QUEUING MODELS OF DEVICE UTILIZATION IN MULTIPROGRAMMED COMPUTER SYSTEMS

by

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CHAPTER I

INTRODUCTION

One of the most important features of the third generation digital computer is the use of multiprogramming techniques where many jobs may reside in central memory and the processing units are shared among them. The computer system itself includes one or more processors and many types of input-output (I/O) devices. Information flows between processors and I/O devices. Since more than one job may share the primary memory units and the computer system includes several devices, there arises the problem of scheduling the devices in a computer system. Device utilization is a measure of system performance. How to achieve greater utilization of the devices in the system is of great concern to system designers. It is this aspect that led to this study.

Because of the probablistic nature of the service time requirements among jobs in the system and because jobs may traverse through different devices in the system, it is reasonable to employ queuing network models for analyzing device utilization. A great deal of research has been concentrated on queuing models of computer systems. Direct preceeedents to the present study include: Gordon and Newell [6] have developed a generalized queuing model of systems with exponential servers and a simple scheme to solve the balance equations. Buzen [2] later adopted this method to analyze computer system with one CPU and many I/O devices. Baskett [1] has developed models for computer systems with various exponential types of devices and service disciplines. Chandy [3] and Chandy, Keller and Browne [4] have developed more general models for systems with exponential type devices under different service disciplines and with nonexponential type devices under processor-sharing service discipline. Gaver [5] has analyzed CPU utilization of systems with exponential type devices.

These studies emphasize either the mathematical points of the models or, in experimental analysis, concentrate on certain parts of the system such as CPU utilization. None has compared the effects of device utilizations under different service disciplines. The present study analyzes systems by constructing and evaluating models parametized by actual data from the UT operating system on a CDC 6600 computer. The goals of this study are: (1) to determine relationships between device utilization and CPU service time by varying parameters characterizing the CPU service time, (2) to determine the effects of different I/O devices on the system as a whole by varying I/O configurations of the system, and (3) to determine effects of several different CPU service disciplines on device utilization. It is hoped that through these analyses a clear view of the system can be obtained.

Chapter II of this study discusses the basic tools of queuing models and reviews the currently available techniques for evaluation of queuing models of computer system.

Chapter III analyzes the utilization of the CPU of a multiprogrammed computer system. CPU utilization is measured by varying the parameters of the models. Effects of two scheduling method, the first-come-first-serve (FCFS) discipline and the processor-sharing (PS) disciplines are compared.

I/O device utilization is evaluated by considering different combinations of hardware configuration which include one CPU and different types of I/O devices. The effects of different I/O configurations on CPU utilization are also obtained. Two scheduling strategies (FCFS and PS) on the same system are compared. All these are discussed in Chapter IV.

In Chapter V various results from the analyses of Chapter III and IV are compared against the hypotheses. Conclusions are drawn both to provide as a better understanding of multiprogrammed computer system and also as a basis for improvement of performance of computer systems.

CHAPTER II

BACKGROUND OF QUEUING ANALYSIS

In a multiprogrammed computer system, jobs in the system flow between processors and I/O devices in a probablistic manner. This suggests the use of queuing models for analyzing the system. It is necessary before going into the actual analysis of the system to acquaint the reader with some background concerning queuing modelling. These include factors or parameters governing the models, the methods of obtaining solutions from the models, and the main points in the results which are useful in the present study.

Service Time Distribution

Jobs in the system flow between processor and I/O devices. Assume it is a closed system, that is, a job may never leave the system during the time the operation of the model is considered. Thus a job in the system may be in either one of the four states: being executed in the central processing unit (CPU), waiting in the CPU queue, performing I/O operations in an I/O device, or waiting service in the I/O queue. The time a job spends in a device, either computing in the CPU or performing I/O operations in I/O device, is called service time. And the time a job waiting in either CPU queue or I/O queue is called waiting time. It is assumed that the service time of a job is completely random and thus is independent of time. This means that the remaining time required to complete service for a job in the processor or I/O device does not depend on the time this particular job has already consumed in the device. It is this memoryless property of the service time which enable the use of Markov processes in constructing models for the system. Three exponential type distributions fell into this category:

1. Exponential distribution

Let $A(x)$ be the probability that the service time for a job will be less than x, then the exponential service time distribution for $A(x)$ is:

$$A(x) = 1 - e^{-ux} \qquad x \geq 0 \qquad (1)$$

The mean service time of this distribution is $1/u$. And the standard deviation of the distribution is also $1/u$. Define coefficient of variation to be the quotient of mean service time divided by standard deviation. Then the exponential distribution always has coefficient of variation equal to 1.

2. Hypoexponential distribution

Some devices in the system, especially I/O devices, typically have service time distributions smaller coefficient of variation than exponential. Then the devices can be modeled by a series of exponential service stations or stages. For a job to receive service from this type of device it must pass sequentially and successfully through each station or stage of the device. No job can receive service unless the present job in the device has completed service in the last stage or station. The service time distribution for such a device is a hypoexponential distribution and has the following form:

$$A(x) = 1 - e^{-nux} * \sum_{i=0}^{n-1} \frac{(nux)^i}{i!} \qquad (2)$$

The mean service time of hypoexponential distribution is

$$\frac{1}{u} = \sum_{i=1}^{n} \frac{1}{u_i} \qquad (3)$$

and the variance is

$$\mathrm{var}(x) = \sum_{i=1}^{n} \frac{1}{u_i^2} \qquad (4)$$

Clearly (4) is less than the square of (3). Thus the hypoexponential distribution has coefficient of variation less than 1.

3. Hyperexponential distribution

Some devices, mainly the CPU, typically have service time distribution more skewed than the exponentially distributed devices. A device like this can be modeled by a set of branches with exponential service time in each branch. When a job is allowed access to this device, it chooses one of the branches. Only one branch is used for each service and only one branch can be in use at one time. Each one of the branches has a distinct exponential service time with different probabilities of entering into each branch. Thus the mean service time of this type of device can be approximated by a weighted sum of exponentials. Morse [7] calls it a hyperexponential distribution. It has the following form:

Let w_i be the probability that a job enters branch i and u_i be the exponential mean service time of this branch. Then the hyperexponential distribution of a device is

$$B(x) = 1 - \sum_{i=1}^{n} w_i e^{-u_i x} \tag{5}$$

The mean service time of x is

$$E(x) = \frac{1}{u} = \sum_{i=1}^{n} \frac{w_i}{u_i} \tag{6}$$

and the expect value of x^2 is

$$E(x^2) = \sum_{i=1}^{n} \frac{2w_i}{u_i^2} \tag{7}$$

So that the variance of x is

$$Var(x) = E(x^2) - (E(x))^2 \tag{8}$$

It is easily to see that the coefficient of variation of hyperexponential distribution is always greater than 1 since (8) is less than the square of (6).

Scheduling

One of the important scheduling strategies in multiprogrammed computer system is to service jobs in fixed time quantum round robin fashion. In this study two types of time quantum are adopted. The first one choose the time quantum to infinity. This scheduling strategy then becomes a first-come-first-serve (FCFS) service discipline. The second one takes the limit zero as the time quantum where zero overhead is assumed. In practice the time quantum can often be set very small in relation to the mean service time and large with respect to service time. Then the scheduling strategy approaches a processor-sharing (PS) discipline. Models under these two scheduling methods are different and so is the utilization of device.

Equations and Solutions

The systems considered in the present study are closed cyclic systems consisting of one CPU and several I/O devices. The service time distributions of the devices in the system are assumed to be of the exponential types as discussed earlier. Jobs in the system flow between CPU and I/O units. A job may be in one of the four states: computing in CPU, doing I/O operations in one of the I/O devices, awaiting service in CPU queue, or awaiting service in I/O queue. Because of the memoryless property of exponential type service distributions, queuing model of the system can be constructed by the method of continuous-time Markov process. Markov model can be characterized by a set of states. Each state describes the status of the system. Let the state i be denoted by $S_i = (n_1, n_2, \ldots\ldots, n_k)$ where n_j is the number of jobs in device j being serviced and awaiting service. Transitions from one state to another state occurs when a job which completes service at device i joins queue of device j. Hardware configuration and service distributions govern the transitions between any two states. The former dictates the permissibility of a transition between two states whereas the latter gives the rate of transition. Let P(i) be the probability that the system stays at the status of state i and let Q_{ij} be the rate of transition from state i to state j. If there are n states for the system, then the queuing model of this system can be expressed as a system of linear equations:

$$\left\{ \sum_{\substack{j=1 \\ j\neq i}}^{n} Q_{ij} \right\} P(i) = \sum_{\substack{j=1 \\ j\neq i}}^{n} Q_{ji}\, P(j) \qquad i=1,2,\ldots\ldots n \tag{9}$$

and

$$\sum_{i=1}^{n} P(i) = 1 \tag{10}$$

These equations are called "detailed balance" equations. The left hand side of (9) describes the rate of transitions from state i to all other states and the right hand side contains the rate of transitions from all other states into state i. Equation (10) simply tells us that the sum of probabilities of all possible states in the system must be 1. Equation (9) form a dependent system of linear equations. Replacing any equation in (9) by (10) the new system of equations is independent and can be solved. The results, the

P(i)s, are called steady-state probabilities. The P(i) are the probability of each state when the system is in equilibrium and provide basis for further analyses.

To obtain all possible transitions between states of computer system takes great effort and time. In order to simplify the effort Gordon and Newell have developed a simple scheme which computes steady-state probabilities by a special solution technique for the detailed balance equations. Their scheme works only when all devices in a system have exponential service distributions. Buzen later adopted their scheme to do some experiments. Chandy has developed a more general method called local balance which can apply to any system of exponential type devices as discussed earlier or any differentiable service distribution with processor-sharing or last-come-first-serve-preemptive-resume discipline. In stead of viewing the system as a whole in which the detailed balance equation states that when the system reaches equilibrium the rate of transition out of each of the devices in the system must equal to the rate of transition into each device, local balance concerns itself with the transition into and out of one device at a time. A closed cyclic computer system can be considered as a closed network in which a job traverses from source to sink via series of devices and back to source in a cyclic manner. A device in the network may have branches and/or stations. Let $P(n_1, n_2, \ldots, n_k)$ be the steady-state probability that there are n_1 jobs in device 1, n_2 jobs in device 2, etc. then the local balance equation (LBE) is:

$$P(n_1,\ldots,n_{i-1}+1,n_i,\ldots,n_k) * u_{i-1}$$

$$=P(n_1,\ldots,n_{i-1},n_i+1,\ldots,n_k) * u_i \qquad (11)$$

Where u_{i-1} and u_i are transition rates of devices i-1 and i.

