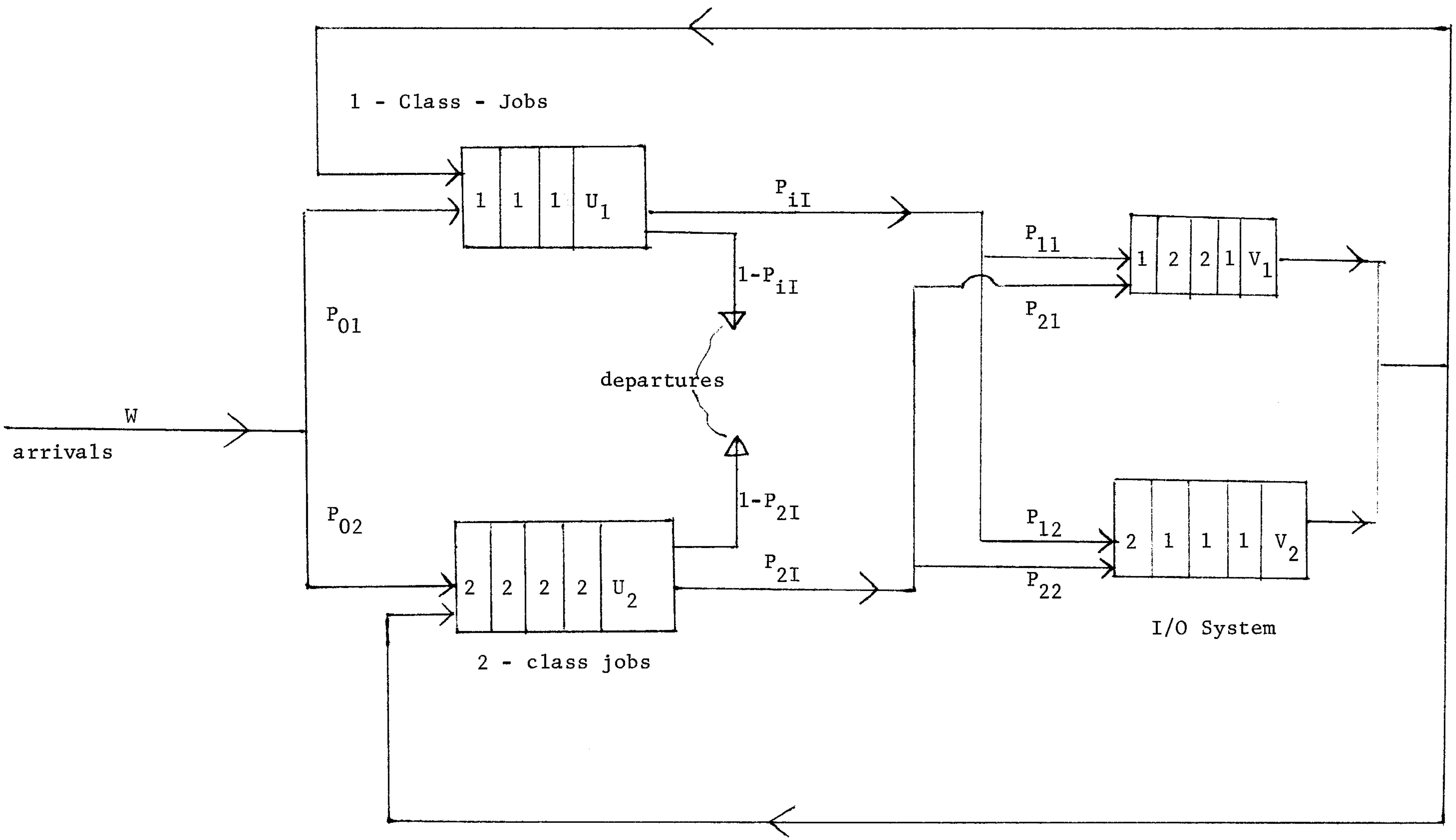

Figure 1. A Dual Processor System.

APPENDIX

Consider a network of n queues where jobs arrive to the network in a Poisson manner with average arrival rate W. On arrival a job joins the i^{th} queue with probability P_{oi}. Let $P_o^{(s)}$ be the probability that a new arrival be an s-class job.

Denote the generic state of the system by $(\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_n)$ where $\bar{Y}_i$ is defined as before. Furthermore assume that the average service time of any job in the i^{th} queue is V_i. Let $P_{ij}^{(s)}$ be the probability that an s-class job joins the j^{th} queue immediately after being serviced in the i^{th} queue. Assume that the last job in the i^{th} queue (represented above by $\bar{Y}_i$) is an s-class job. The LBE for this queue are:

(4) $$V_i\, P(\bar{Y}_1, \ldots, \bar{Y}_i, \ldots, \bar{Y}_n) =$$

$$\sum_{j\neq i}^{n} P_{ji}^{(s)}\, V_j\, P(\bar{Y}_1, \ldots, \bar{Y}_i^{(-s)}, \ldots, \bar{Y}_j^{(+s)}, \ldots \bar{Y}_n)$$
$$+ V_i\, P_{ii}^{(s)}\, P(\bar{Y}_1, \ldots \bar{Y}_i^{(+s,\,-s)}, \ldots \bar{Y}_n)$$
$$+ W\, P_{oi}\, P_o^{(s)}\, P(\bar{Y}_1, \ldots, \bar{Y}_i^{(-s)}, \ldots \bar{Y}_n)$$

for $s = 1, \ldots, m$

where $\bar{Y}_i^{(-s)}$ and $\bar{Y}_i^{(+s)}$ are defined as before and $\bar{Y}_i^{(+s,-s)}$ is identical to $\bar{Y}_i$ except for the presence of an additional s-class job at its head and the absence of an s-class job at its rear.

Let $BT_{(0)}^{(s)} = P_0^{(s)}$

and $$BT_{(i)}^{(s)} = \sum_{j=0}^{n} P_{ji}^{(s)}\, BT_{(j)}^{(s)}$$

for $s = 1, \ldots, m$ and $i = 1, \ldots, n$.

It is trivial to verify that

(5) $$h\, P(Y_1, \ldots Y_i, \ldots, Y_n) = \text{NORN} \prod_{i=1}^{n} \prod_{s=1}^{m} \left[\frac{BT_{(i)}^{(s)}\, W}{V_i}\right]^{Y_{is}}$$

is the unique solution to eq. (4) and then the equilibrium solution for the Markov Process.

NORN is a normalization constant defined by equation (3)

It is worth noticing that equation (5) is totally independent of the other of the jobs in the queue.

A similar extension can be carried out in a straightforward manner to obtain the equilibrium solution to cyclic closed networks.

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