Polarograms: A new tool for image texture analysis

Larry S. Davis
University of Texas
Austin, Texas 78712

November 1979 TR-116

This research was supported in part by funds derived from the Air Force Office of Scientific Research under contract F49620-79-C-0043.

ABSTRACT

This paper introduces a new tool for image texture analysis called a polarogram. A polarogram is a polar plot of an orientation sensitive texture statistic. Polarograms give rise to a class of texture descriptors which are sensitive to both texture coarseness and directionality, but yet which are invariant to rotations of the image textures. An experiment is described in which polarograms are applied to the classification of texture samples.

1. INTRODUCTION

This paper introduces a new tool for image texture analysis called a polarogram. A polarogram is a polar representation of an orientation specific texture statistic. It can be used to obtain information about both texture coarseness and texture directionality.

Texture coarseness, or the size of the texture elements, is ordinarily related to the autocorrellation of the texture. For example, the contrast feature for grey level cooccurrence matrices (Haralick[1]) or grey level difference histograms (Weszka et al[2]) is, essentially, a measurement of a single value of the image autocorrelation. It is ordinarily computed for pairs of image points which are close to one another in the image. High values of contrast usually indicate fine textures (where there are relatively many edge points) and low values of contrast indicate coarse textures. Of course, other factors such as edge sharpness also effect the value of the contrast statistic.

Texture directionality has not been handled as adequately as coarseness. It has been suggested that directionally specific statistics be computed. For example, suppose that $P(\theta,r)$ is a polar representation of the power spectrum of a texture. Then Weszka et al[2] suggested "wedges" defined as:

$$W(\theta_1,\theta_2) = \int_{\theta_1}^{\theta_2} P(\theta,r)d\theta dr$$

as direction sensitive texture statistics. A difficulty with such statistics is that they are not rotation invariant - i.e., rotating the original texture changes the statistics. Since orientation cannot, in general, be controlled, and might even vary across a single "homogeneous" field, it is important that even statistics which measure directionality be rotation invariant. Polarograms can be used to generate rotation invariant statistics which are sensitive to texture directionality.

In Section 2 we define the polarogram, and present some examples. Section 3 contains an experimental study using the texture database employed in [3]. Finally, Section 4 contains conclusions and a discussion.

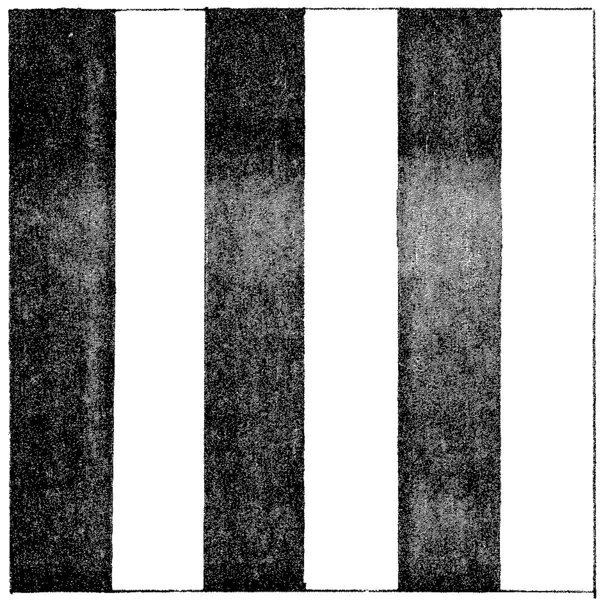
2. POLAROGRAMS

Polarograms are a new tool for describing image textures. A polarogram is a polar plot of a texture statistic as a function of orientation. For example, let D(a) be a displacement vector of fixed magnitude, d, and variable orientation, a, and $C_{D(a)}$ be the cooccurrence matrix for displacement D(a). Let f be some statistic, such as contrast, defined for a cooccurrence matrix. Then we can define the polarogram, P_f, by

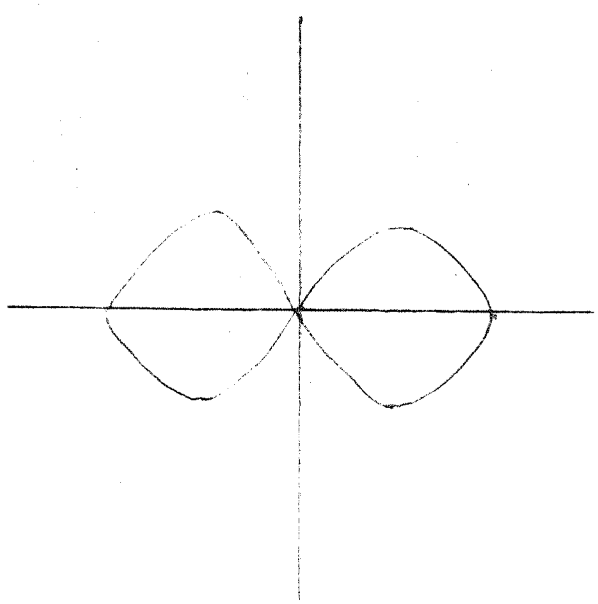
$$P_f(a) = f(C_{D(a)})$$

As a simple example, consider the vertical bar texture in Figure 1a. The bars are five pixels wide. Let D(a) have magnitude 1, and let f be the contrast statistic. Then Figure 1b displays the polarogram P_f. Note that $P_f(\pi/2) = 0$ since all pairs of adjacent vertical points have identical grey levels. $P_f(0)$, on the other hand, has value .4, since 2 out of every 5 one's are adjacent to a zero. In general, if we regard the texture in Figure 1a as a continuous image, then $P_f(a) = 2\cos(a)/5$. In practice, $P_f(a)$ is only computed for a discrete set of values for a. An interpolation function is used to compute $P_f(a)$ for intermediate values of a. The simplest interpolation scheme connects consecutive values of $P_f(a)$ with straight lines.