And the solution can be easily obtained:

$$P(n_1,\ldots,n_k) = \text{NORM} * \prod_{i=1}^{k} \text{TERM}(i,y_i) \qquad (12)$$

Here TERM (i,y_i) is the probability contribution of device i and

$$\text{TERM}(i,y_i) = \frac{n_i!}{n_{1i}!\ldots n_{s_i i}!} \prod_{j=1}^{s_i} (\text{BTERM}(j,i) * \text{PTERM}(j,i))^{n_{ji}} \qquad (13)$$

In eq(13) n_i is the number of jobs in device i, s_i is the number of branches and/or stations in device i, $n_{1i},\ldots,n_{s_i i}$ are numbers of jobs in each station and/or branch of device i. BTERM(j,i) is the arrival probability for a job from source to station or branch j of device i and PTERM(j,i) is the mean service time of this branch or station. And:

$$\text{NORM} = 1/(\sum P(n_1,n_2,\ldots n_k))$$

where $n = n_1+n_2+\ldots+n_k$ is the total number of jobs in the system.

Because the systems considered in this study all have hyperexponential CPU service time and because local balance does not work under the hyperexponentially distributed device in FCFS discipline, the FCFS models are solved by direct solutions of the detailed balance equations.

Points of Interests

The steady-state probabilities so obtained in last section provide basis for system performance evaluation. The following are the measures this study seeks:

1. Idle time

Idle time of each device in the system tells the probability that a device is idle and is measured by adding all steady-state probabilities where the number of queue entries is zero for this device. Device utilization can be obtained by subtracting idle time from unity. In general, the longer the mean service time of a device, the more utilization and hence the less idle time this device will have.

2. Average queue length

If $P_i(n_i)$ is the probability that there are n_i jobs in device i, the average queue length QL_i can be measured by:

$$QL_i = \sum_{j=0}^{n} (j * P_i(j)) \tag{14}$$

Average queue length reflects average number of jobs in a particular device. Define utilization factor R=a/u where a be mean arrival rate and u be mean service rate. Then according to Morse, the larger the ratio R, the larger the average queue length QL.

3. Standard deviation of average queue length

The standard deviation of average queue length describes the degree of dispersion about the average queue length of a device. It is measured by:

$$SD_i = \left(\sum_{j=0}^{n} j^2 p_i(j) - QL_i^2 \right)^{1/2} \tag{15}$$

Morse has stated that SD_i is large compared to QL_i itself when R is small.

Varying the coefficient of variation, the arrival probabilities, or the mean service time of each device will cause changes in device utilization. Device utilization will be different under different service disciplines and/or system organizations. All these changes provide interesting points in analyzing system performances. The actual models and results are presented in the next two chapters.

CHAPTER III

QUEUING MODELS FOR CPU UTILIZATION

One of the main goals of system designers is to keep all devices of a computer as fully utilized as possible. Because of the different characteristics of devices in the system it is not possible to reach this goal; the policy is then to keep CPU busy even at the expense of inactivity of I/O devices. Since the CPU plays such an important role in system performance it is first necessary to analyze the effects of CPU scheduling on the system. These include CPU service time and service disciplines.

The System

The computer system under consideration is a closed system and consists of one CPU and two I/O devices. The CPU service time is assumed to be hyperexponentially distributed and there are two branches in the CPU, each having distinct service time. The two I/O devices are assumed to have exponentially distributed service time. A job finishing service in one of the branch of CPU immediately joins an I/O queue and waits to serviced. A job completing I/O service at one of the I/O devices goes directly to the CPU queue and waits for service by one of the CPU branches. Thus jobs run cyclically in the system.

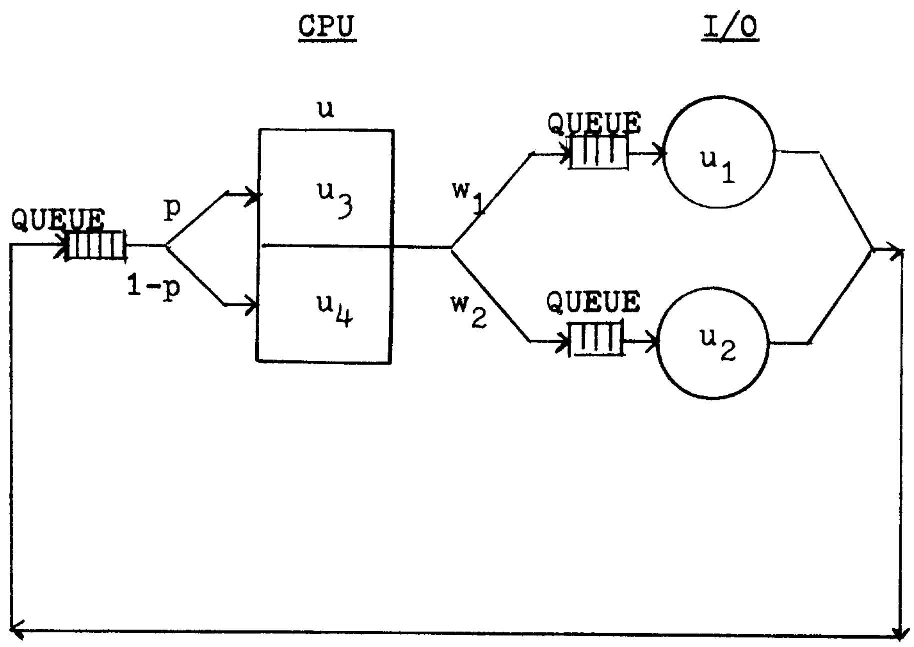

Figure 1. System Configuration of Models A and B

The Models

Two models are constructed for this computer system, model A corresponds to the system when the service discipline of CPU is first-come-first-serve (FCFS) and model B corresponds to the system where CPU service discipline is processor-sharing (PS).

Model A

Let:

w_1= Probability that a job finishing service in CPU will join the queue of I/O device 1.

w_2= Probability that a job finishing service in CPU will join the queue of I/O device 2.

$(w_1 + w_2 = 1)$

u_1= Mean service rate (exponential) of I/O 1.

u_2= Mean service rate (exponential) of I/O 2.

u = Mean service rate (hyperexponential) of CPU

u_3= Mean service rate of branch a of the CPU

u_4= Mean service rate of branch b of the CPU

p = Probability that a job finishing I/O operation will require the service of branch a of the CPU

$1-p$= Probability that a job finishing I/O operation will require the service of branch b of the CPU

Then the probability density functions of I/O 1 and I/O 2 are in the same form of (1) as shown in Chapter II with rate u_1 and u_2 respectively while the CPU has probability density function

$$B(x) = 1 - pe^{-u_3 x} - (1 - p)e^{-u_4 x} \qquad (16)$$

The mean service time and its variance of the CPU are :

$$E(x) = \frac{p}{u_3} + \frac{1-p}{u_4} = \frac{1}{u} \qquad (17)$$

and

$$\begin{aligned} var(x) &= E(x^2) - (E(x))^2 \\ &= \frac{2p}{u_3^2} + \frac{2(1-p)}{u_4^2} - \left(\frac{p}{u_3} + \frac{1-p}{u_4}\right)^2 \\ &= \frac{k^2}{u^2} \end{aligned} \qquad (18)$$

k is defined to be the coefficient of variation.

Let N be the total number jobs in the system and let $P(n_1, n_2, n_3, n_4)$ be the steady-state probability that there are n_1 jobs in I/O 1 queue, n_2 jobs in I/O 2 queue, n_3 jobs in branch a of CPU, n_4 jobs in branch b of CPU, and $n_3 + n_4$

$= n_3$ or n_4, then $n_1+n_2+n_3+n_4=N$. Let δ_{ij} be 1 if $i=j$ and 0 if $i\neq j$, let $w_3=p$ and $w_4=1-p$, and let $m=n_3+n_4$, then the detailed balance equations of this system under FCFS discipline are of the form:

$$\Big(\sum_{i=1}^{4}(1-\delta_{0n_i})u_i\Big)P(n_1,n_2,n_3,n_4)$$
$$= \sum_{i=1}^{2}\sum_{j=3}^{4}(1-\delta_{0n_j})\Big[(1-\delta_{1n_j})+\delta_{1n_j}w_j\Big]u_iP(n_i+1,\ldots,n_j-1,\ldots) + (1-\delta_{Nm})\sum_{i=1}^{2}\sum_{j=3}^{4}(1-\delta_{0n_i})w_i\Big\{\sum_{k=3}^{4}((1-\delta_{0m})w_k+\delta_{om})u_kP(n_i-1,\ldots,n_k+1,\ldots)\Big\} \tag{19}$$

There are $2 * C_2^{n+2} - C_1^{n+1}$ different states and therefore same number of detailed balance equations of the form (19). Replace any of these balance equations by the relation:

$$\sum_i P_i(n_1,\ n_2,\ n_3,\ n_4) = 1$$

The system of equations is then independent and the steady-state probabilities can be solved from it.

<u>Model B</u>

When CPU scheduling strategy is processor-sharing the model for this system can be expressed in terms of local balance equations (LBE). Again $P(n_1,\ n_2,\ n_3,\ n_4)$ is the steady-state probability as defind in model A. Contrary to the conditions imposed on FCFS discipline where $n_3 + n_4$ must be either n_3 or n_4, it is possible in PS to have both n_3 and n_4

greater than zero because under PS all jobs in the CPU queue are processed simultaneously. The total number of states in PS is C_3^{n+3} and so is the number of LBEs. Then according to the definition of LBE in Chapter II.

PTERM(i) = $1/u_i$ (i = 1, 2, 3, 4) and

BTERM(i) = w_i (i = 1, 2, 3, 4)

The LBE of state $P(n_1, n_2, n_3, n_4)$ is :

$$P(n_1,n_2,n_3,n_4)=\text{NORM} * \frac{(n_3+n_4)!}{n_3!n_4!} \prod_{i=1}^{4} (\text{BTERM}(i)*\text{PTERM}(i))^{n_i} \quad (20)$$

Where

$$\text{NORM}=\left\{ \sum_{\sum n_i=N} \frac{(n_3+n_4)!}{n_3!n_4!} \prod_{i=1}^{4} (\text{BTERM}(i)*\text{PTERM}(i))^{n_i} \right\}^{-1} \quad (21)$$

The steady-state probabilities can be obtained quite easily from (20) and (21).

The Experiments

We will keep w_1, w_2, u_1, u_2 constant while varying the value of N and mean CPU service time u. The steady-state probabilities will vary as N, the scheduling discipline, u_3, u_4, p and k vary. This study analyzes device utilizations, average queue lengths of each device, and standard deviations of average queue lengths by letting w_1=0.75, w_2=0.25, $1/u_1$=0.078 second, $1/u_2$=0.048 second while changing mean CPU service time 1/u at 0.025, 0.05, 0.075 and 0.1 second and with different jobs in the system from 2 to 4 and also varing p and

k. Thus for each device in the system there are many different utilization levels.

As mentioned earlier, the CPU service time in the system is hyperexponentially distributed with two branches and has the probability distribution function of formula (16) where $0 \leq p \leq 0.5$, u_3 is the slow CPU service rate and u_4 is the fast CPU service rate. From (17):

$$u_3 = puu_4 \;/\; (u_4 - (1-p)u) \qquad (22)$$

Replace u_3 in (18) by the relation (22), the value of u_4 is:

$$u_4 = \frac{2(1-p)u + u\sqrt{2(p^2 - p + pk^2 - p^2k^2}}{2 - p - pk^2} \qquad (23)$$

under predetermined p, k, and u values. The value of u_3 can be obtained by inserting u_4 of (23) into (22). It is obvious that changes in either p or k will result in different rates of u_3 and u_4. Hence for given u and N, the effects of p and k on device utilizations are also analyzed. In this system the values of k are set from 1 to 5 and the values of p are set from 0.05 to 0.40. However, in order that both u_3 and u_4 must be positive to be meaningful, it is necessary to have $p < 2/(1+k^2)$. Therefore for any combination of p and k such that u_3 or u_4 is negative, the effect on device utilization is not analyzed. If k=1, $u_3=u_4=u$. The CPU service time distribution in fact reduces to exponential.

In stead of depending upon the values of p and k as in eqs. (22) and (23), mean CPU service rates u_3 and u_4 may depend on k only. In this case the probability density function of CPU service time is:

$$B(x) = 1 - se^{-2sux} - (1-s)e^{-2(1-s)ux} \tag{24}$$

and $u_3=2su$, $u_4=2(1-s)u$, and $0 \leq s \leq 0.5$. The variance of B(x) is:

$$var(x) = \frac{1}{u^2}\left(1 + \frac{(1-2s)^2}{2s(1-s)}\right) = \frac{k^2}{u^2} \tag{25}$$

Where k is the coefficient of variation. For any predetermined value of k,

$$s = \frac{1+k^2-\sqrt{k^4-1}}{2(1+k^2)} \tag{26}$$

Then u_3 and u_4 can be easily calculated from (26) and u. When k=1, s is 0.5 and B(x) reduces to an exponential distribution function. Since k affects the rates of u_3 and u_4, this study also analyzes device utilizations, for each N and u, of CPU service distribution in the form of (24) under different k values from 1 to 5.

When CPU service discipline is processor-sharing, the jobs in the CPU share the computing resource equally. The changes of p and k and hence u_3 and u_4 do not cause changes in device utilization. Therefore only in model A the effects of p and k are analyzed. In model B only utilizations under different N and u values are analyzed. The results from these various cases are shown in Table 1 through Table 8.

Results

Based on various assumptions as discussed in the last section, solutions have been obtained by changing different parameters of the models. The system idle times are listed in Table 1 through Table 4; the mean queue lengths and the standard deviations of mean queue lengths of all devices in the system are listed in Table 5 through Table 8.

It is very obvious from Table 1 to Table 4 that increassing the total number of jobs in the system will result in less idle time and hence more utilization of all devices in the system. The mean queue length(QL) and the standard deviations of queue length(SD) also increases as N increases from 2 to 4, as shown in Table 5 through Table 8.

Under FCFS CPU service discipline and given any values of N, p, and mean CPU service rate u, the coefficient of variation k plays an important role in over all system performance. Device idle time increases as k increases. When N is 4 and the mean CPU service time is 0.025 second, and if CPU service distribution is form (24), the increase of idle time for CPU is 5% if k increases from 1 to 5. For I/O 1 it is 11% and for I/O 2 it is 2%. The changes are 10% for CPU, 13% for I/O 1, 3% for I/O 2 when mean CPU service time is 0.05 second; 11, 9, 2 at 0.075 second and 9, 5, 1 at 0.1 second. When form (16) is used for CPU service distribution and when p is set at 0.05, the percentage increases

TABLE 1

SYSTEM IDLE TIME OF MODELS A AND B

(1/u=0.025 sec.)

P	N	CPU FCFS K=1	CPU FCFS K=2	CPU FCFS K=3	CPU FCFS K=4	CPU FCFS K=5	CPU PS	I/O 1 FCFS K=1	I/O 1 FCFS K=2	I/O 1 FCFS K=3	I/O 1 FCFS K=4	I/O 1 FCFS K=5	I/O 1 PS	I/O 2 FCFS K=1	I/O 2 FCFS K=2	I/O 2 FCFS K=3	I/O 2 FCFS K=4	I/O 2 FCFS K=5	I/O 2 PS
	2	0.641	0.655	0 562	0.666	0.668	*	0.161	0.191	0.209	0.218	0.224		0.828	0.834	0.838	0.840	0.841	
[illegible]	3	0.602	0.620	0.632	0.640	0.644		0.068	0.110	0.140	0.158	0.168		0.809	0.817	0.824	0.827	C.829	
	4	0.585	0.602	0.617	0.628	0.634		0.029	0.067	0.105	0.128	0.143		0.801	0.809	0.816	0.821	0.824	
	2	0.641	0.651	0.662	0.672	0.683	0.641	0.161	0.184	0.208	0.233	0.259	0.161	0.828	0.833	0.838	0.843	0.848	0.828
.05	3	0.602	0.617	0.632	0.648	0.663	0.602	0.068	0.103	0.139	0.175	0.212	0.068	0.809	0.816	0.823	0.831	0.838	0.809
	4	0.585	0.600	0.617	0.634	0.651	0.585	0.029	0.064	0.104	0.144	0.183	0.029	0.801	0.808	0.816	0.824	0.832	0.801
	2	0.641	0.654	0.668	0.684		0.641	0.161	0.190	0.224	0.260		0.161	C 828	0.834	0.841	0.848		0.828
.10	3	0.602	0.619	0.639	0.659		0.602	0.068	0.109	0.155	0.203		0.068	0.809	0.817	0.827	0.837		0.809
	4	0.585	0.602	0.622	0.643		0.585	0.029	0.068	0.115	0.165		0.029	0.801	0.809	0.819	0.829		0.801
	2	0.641	0.656	0.673			0.641	0.161	0.194	0.235			0.161	0.828	0.835	0.843			0.828
.15	3	0.602	0.621	0.643			0.602	0.068	0.112	0.165			0.068	0.809	0.818	0.829			0.809
	4	0.585	0.603	0.625			0.585	0.029	0.070	0.122			0.029	0.801	0.809	0.820			0.801
	2	0.641	0.661				0.641	0.161	0.206				0.161	0.828	0.837				0.828
.35	3	0.602	0.624				0.602	0.068	0.121				0.068	0.809	0.820				0.809
	4	0.585	0.604				0.585	0.029	0.073				0.029	0.801	0.810				0.801

*Not analyzed

TABLE 2

SYSTEM IDLE TIME OF MODELS A AND B

(1/u=0.050 sec.)

P	N	CPU						I/O 1						I/O 2					
		FCFS						FCFS						FCFS					
		K=1	K=2	K=3	K=4	K=5	PS	K=1	K=2	K=3	K=4	K=5	PS	K=1	K=2	K=3	K=4	K=5	PS
	2	0.415	0.444	0.457	0.463	0.466	*	0.315	0.350	0.365	0.372	0.375		0.860	0.867	0.870	0.871	0.872	
■ ■	3	0.328	0.375	0.399	0.411	0.417		0.214	0.269	0.297	0.311	0.318		0.839	0.850	0.856	0.859	0.860	
	4	0.278	0.333	0.366	0.383	0.393		0.155	0.220	0.258	0.278	0.290		0.827	0.840	0.848	0.852	0.854	
	2	0.415	0.436	0.456	0.478	0.502	0.415	0.315	0.340	0.363	0.389	0.417	0.315	0.860	0.865	0.869	0.875	0.880	0.860
.05	3	0.328	0.363	0.397	0.433	0.470	0.328	0.214	0.255	0.295	0.337	0.380	0.214	0.839	0.847	0.855	0.864	0.873	0.839
	4	0.278	0.321	0.364	0.408	0.451	0.278	0.155	0.205	0.256	0.307	0.357	0.155	0.827	0.837	0.847	0.858	0.868	0.827
	2	0.415	0.443	0.473	0.509		0.415	0.315	0.348	0.384	0.426		0.315	0.860	0.867	0.874	0.882		0.860
.10	3	0.328	0.373	0.422	0.474		0.328	0.214	0.267	0.324	0.384		0.214	0.839	0.850	0.861	0.874		0.839
	4	0.278	0.331	0.390	0.448		0.278	0.155	0.218	0.286	0.354		0.155	0.827	0.840	0.854	0.868		0.827
	2	0.415	0.448	0.489			0.415	0.315	0.354	0.402			0.315	0.860	0.868	0.877			0.860
.15	3	0.328	0.380	0.441			0.328	0.214	0.275	0.346			0.214	0.839	0.851	0.866			0.839
	4	0.278	0.338	0.407			0.278	0.155	0.226	0.306			0.155	0.827	0.841	0.858			0.827
	2	0.415	0.467				0.415	0.315	0.376				0.315	0.860	0.872				0.860
.35	3	0.328	0.401				0.328	0.214	0.299				0.214	0.839	0.856				0.839
	4	0.278	0.356				0.278	0.155	0.246				0.155	0.827	0.845				0.827

*Not analyzed

TABLE 3

SYSTEM IDLE TIME OF MODELS A AND B

(1/u=0.075 sec.)

P	N	CPU FCFS K=1	CPU FCFS K=2	CPU FCFS K=3	CPU FCFS K=4	CPU FCFS K=5	CPU PS	I/O 1 FCFS K=1	I/O 1 FCFS K=2	I/O 1 FCFS K=3	I/O 1 FCFS K=4	I/O 1 FCFS K=5	I/O 1 PS	I/O 2 FCFS K=1	I/O 2 FCFS K=2	I/O 2 FCFS K=3	I/O 2 FCFS K=4	I/O 2 FCFS K=5	I/O 2 PS
	2	0.281	0.316	0.330	0.336	0.339	*	0.439	0.467	0.478	0.482	0.485		0.885	0.891	0.893	0.894	0.894	
∞	3	0.181	0.234	0.258	0.269	0.275		0.361	0.402	0.421	0.430	0.435		0.869	0.877	0.881	0.883	0.884	
	4	0.124	0.183	0.215	0.231	0.240		0.317	0.363	0.388	0.400	0.407		0.860	0.869	0.874	0.877	0.878	
	2	0.281	0.305	0.329	0.355	0.387	0.281	0.439	0.458	0.476	0.497	0.522	0.439	0.885	0.889	0.893	0.897	0.902	0.885
.05	3	0.181	0.218	0.256	0.300	0.349	0.181	0.361	0.390	0.420	0.454	0.492	0.361	0.869	0.875	0.881	0.888	0.896	0.869
	4	0.124	0.166	0.213	0.267	0.327	0.124	0.317	0.349	0.386	0.429	0.475	0.317	0.860	0.867	0.874	0.883	0.892	0.860
	2	0.281	0.314	0.352	0.401		0.281	0.439	0.465	0.495	0.533		0.439	0.885	0.890	0.896	0.904		0.885
.10	3	0.181	0.231	0.291	0.363		0.181	0.361	0.400	0.447	0.503		0.361	0.869	0.877	0.887	0.898		0.869
	4	0.124	0.180	0.252	0.335		0.124	0.317	0.361	0.417	0.481		0.317	0.860	0.869	0.880	0.894		0.860
	2	0.281	0.322	0.375			0.281	0.439	0.471	0.513			0.439	0.885	0.892	0.900			0.885
.15	3	0.181	0.241	0.321			0.181	0.361	0.408	0.471			0.361	0.869	0.879	0.891			0.869
	4	0.124	0.191	0.284			0.124	0.317	0.369	0.441			0.317	0.860	0.871	0.885			0.860
	2	0.281	0.352				0.281	0.439	0.495				0.439	0.885	0.896				0.885
.35	3	0.181	0.278				0.181	0.361	0.439				0.361	0.869	0.885				0.869
	4	0.124	0.227				0.124	0.317	0.397				0.317	0.860	0.876				0.860

*Not analyzed

TABLE 4

SYSTEM IDLE TIME OF MODELS A AND B

(1/u=0.100 sec.)

P	N	CPU						I/O 1						I/O 2					
		FCFS						FCFS						FCFS					
		K=1	K=2	K=3	K=4	K=5	PS	K=1	K=2	K=3	K=4	K=5	PS	K=1	K=2	K=3	K=4	K=5	PS
	2	0.200	0.235	0.248	0.253	0.256		0.532	0.552	0.560	0.563	0.565		0.904	0.908	0.909	0.910	0.911	
= *	3	0.106	0.152	0.172	0.182	0.187		0.477	0.504	0.516	0.521	0.524		0.893	0.898	0.901	0.902	0.903	
	4	0.058	0.104	0.128	0.141	0.147		0.449	0.476	0.490	0.497	0.501		0.887	0.893	0.895	0.897	0.898	
	2	0.200	0.223	0.247	0.274	0.308	0.200	0.532	0.546	0.559	0.575	0.595	0.532	0.904	0.907	0.910	0.913	0.917	0.904
.05	3	0.106	0.136	0.170	0.213	0.267	0.106	0.477	0.495	0.514	0.540	0.571	0.477	0.893	0.896	0.900	0.906	0.912	0.893
	4	0.058	0.088	0.126	0.178	0.244	0.058	0.449	0.467	0.489	0.519	0.558	0.449	0.887	0.891	0.895	0.901	0.909	0.887
	2	0.200	0.233	0.272	0.328		0.200	0.532	0.551	0.574	0.607		0.532	0.904	0.908	0.913	0.919		0.904
.10	3	0.106	0.149	0.208	0.289		0.106	0.477	0.502	0.537	0.584		0.477	0.893	0.898	0.905	0.915		0.893
	4	0.058	0.101	0.168	0.262		0.058	0.449	0.474	0.513	0.568		0.449	0.887	0.892	0.900	0.911		0.887
	2	0.200	0.241	0.299			0.200	0.532	0.556	0.590			0.532	0.904	0.909	0.916			0.904
.15	3	0.106	0.160	0.245			0.106	0.477	0.508	0.558			0.477	0.893	0.899	0.909			0.893
	4	0.058	0.112	0.207			0.058	0.449	0.481	0.536			0.449	0.887	0.893	0.905			0.887
	2	0.200	0.278				0.200	0.532	0.578				0.532	0.904	0.913				0.904
.35	3	0.106	0.205				0.106	0.477	0.535				0.477	0.893	0.905				0.893
	4	0.058	0.155				0.058	0.449	0.506				0.449	0.887	0.899				0.887

*Not analyzed

TABLE 5

AVERAGE QUENE LENGTH AND STANDARD DEVIATION OF MODELS A AND B

(1/u=0.025 sec.)

P	N	CPU						I/O 1						I/O 2					
		FCFS					PS	FCFS					PS	FCFS					PS
		K=1	K=2	K=3	K=4	K=5		K=1	K=2	K=3	K=4	K=5		K=1	K=2	K=3	K=4	K=5	
5	2	0.453* (0.660)**	0.479 (0.719)	0.494 (0.750)	0.502 (0.765)	0.506 (0.774)	***	1.354 (0.741)	1.332 (0.778)	1.321 (0.797)	1.315 (0.808)	1.312 (0.813)		0.194 (0.447)	0.188 (0.445)	0.185 (0.443)	0.183 (0.442)	0.182 (0.441)	
	3	0.579 (0.824)	0.641 (0.959)	0.676 (1.036)	0.696 (1.077)	0.706 (1.100)		2.193 (0.929)	2.139 (1.021)	2.109 (1.077)	2.094 (1.107)	2.086 (1.125)		0.228 (0.508)	0.221 (0.507)	0.214 (0.503)	0.210 (0.500)	0.208 (0.498)	
	4	0.655 (0.942)	0.760 (1.161)	0.823 (1.297)	0.858 (1.372)	0.877 (1.415)		3.100 (1.061)	3.001 (1.223)	2.946 (1.329)	2.918 (1.389)	2.902 (1.425)		0.245 (0.540)	0.239 (0.544)	0.231 (0.539)	0.224 (0.533)	0.220 (0.528)	
.05	2	0.453 (0.660)	0.473 (0.706)	0.493 (0.748)	0.515 (0.790)	0.537 (0.830)	0.453 (0.660)	1.354 (0.741)	1.337 (0.770)	1.321 (0.796)	1.305 (0.823)	1.287 (0.858)	1.354 (0.741)	0.194 (0.447)	0.190 (0.445)	0.185 (0.443)	0.181 (0.441)	0.176 (0.438)	0.194 (0.447)
	3	0.579 (0.824)	0.626 (0.934)	0.674 (1.033)	0.725 (1.123)	0.778 (1.205)	0.579 (0.824)	2.193 (0.929)	2.152 (1.004)	2.111 (1.074)	2.068 (1.141)	2.023 (1.202)	2.193 (0.929)	0.228 (0.508)	0.222 (0.507)	0.215 (0.503)	0.207 (0.500)	0.199 (0.495)	0.228 (0.508)
	4	0.655 (0.942)	0.737 (1.128)	0.820 (1.292)	0.906 (1.435)	0.995 (1.559)	0.655 (0.942)	3.100 (1.061)	3.024 (1.199)	2.949 (1.325)	2.871 (1.439)	2.790 (1.540)	3.100 (1.061)	0.245 (0.540)	0.239 (0.541)	0.231 (0.539)	0.223 (0.535)	0.214 (0.531)	0.245 (0.540)
.10	2	0.453 (0.660)	0.478 (0.717)	0.507 (0.775)	0.539 (0.833)		0.453 (0.660)	1.354 (0.741)	1.333 (0.776)	1.310 (0.813)	1.285 (0.851)		1.354 (0.741)	0.194 (0.447)	0.189 (0.445)	0.183 (0.442)	0.176 (0.440)		0.194 (0.447)
	3	0.579 (0.824)	0.638 (0.955)	0.706 (1.080)	0.780 (1.195)		0.579 (0.824)	2.193 (0.929)	2.141 (1.018)	2.082 (1.108)	2.017 (1.192)		2.193 (0.929)	0.228 (0.508)	0.221 (0.507)	0.212 (0.505)	0.203 (0.501)		0.228 (0.508)
	4	0.655 (0.942)	0.756 (1.156)	0.870 (1.354)	0.992 (1.525)		0.655 (0.942)	3.100 (1.061)	3.005 (1.220)	2.900 (1.373)	2.786 (1.508)		3.100 (1.061)	0.245 (0.540)	0.239 (0.543)	0.230 (0.544)	0.222 (0.543)		0.245 (0.540)
.15	2	0.453 (0.660)	0.482 (0.725)	0.518 (0.795)			0.453 (0.660)	1.354 (0.741)	1.330 (0.781)	1.302 (0.825)			1.354 (0.741)	0.194 (0.447)	0.188 (0.445)	0.181 (0.442)			0.194 (0.447)
	3	0.579 (0.824)	0.647 (0.968)	0.729 (1.111)			0.579 (0.824)	2.193 (0.929)	2.133 (1.027)	2.060 (1.129)			2.193 (0.929)	0.228 (0.508)	0.221 (0.508)	0.211 (0.506)			0.228 (0.508)
	4	0.655 (0.942)	0.769 (1.172)	0.906 (1.391)			0.655 (0.942)	3.100 (1.061)	2.992 (1.231)	2.863 (1.400)			3.100 (1.061)	0.245 (0.540)	0.239 (0.545)	0.231 (0.549)			0.245 (0.540)
.35	2	0.453 (0.660)	0.493 (0.747)				0.453 (0.660)	1.354 (0.741)	1.321 (0.794)				1.354 (0.741)	0.194 (0.447)	0.186 (0.445)				0.194 (0.447)
	3	0.579 (0.824)	0.671 (1.001)				0.579 (0.824)	2.193 (0.929)	2.109 (1.048)				2.193 (0.929)	0.228 (0.508)	0.220 (0.511)				0.228 (0.508)
	4	0.655 (0.942)	0.805 (1.208)				0.655 (0.942)	3.100 (1.061)	2.954 (1.256)				3.100 (1.061)	0.245 (0.540)	0.241 (0.551)				0.245 (0.540)

*Average quene length **Standard deviation ***Not analyzed

TABLE 6

AVERAGE QUENE LENGTH AND STANDARD DEVIATION OF MODELS A AND B

(1/u=0.050 sec.)

P	M	CPU FCFS K=1	CPU FCFS K=2	CPU FCFS K=3	CPU FCFS K=4	CPU FCFS K=5	CPU PS	I/O 1 FCFS K=1	I/O 1 FCFS K=2	I/O 1 FCFS K=3	I/O 1 FCFS K=4	I/O 1 FCFS K=5	I/O 1 PS	I/O 2 FCFS K=1	I/O 2 FCFS K=2	I/O 2 FCFS K=3	I/O 2 FCFS K=4	I/O 2 FCFS K=5	I/O 2 PS
= 5	2	0.828*	0.846	0.853	0.856	0.858	***	1.017	1.006	1.001	0.999	0.998		0.154	0.149	0.146	0.145	0.144	
		(0.792)**	(0.843)	(0.863)	(0.872)	(0.877)		(0.805)	(0.839)	(0.855)	(0.861)	(0.865)		(0.398)	(0.397)	(0.395)	(0.394)	(0.394)	
	3	1.228	1.251	1.260	1.263	1.265		1.586	1.572	1.569	1.568	1.568		0.186	0.177	0.171	0.168	0.147	
		(1.076)	(1.184)	(1.235)	(1.258)	(1.271)		(1.092)	(1.173)	(1.213)	(1.232)	(1.243)		(0.454)	(0.453)	(0.450)	(0.447)	(0.446)	
	4	1.609	1.634	1.638	1.638	1.638		2.185	2.170	2.174	2.179	2.183		0.206	0.195	0.187	0.182	0.180	
		(1.354)	(1.518)	(1.602)	(1.645)	(1.668)		(1.373)	(1.500)	(1.571)	(1.607)	(1.627)		(0.487)	(0.490)	(0.485)	(0.480)	(0.476)	
.05	2	0.828	0.841	0.852	0.865	0.879	0.828	1.017	1.009	1.002	0.993	0.984	1.017	0.154	0.150	0.146	0.142	0.137	0.154
		(0.792)	(0.829)	(0.861)	(0.896)	(0.932)	(0.792)	(0.805)	(0.830)	(0.853)	(0.878)	(0.904)	(0.805)	(0.398)	(0.397)	(0.395)	(0.393)	(0.391)	(0.398)
	3	1.228	1.242	1.258	1.282	1.314	1.228	1.586	1.579	1.570	1.554	1.531	1.586	0.186	0.179	0.172	0.164	0.156	0.186
		(1.076)	(1.157)	(1.231)	(1.304)	(1.374)	(1.076)	(1.092)	(1.153)	(1.210)	(1.267)	(1.323)	(1.092)	(0.454)	(0.452)	(0.450)	(0.446)	(0.442)	(0.454)
	4	1.609	1.617	1.636	1.673	1.731	1.609	2.185	2.186	2.176	2.149	2.102	2.185	0.206	0.197	0.188	0.178	0.168	0.206
		(1.354)	(1.480)	(1.598)	(1.710)	(1.811)	(1.354)	(1.373)	(1.472)	(1.567)	(1.659)	(1.744)	(1.373)	(0.487)	(0.487)	(0.485)	(0.481)	(0.476)	(0.487)
.10	2	0.828	0.845	0.863	0.884		0.828	1.017	1.006	0.994	0.980		1.017	0.154	0.149	0.143	0.136		0.154
		(0.792)	(0.840)	(0.889)	(0.943)		(0.792)	(0.805)	(0.838)	(0.873)	(0.912)		(0.805)	(0.398)	(0.397)	(0.394)	(0.392)		(0.398)
	3	1.228	1.250	1.282	1.332		1.228	1.586	1.573	1.551	1.542		1.586	0.186	0.177	0.167	0.156		0.186
		(1.076)	(1.180)	(1.283)	(1.382)		(1.076)	(1.092)	(1.170)	(1.249)	(1.326)		(1.092)	(0.454)	(0.453)	(0.451)	(0.447)		(0.454)
	4	1.609	1.631	1.680	1.768		1.609	2.185	2.173	2.136	2.060		2.185	0.206	0.196	0.184	0.172		0.206
		(1.354)	(1.512)	(1.667)	(1.808)		(1.354)	(1.373)	(1.496)	(1.621)	(1.733)		(1.373)	(0.487)	(0.489)	(0.489)	(0.490)		(0.487)
.15	2	0.828	0.848	0.872			0.828	1.017	1.004	0.988			1.017	0.154	0.148	0.140			0.154
		(0.792)	(0.849)	(0.913)			(0.792)	(0.805)	(0.844)	(0.889)			(0.805)	(0.398)	(0.397)	(0.394)			(0.398)
	3	1.228	1.257	1.306			1.228	1.586	1.567	1.529			1.586	0.186	0.176	0.164			0.186
		(1.076)	(1.196)	(1.320)			(1.076)	(1.092)	(1.181)	(1.276)			(1.092)	(0.454)	(0.454)	(0.453)			(0.454)
	4	1.609	1.644	1.727			1.609	2.185	2.161	2.091			2.185	0.206	0.195	0.183			0.206
		(1.354)	(1.533)	(1.713)			(1.354)	(1.373)	(1.511)	(1.653)			(1.373)	(0.487)	(0.492)	(0.497)			(0.487)
.35	2	0.828	0.860				0.828	1.017	0.995				1.017	0.154	0.145				0.154
		(0.792)	(0.880)				(0.792)	(0.805)	(0.864)				(0.805)	(0.398)	(0.398)				(0.398)
	3	1.228	1.288				1.228	1.586	1.538				1.586	0.186	0.174				0.186
		(1.076)	(1.242)				(1.076)	(1.092)	(1.211)				(1.092)	(0.454)	(0.460)				(0.454)
	4	1.609	1.703				1.609	2.185	2.102				2.185	0.206	0.196				0.206
		(1.354)	(1.585)				(1.354)	(1.373)	(1.544)				(1.373)	(0.487)	(0.505)				(0.487)

*Average quene length **Standard deviation ***Not analyzed

TABLE 7

AVERAGE QUENE LENGTH AND STANDARD DEVIATION OF MODELS A AND B

(1/u=0.075 sec.)

P	N	CPU FCFS K=1	CPU FCFS K=2	CPU FCFS K=3	CPU FCFS K=4	CPU FCFS K=5	CPU PS	I/O 1 FCFS K=1	I/O 1 FCFS K=2	I/O 1 FCFS K=3	I/O 1 FCFS K=4	I/O 1 FCFS K=5	I/O 1 PS	I/O 2 FCFS K=1	I/O 2 FCFS K=2	I/O 2 FCFS K=3	I/O 2 FCFS K=4	I/O 2 FCFS K=5	I/O 2 PS
.5	2	1.089*	1.093	1.095	1.095	1.096	***	0.786	0.786	0.787	0.787	0.787		0.124	0.120	0.118	0.118	0.117	
		(0.802)**	(0.847)	(0.864)	(0.871)	(0.874)		(0.787)	(0.821)	(0.834)	(0.840)	(0.843)		(0.358)	(0.357)	(0.356)	(0.356)	(0.356)	
	3	1.711	1.686	1.674	1.668	1.665		1.141	1.173	1.188	1.195	1.199		0.147	0.142	0.138	0.137	0.136	
		(1.084)	(1.185)	(1.228)	(1.248)	(1.258)		(1.064)	(1.148)	(1.185)	(1.202)	(1.211)		(0.401)	(0.404)	(0.403)	(0.402)	(0.401)	
	4	2.376	2.291	2.249	2.228	2.217		1.463	1.553	1.600	1.623	1.636		0.161	0.155	0.151	0.148	0.147	
		(1.351)	(1.515)	(1.592)	(1.629)	(1.648)		(1.329)	(1.470)	(1.539)	(1.572)	(1.590)		(0.426)	(0.434)	(0.432)	(0.430)	(0.428)	
.05	2	1.089	1.092	1.095	1.097	1.101	1.089	0.786	0.786	0.787	0.787	0.788	0.786	0.124	0.122	0.119	0.115	0.112	0.124
		(0.802)	(0.833)	(0.862)	(0.894)	(0.930)	(0.802)	(0.787)	(0.810)	(0.833)	(0.858)	(0.886)	(0.787)	(0.358)	(0.357)	(0.356)	(0.356)	(0.355)	(0.358)
	3	1.711	1.691	1.674	1.664	1.664	1.711	1.141	1.166	1.187	1.203	1.210	1.141	0.147	0.143	0.139	0.133	0.126	0.141
		(1.084)	(1.156)	(1.224)	(1.296)	(1.370)	(1.084)	(1.064)	(1.124)	(1.182)	(1.242)	(1.304)	(1.064)	(0.401)	(0.402)	(0.403)	(0.402)	(0.400)	(0.401)
	4	2.376	2.306	2.251	2.215	2.213	2.376	1.463	1.537	1.598	1.640	1.652	1.463	0.161	0.157	0.151	0.144	0.136	0.161
		(1.351)	(1.472)	(1.586)	(1.702)	(1.811)	(1.351)	(1.329)	(1.433)	(1.534)	(1.635)	(1.730)	(1.329)	(0.426)	(0.430)	(0.432)	(0.432)	(0.430)	(0.426)
.10	2	1.089	1.093	1.097	1.103		1.089	0.786	0.786	0.787	0.787		0.786	0.124	0.121	0.116	0.110		0.124
		(0.802)	(0.844)	(0.890)	(0.946)		(0.802)	(0.787)	(0.819)	(0.855)	(0.898)		(0.787)	(0.358)	(0.357)	(0.356)	(0.356)		(0.358)
	3	1.711	1.686	1.671	1.678		1.711	1.141	1.172	1.194	1.196		1.141	0.147	0.142	0.135	0.126		0.147
		(1.084)	(1.180)	(1.282)	(1.387)		(1.084)	(1.064)	(1.144)	(1.228)	(1.315)		(1.064)	(0.401)	(0.404)	(0.405)	(0.405)		(0.401)
	4	2.376	2.293	2.240	2.251		2.376	1.463	1.551	1.612	1.611		1.463	0.161	0.156	0.148	0.138		0.161
		(1.351)	(1.508)	(1.669)	(1.818)		(1.351)	(1.329)	(1.464)	(1.603)	(1.728)		(1.329)	(0.426)	(0.433)	(0.438)	(0.443)		(0.426)
.15	2	1.089	1.094	1.100			1.089	0.786	0.786	0.786			0.786	0.124	0.120	0.113			0.124
		(0.802)	(0.854)	(0.917)			(0.802)	(0.787)	(0.826)	(0.875)			(0.787)	(0.358)	(0.357)	(0.357)			(0.358)
	3	1.711	1.685	1.678			1.711	1.141	1.174	1.191			1.141	0.147	0.141	0.132			0.147
		(1.084)	(1.198)	(1.327)			(1.084)	(1.064)	(1.158)	(1.263)			(1.064)	(0.401)	(0.405)	(0.408)			(0.401)
	4	2.376	2.288	2.257			2.376	1.463	1.557	1.579			1.463	0.161	0.155	0.146			0.161
		(1.351)	(1.533)	(1.725)			(1.351)	(1.329)	(1.485)	(1.645)			(1.329)	(0.426)	(0.436)	(0.447)			(0.426)
.35	2	1.089	1.099				1.089	0.786	0.785				0.786	0.124	0.117				0.124
		(0.802)	(0.890)				(0.802)	(0.787)	(0.853)				(0.787)	(0.358)	(0.360)				(0.358)
	3	1.711	1.693				1.711	1.141	1.168				1.141	0.147	0.139				0.147
		(1.084)	(1.257)				(1.084)	(1.064)	(1.200)				(1.064)	(0.401)	(0.414)				(0.401)
	4	2.376	2.305				2.376	1.463	1.540				1.463	0.161	0.155				0.161
		(1.351)	(1.603)				(1.351)	(1.329)	(1.533)				(1.329)	(0.426)	(0.454)				(0.426)

*Average quene length **Standard deviation ***Not analyzed

TABLE 8

AVERAGE QUENE LENGTH AND STANDARD DEVIATION OF MODELS A AND B

(1/u=0.100 sec.)

P	N	CPU FCFS K=1	CPU FCFS K=2	CPU FCFS K=3	CPU FCFS K=4	CPU FCFS K=5	CPU PS	I/O 1 FCFS K=1	I/O 1 FCFS K=2	I/O 1 FCFS K=3	I/O 1 FCFS K=4	I/O 1 FCFS K=5	I/O 1 PS	I/O 2 FCFS K=1	I/O 2 FCFS K=2	I/O 2 FCFS K=3	I/O 2 FCFS K=4	I/O 2 FCFS K=5	I/O 2 PS
= 5	2	1.269*	1.265	1.263	1.262	1.262	***	0.628	0.636	0.638	0.640	0.640		0.103	0.100	0.099	0.098	0.098	
		(0.773)**	(0.815)	(0.830)	(0.837)	(0.840)		(0.745)	(0.780)	(0.792)	(0.798)	(0.801)		(0.325)	(0.326)	(0.325)	(0.325)	(0.325)	
	3	2.030	1.982	1.963	1.954	1.949		0.852	0.902	0.924	0.933	0.938		0.118	0.116	0.114	0.113	0.112	
		(1.010)	(1.115)	(1.157)	(1.175)	(1.185)		(0.976)	(1.068)	(1.105)	(1.122)	(1.131)		(0.358)	(0.363)	(0.364)	(0.363)	(0.363)	
	4	2.853	2.735	2.680	2.654	2.640		1.020	1.140	1.197	1.224	1.239		0.126	0.128	0.123	0.122	0.121	
		(1.210)	(1.392)	(1.472)	(1.509)	(1.528)		(1.172)	(1.338)	(1.412)	(1.447)	(1.465)		(0.374)	(0.385)	(0.387)	(0.387)	(0.386)	
.05	2	1.269	1.266	1.263	1.259	1.255	1.269	0.628	0.633	0.638	0.644	0.651	0.628	0.103	0.101	0.099	0.097	0.094	0.103
		(0.773)	(0.801)	(0.829)	(0.860)	(0.898)	(0.773)	(0.745)	(0.768)	(0.791)	(0.817)	(0.849)	(0.745)	(0.325)	(0.325)	(0.325)	(0.326)	(0.326)	(0.325)
	3	2.030	1.996	1.964	1.933	1.908	2.030	0.852	0.888	0.922	0.956	0.987	0.852	0.118	0.116	0.114	0.110	0.106	0.118
		(1.010)	(1.082)	(1.153)	(1.231)	(1.316)	(1.010)	(0.976)	(1.039)	(1.102)	(1.171)	(1.246)	(0.976)	(0.358)	(0.361)	(0.363)	(0.365)	(0.366)	(0.358)
	4	2.853	2.768	2.684	2.604	2.550	2.853	1.020	1.107	1.192	1.277	1.337	1.020	0.126	0.125	0.123	0.119	0.113	0.126
		(1.210)	(1.338)	(1.465)	(1.602)	(1.737)	(1.210)	(1.172)	(1.289)	(1.406)	(1.531)	(1.653)	(1.172)	(0.374)	(0.381)	(0.386)	(0.391)	(0.393)	(0.374)
.10	2	1.269	1.265	1.260	1.253		1.269	0.628	0.635	0.643	0.654		0.628	0.103	0.100	0.097	0.092		0.103
		(0.773)	(0.813)	(0.859)	(0.919)		(0.773)	(0.745)	(0.778)	(0.815)	(0.865)		(0.745)	(0.325)	(0.326)	(0.326)	(0.327)		(0.325)
	3	2.030	1.985	1.940	1.912		2.030	0.852	0.900	0.948	0.984		0.852	0.118	0.116	0.111	0.105		0.118
		(1.010)	(1.109)	(1.220)	(1.343)		(1.010)	(0.976)	(1.063)	(1.160)	(1.266)		(0.976)	(0.358)	(0.363)	(0.367)	(0.371)		(0.358)
	4	2.853	2.740	2.629	2.573		2.853	1.020	1.135	1.250	1.312		1.020	0.126	0.125	0.121	0.114		0.126
		(1.210)	(1.383)	(1.574)	(1.757)		(1.210)	(1.172)	(1.329)	(1.502)	(1.663)		(1.172)	(0.374)	(0.385)	(0.394)	(0.405)		(0.374)
.15	2	1.269	1.264	1.257			1.269	0.628	0.637	0.648			0.628	0.103	0.099	0.095			0.103
		(0.773)	(0.822)	(0.888)			(0.773)	(0.745)	(0.785)	(0.839)			(0.745)	(0.325)	(0.326)	(0.328)			(0.325)
	3	2.030	1.976	1.929			2.030	0.852	0.908	0.962			0.852	0.118	0.115	0.109			0.118
		(1.010)	(1.131)	(1.277)			(1.010)	(0.976)	(1.081)	(1.208)			(0.976)	(0.358)	(0.364)	(0.372)			(0.358)
	4	2.853	2.772	2.610			2.853	1.020	1.154	1.270			1.020	0.126	0.125	0.120			0.126
		(1.210)	(1.417)	(1.652)			(1.210)	(1.172)	(1.360)	(1.567)			(1.172)	(0.374)	(0.388)	(0.405)			(0.374)
.35	2	1.269	1.260				1.269	0.628	0.643				0.628	0.103	0.097				0.103
		(0.773)	(0.865)				(0.773)	(0.745)	(0.819)				(0.745)	(0.325)	(0.329)				(0.325)
	3	2.030	1.960				2.030	0.852	0.926				0.852	0.118	0.114				0.118
		(1.010)	(1.208)				(1.010)	(0.976)	(1.142)				(0.976)	(0.358)	(0.376)				(0.358)
	4	2.853	2.690				2.853	1.020	1.184				1.020	0.126	0.126				0.126
		(1.210)	(1.522)				(1.210)	(1.172)	(1.442)				(1.172)	(0.374)	(0.410)				(0.374)

*Average quene length **Standard deviation ***Not analyzed

are 6 for CPU, 15 for I/O 1, 3 for I/O 2 at 0.025 second, 18, 21, 4 at 0.05 second, 21, 16, 3 at 0.075 second, and 18, 11,2 at 0.1 second. The relative small increments of I/O 2 is a result of the low percentage of time (25%) that a job finishing computing in CPU will join I/O queue.

Increment of the probability p that a job in CPU will require longer CPU service time will cause the increment of idle time of all devices. Setting N=4, k=2 and from (16), the percentages of idle time increment from p=0.05 to 0.35 for CPU are 1 at 0.025 second, 4 at 0.05 second, 6 at 0.075 second, and 7 at 0.1 second. The percentages are 2, 4, 5, 4 for I/O 1 and 0.5, 1, 1, 1 for I/O 2. Thus idle time changes for p are rather insignificient compared to k. And p has no effect on device utilization when k=1.

A job leaving CPU has three times the probability of going to the queue of I/O 1 than of I/O 2. Therefore inactivity in I/O 2 is high. This can be verified by the larger idle time percentages of this device shown in Table 1 through 4.

Increasing the CPU mean service time will improve the relative performance of CPU while at the same time reducing the I/O device utilization. The average percentages of improvement in CPU utilization are 20 when CPU mean service time changes from 0.025 second to 0.05 second, 14 from 0.05 to 0.075 second, and 8 from 0.075 to 0.1 second. At the same time, the average percentages of decrement in device utilization for I/O 1 are 15, 14 and 10. The values are 4, 3 and 2 for I/O 2. These figures can be explained by utilization factors of each device in the system. As mentioned earlier, the utilization factor for each device is defined to be the quotient of mean arrival rate over mean service rate. The factors, under CPU mean service times of 0.025 to 0.1 second, are 0.37, 0.74, 1.11 and 1.48 for CPU, 2.35, 1.17, 0.78 and 0.59 for I/O 1, and 0.48, 0.24, 0.16 and 0.12 for I/O 2. Thus under increasing CPU mean service time, utilization factors are increasing for CPU and decreasing for I/O devices. The low values of I/O 2 also helps to explain the large numbers of I/O 2 idle time percentages.

It is interesting to note that system idle time values under PS discipline are the same under FCFS discipline when k is 1. This means that PS behaves the same when CPU is exponentially distributed and under FCFS discipline. It is clear that PS yields better performance than FCFS when $k>1$.

While idle time is used as the measure of device utilization, the mean queue length(QL) reflects bottlenecks among devices in the system and the standard deviation(SD) of QL measures dispersal of the queue length of each device. Effects of N on QL and SD have discussed earlier. As shown in Table 5 through 8, increasing the CPU mean service time causes increments of QL and SD for CPU and decrements for I/O 1 and I/O 2. The decrements for I/O 2 is insignificant because of the low arrival rate of this device. Values of

SD in I/O 2 is at least twice as great as the QL measures. This is a direct result of the high percentage of time that I/O 2 is idle. For CPU and I/O 1 the increments and decrements are quite significant. The average change is more than 20 percent with each increment of CPU service time being 0.025 second. The bottleneck remains at the queue of the device with the highest utilization factor. Therefore when CPU mean service time is at 0.05 second or less, jobs queue up at I/O 1. When CPU service time is greater than 0.05 second, QL is the largest at CPU queue.

Under FCFS service discipline, for any given N, and under the service distribution of either (16) or (24), the changes of QL and SD of any device due to changes of k values are not significant for any CPU mean service time. Aside from the relatively very small changes of QL and SD in I/O 2 queue, increment of k values from 1 to 5 will cause increases in QL and SD for CPU and decreases in QL but increases SD for I/O 1 if 1/u is less than 0.1 second. When 1/u is 0.1 second, the SD increases for either CPU or I/O 1 while QL is increasing for the I/O devices but decreasing for the CPU.

The effect of p on QL and SD are not significant, particularly when CPU mean service time is greater than 0.05 second. When 1/u is less than 0.075 second, the changes of p from 0.05 to 0.35 will cause increases of QL and SD for CPU but decreases for I/O devices. The results are reversed when service time is greater than 0.075 second.

Again the values of QL and SD under PS discipline are the same as the values under FCFS discipline with k equals 1.

In general, mean CPU service time and utilization factors of each device are important elements in determining device utilizations and bottlenecks. The value of k is also important in terms of device utilization while the value of p is not. PS discipline behaves just as FCFS discipline with CPU service distribution being exponential; and PS is superior in system performance than hyperexponentially distributed CPU service time under FCFS discipline. Therefore it is strongly suggested that when CPU service distribution is hyperexponential and when round robin scheduling strategy is used, the best policy is to keep time quantum as small as possible subject to incurring of excessive overhead.

CHAPTER IV

QUEUING MODELS FOR I/O UTILIZATION

In addition to the important role of the CPU in the computer system, input-output devices also contribute to the over all system performance. Badly organized I/O devices in a system will degrade the effectiveness of the entire system. Therefore it is worthwhile to study the effects of various I/O configurations in the system upon CPU utilization and upon the whole system utilizations. The effects of different CPU service disciplines (FCFS and PS) with each I/O configuration are compared.

The System

The computer system considered in this chapter resembles a partial configuration of the University of Texas at Austin CDC 6600 computer system which includes one CPU and four disk units. In the UT-2 operating system for the CDC 6600 computer, one of the disks is used for storing system libraries while the other three are scratch disks used for storing users' jobs. Because the system libraries only occupy a small fraction of the tracks of the system disk(SD), there results in large waste of space in the SD. The probability of jobs in the system requesting the service of SD is three times as frequent than the probability of requesting the service of user disks(UD), thereby leaving high inactivity of the UDs and queuing for the SD. Aside from the space utilization problem, the question arises as to whether reorganization of the disks would improve the overall system performance. This study seeks to obtain an answer to this question by comparing system utilizations of these four disk units.

The CPU service time under the present system models is hyperexponentially distributed with two branches; one with low arrival probability and slower service time and the other with high arrival probability and faster service time. The four disks are assumed to have exponentially distributed service time. Jobs in the system traverse between the CPU and one of the disks. There is no path between any two disks. The system is both cyclic and closed, as shown in the figure below:

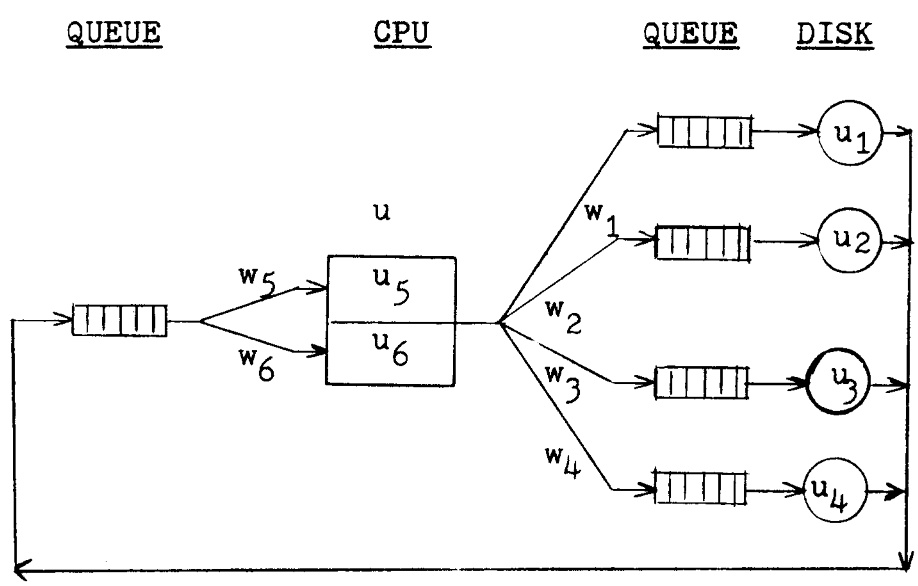

Figure 2. System Configuration of Models C and D.

The Models

Depending upon the CPU scheduling strategies, two different models are formulated for analysis. Model C is used when CPU scheduling is FCFS and Model D is used when CPU scheduling is PS. Jobs in I/O queues are serviced in FCFS.

Let:

w_1 = Arrival probability of queue i, with queue 1 through 4 being queues of disks, i=5 being the CPU branch with slower service time, and i=6 being the CPU branch with faster service time

u = Mean CPU service rate

u_i = Mean service rate of queue i or branch i

N = Total number of jobs in the system

n_i = Number of jobs in queue i or branch i of CPU and

$$\sum_{i=1}^{6} n_i = N$$

m = Number of jobs in CPU queue and $m=n_5 + n_6$

δ_{ij} = 1 if i=j and 0 if $i \neq j$

$P(n_1, n_2, \ldots\ldots n_6)$ = State that there are n_i jobs in queue i or branch i

<u>Model C</u>

Under FCFS CPU service discipline and under CPU probability density function of form (16), at least n_5 or n_6 must be o. The detailed balance equations are of the form:

$$\left(\sum_{i=1}^{6}(1-\delta_{0n_i})u_i\right) p(n_1, n_2,\ldots,n_6)$$

$$=\sum_{i=1}^{4}\sum_{j=5}^{6}(1-\delta_{0n_j})\left[(1-\delta_{1n_j})+\delta_{1n_j}w_j\right]u_iP(\ldots,n_i+1,\ldots,n_j-1,\ldots)$$

$$+(1-\delta_{Nm})\sum_{i=1}^{4}\sum_{j=5}^{6}(1-\delta_{0n_i})w_i\left\{\sum_{k=5}^{6}\left[(1-\delta_{0m})w_k+\delta_{0m}\right]u_k\ P(\ldots,n_i-1,\ldots,n_k+1,\ldots)\right\} \qquad (27)$$

and $\sum_i P_i(n_1,\ldots,n_6) = 1$

Solutions can be obtained by the same method of Model A.

Model D

Define PTERM(i) = $1/u_i$ and BTERM(i) = w_i, LBE of state $P(n_1, n_2,\ldots,n_6)$ is:

$$P(n_1,\ldots,n_6)=\text{NORM}*\frac{(n_5+n_6)!}{n_5!n_6!}\prod_{i=1}^{6}(\text{BTERM}(i)*\text{PTERM}(i))^{n_i} \qquad (28)$$

where

$$\text{NORM}=\left(\sum_{\sum_i n_i=N}\frac{(n_5+n_6)!}{n_5!n_6!}\prod_{i=1}^{6}(\text{BTERM}(i)*\text{PTERM}(i))^{n_i}\right)^{-1} \qquad (29)$$

and $n_5 + n_6 \geq n_5$ or n_6.

The Experiments

There are three types of disks: the system disk(SD) which stores system routines and libraries, the user disk(UD) which storẽs users' jobs, and the system-user disk(SUD) which stores both system routines and users' jobs. I/O requests for SD are three times the requests for UD. Then SUD requests are more than three times of UD requests. The mean service times for SD, UD and SUD are 0.078, 0.048 and 0.085 second. Mean CPU service time is assumed to be 0.025 second with coefficient of variation being 5, w_5 is 0.05, w_6 is 0.095, u_5 is 0.4035 second and u_6 is 0.0051 second. Then for each N and under each CPU service discipline, device utilizations are computed for five configurations of the four disks:

	Disk 1	Disk 2	Disk 3	Disk 4
Case 1	SD	UD	UD	UD
Case 2	SD	SD	UD	UD
Case 3	SUD	SUD	UD	UD
Case 4	SUD	SUD	SUD	UD
Case 5	SUD	SUD	SUD	SUD

The values of N are set at 3, 4, and 5.

Results

The idle time measures of all devices in the system under five different cases and under three different N values are listed in Table 9; and measures of mean queue lengths and its standard deviations are listed in Table 10.

In case 1 of Table 9, SD(I/O 1) is highly utilized

TABLE 9

SYSTEM IDLE TIME OF MODELS C AND D

Device	N	Case 1		Case 2		Case 3		Case 4		Case 5	
		FCFS	PS	FCFS	PS	FCFS	PS	FCFS	PS	FCFS	PS
CPU	3	0.567	0.479	0.572	0.489	0.608	0.537	0.596	0.524	0.586	0.514
	4	0.540	0.431	0.533	0.424	0.573	0.482	0.552	0.454	0.535	0.433
	5	0.520	0.403	0.504	0.378	0.547	0.443	0.518	0.402	0.496	0.371
I/O 1	3	0.324	0.187	0.499	0.403	0.466	0.370	0.576	0.502	0.648	0.587
	4	0.281	0.112	0.453	0.326	0.418	0.295	0.530	0.429	0.604	0.518
	5	0.251	0.069	0.419	0.272	0.383	0.242	0.495	0.373	0.571	0.465
I/O 2	3	0.861	0.833	0.499	0.403	0.466	0.370	0.576	0.502	0.648	0.587
	4	0.852	0.818	0.453	0.326	0.418	0.295	0.530	0.429	0.604	0.518
	5	0.846	0.809	0.419	0.272	0.383	0.242	0.495	0.373	0.571	0.465
I/O 3	3	0.861	0.833	0.897	0.877	0.925	0.911	0.576	0.502	0.648	0.587
	4	0.852	0.818	0.888	0.862	0.918	0.900	0.530	0.429	0.604	0.518
	5	0.846	0.809	0.881	0.851	0.913	0.893	0.495	0.373	0.571	0.465
I/O 4	3	0.862	0.834	0.897	0.877	0.925	0.911	0.941	0.931	0.648	0.587
	4	0.853	0.819	0.888	0.862	0.918	0.900	0.935	0.920	0.604	0.518
	5	0.847	0.810	0.881	0.851	0.913	0.893	0.930	0.913	0.571	0.465

TABLE 10

AVERAGE QUEUE LENGTH AND STANDARD DEVIATION OF MODELS C AND D

Device	N	Case 1		Case 2		Case 3		Case 4		Case 5	
		FCFS	PS	FCFS	PS	FCFS	PS	FCFS	PS	FCFS	PS
CPU	3	1.022* (1.299)**	0.813 (0.930)	1.005 (1.291)	0.778 (0.904)	0.909 (1.256)	0.682 (0.860)	0.937 (1.264)	0.697 (0.859)	0.960 (1.272)	0.712 (0.860)
	4	1.363 (1.705)	1.031 (1.139)	1.383 (1.709)	1.024 (1.115)	1.240 (1.659)	0.872 (1.044)	1.311 (1.682)	0.926 (1.057)	1.367 (1.698)	0.970 (1.067)
	5	1.684 (2.097)	1.213 (1.330)	1.757 (2.117)	1.259 (1.317)	1.564 (2.051)	1.043 (1.213)	1.688 (2.092)	1.153 (1.249)	1.783 (2.118)	1.240 (1.274)
I/O 1	3	1.482 (1.209)	1.611 (1.050)	0.881 (1.033)	0.976 (0.979)	0.963 (1.064)	1.063 (1.007)	0.667 (0.903)	0.743 (0.882)	0.510 (0.789)	0.572 (0.783)
	4	2.087 (1.571)	2.317 (1.288)	1.177 (1.316)	1.331 (1.266)	1.288 (1.357)	1.455 (1.265)	0.873 (1.128)	0.996 (1.090)	0.658 (0.970)	0.757 (0.955)
	5	2.723 (1.927)	3.090 (1.500)	1.478 (1.596)	1.697 (1.473)	1.618 (1.644)	1.860 (1.523)	1.078 (1.346)	1.251 (1.295)	0.804 (1.144)	0.940 (1.121)
I/O 2	3	0.166 (0.444)	0.193 (0.459)	0.881 (1.033)	0.976 (0.979)	0.963 (1.064)	1.063 (1.007)	0.667 (0.903)	0.743 (0.882)	0.510 (0.789)	0.572 (0.783)
	4	0.184 (0.485)	0.218 (0.501)	1.177 (1.316)	1.331 (1.266)	1.288 (1.357)	1.455 (1.265)	0.873 (1.128)	0.996 (1.090)	0.658 (0.970)	0.757 (0.955)
	5	0.198 (0.518)	0.233 (0.527)	1.478 (1.596)	1.697 (1.473)	1.618 (1.644)	1.860 (1.523)	1.078 (1.346)	1.251 (1.295)	0.804 (1.144)	0.940 (1.121)
I/O 3	3	0.166 (0.444)	0.193 (0.459)	0.116 (0.363)	0.136 (0.380)	0.082 (0.301)	0.096 (0.317)	0.667 (0.903)	0.743 (0.882)	0.510 (0.789)	0.572 (0.783)
	4	0.184 (0.485)	0.218 (0.501)	0.131 (0.397)	0.157 (0.417)	0.092 (0.324)	0.109 (0.343)	0.873 (1.128)	0.996 (1.090)	0.658 (0.970)	0.757 (0.955)
	5	0.198 (0.518)	0.233 (0.527)	0.144 (0.424)	0.173 (0.443)	0.699 (0.342)	0.119 (0.361)	1.078 (1.346)	1.251 (1.295)	0.804 (1.144)	0.940 (1.121)
I/O 4	3	0.162 (0.442)	0.191 (0.457)	0.116 (0.363)	0.136 (0.380)	0.082 (0.301)	0.096 (0.317)	0.063 (0.260)	0.073 (0.276)	0.510 (0.789)	0.572 (0.783)
	4	0.183 (0.483)	0.216 (0.499)	0.131 (0.397)	0.157 (0.417)	0.092 (0.324)	0.109 (0.343)	0.071 (0.281)	0.085 (0.301)	0.658 (0.970)	0.757 (0.955)
	5	0.197 (0.516)	0.231 (0.525)	0.144 (0.424)	0.173 (0.443)	0.099 (0.342)	0.119 (0.361)	0.078 (0.298)	0.095 (0.319)	0.804 (1.144)	0.940 (1.121)

*Average queue length **Standard deviation while the three UDs (I/O 2,3,4,) are idle most of the time (86%). The inactivity of user disk compared to SD or SUD is also revealed by the large idle time percentages in case 2 to case 4, with the highest number being 0.941 in case 4 when N is 3 and under FCFS discipline. In case 5 all disks are equally utilized.

The effects of I/O devices on CPU utilization is not obvious if CPU service time is large. When CPU service time is set at 0.025 second, significant facts are developed. Firstly, CPU utilization is worst in case 3 which has the largest idle time fractions than any of the other four cases. Secondly, although increasing N will improve all device utilizations, the effect is more significant on CPU utilization in case 5. There is (in case 5) a 9 percent improvement in CPU utilization from 3 to 5 of N under FCFS and a 14 percent improvement under PS. The figures 0.496 and 0.371 when N is 5 are the lowest idle times among all other cases. Thirdly, devices are better utilized under PS than FCFS, which corresponds to the conclusion of the previous chapter.

Measures of mean queue length(QL) and its standard deviation (SD) for all devices as shown in Table 10 indicate that jobs in the system queue up between CPU and SD or SUD. Thus the mean queue length of UD is small in case 1 through case 4 and its SD is about three times larger than QL, which reflects the inactivity of the UD. QL and SD are evenly utilized among all disks in case 5.

In all cases, QL and SD of CPU under FCFS are larger than under PS. For SD and SUD, QL in FCFS is larger but SD is smaller than QL and SD under PS, and for UD both QL and SD of FCFS are smaller than of PS.

The importance of balancing the I/O load is clear. Case 1 through 5 involve the sequential replacing of faster disk service time with slower disk service time and yet the over all performance of the system improves (except for case 3). This improvement is obviously a result of the more balanced load in the larger numbered cases where queues decrease rapidly as I/O service time is increasing.

CHAPTER V

SUMMARIES AND CONCLUSIONS

Two models were constructed to analyze the effect of CPU service time on a computer system model with one CPU having hyperexponentially distributed service time and two I/O devices having exponentially distributed service times. The mean CPU service time, the coefficient of variation of the CPU service time, and the probability of choosing the faster branch of the hyperexponential server (by holding the mean service rate constant) were varriants. Two models were constructed in analyzing the effects of I/O configuration on a computer system with one hyperexponentially distributed CPU and four exponentially distributed I/O devices. Five different conbinations of disk service times are evaluated. Results are obtained and compared by varing the parameters of the models. Through the analyzes of the last two chapters summaries and conclusions are drawn as follows:

1. Increasing N, the number of jobs in the system, will increase the utilization, the mean queue lengths and standard deviations of queue lengths of all devices in the system.
2. Under FCFS-CPU service discipline, increasing k, the coefficient of variation of CPU mean service time, will cause increments of idle times of all devices and the increments are significant. Variations of p, the arrival

probability of the slow CPU branch, will result in variations in idle time of all devices; however, the changes are not as large as those resulting from variations in k. This suggests that the general properties of the distribution are more significant than local properties.

3. Changes of average queue length and standard deviation of queue length of each device in the system depend more on the utilization factor of each device. Device with large utilization factor will have large values of queue length and standard deviation. The effects of k and p under FCFS discipline upon the queue length and its standard deviation are not significant compared to the effect of the utilization factor.
4. Effects of various parameters in the models have little impact upon the performance of any device with low arrival probability. The standard deviation is at least twice as the mean queue length for such a device. However, the differences of these two measures are not large enough as expected because as the coefficient of variation k increases the difference between the fast CPU service rate and the slow CPU service rate widens and thereby causes large variation of jobs staying in the queue than average from time to time.
5. A system with hyperexponentially distributed CPU service time under PS service discipline behaves the same as a system with exponentially distributed CPU service time under FCFS discipline. Since increasing the value

of k will degrade the system performance under FCFS, PS yields better system utilization than FCFS, particularly when k is much larger than 1.

6. Balanced utilization of all I/O devices in computer system not only reduces the bottleneck of jobs queuing at certain parts of the system but also improves the CPU utilization and hence the whole system. This effect increases with N.

The above points provide some basic information about the characteristics of a simple computer system. Through these conclusions, efforts can be done to improve the system performance. These include the control of the CPU service time, the adoption of the PS scheduling for the CPU, and the use of a balanced I/O configuration. The systems under study are by no means simple but do not span the range of existing computer systems. There are certain points which are interesting but are not analyzed here. These may include, for example, the performance of a system with two or more CPUs and I/O devices with service distributions other than exponential. Many significant problems on computer system of moderate complexity remain to be studied by queuing network models.

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