Texture statistics are derived from polarograms by computing size and shape features of the polarogram. The shape features will ordinarily not only depend on the shape of



a) vertical bar texture



b) polarogram for the texture in (a)

Figure 1. Example of a polarogram.

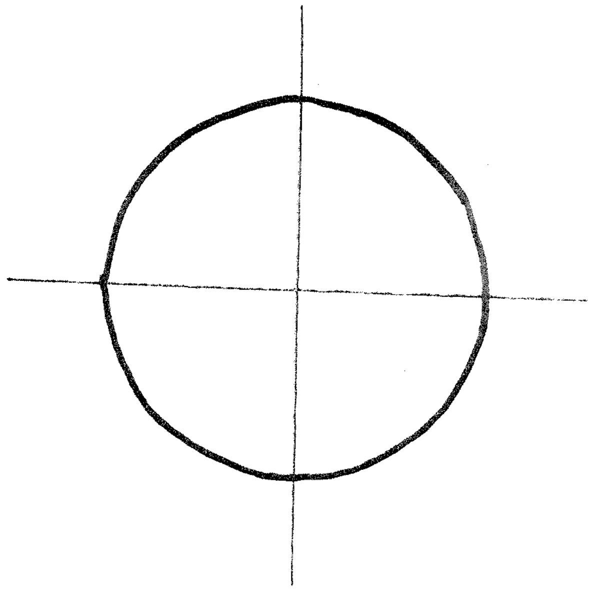
the boundary of the polarogram, but also on the position of the origin of the polarogram within that shape. For example, the polarograms in Figures 2a and 2b have the same boundary shapes (circles), but the polarogram in Figure 2a is centered at the circle center, while the polarogram in Figure 2b is centered away from the circle center. This difference indicates a crucial difference in the two underlying textures. The texture corresponding to the polarogram in Figure 2a is isotropic with respect to the statistic of the polarogram, while the texture corresponding to Figure 2b is not. The <u>skewness</u> feature defined below measures the extent of such isotropy.

An important class of statistics which can be computed from a polarogram are its <u>moments</u> from the origin (note that these are not the same as the central moments of the polarogram). The p'th moment of P_f is :

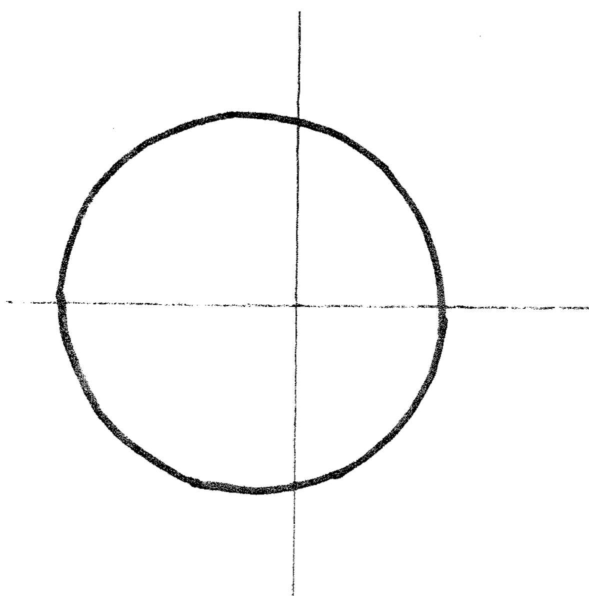
$$u_p = 1/2\pi \int_0^{2\pi} (P_f(a) - \bar{P}_f)^p \, da$$

where

$$\bar{P}_f = 1/2\pi \int_0^{2\pi} P_f(a)\, da$$



a) radially symmetric polarogram



b) non-symmetric polarogram

Figure 2. The polarogram of an isotropic and non-isotropic texture.

In the special case where the boundary of P_f is a polygon, it can be shown that:

$$u_1 = a_1$$
$$u_2 = a_2 - u_1^2$$
$$u_3 = a_3 - 3u_1a_2 + 2u_1^3$$

where

$$a_p = \sum_{i=1}^{n} \gamma_i/2\pi \, a_p(i)$$

and

$$a_1(i) = \frac{a_ib_i}{c_i} \frac{\sin\gamma_i}{\gamma_i} \log \left| \frac{(c_i+a_i - b_i\cos\gamma_i)\ (c_i+b_i - a_i\cos\gamma_i)}{a_ib_i\sin^2\gamma_i} \right|$$

$$a_2(i) = \frac{1}{\gamma_i} \left(\frac{a_ib_i}{c_i} \sin\gamma_i \right)^2 (\cot\alpha_i + \cot\beta_i)$$

$$a_3(i) = \frac{1}{2\gamma_i} \left(\frac{a_ib_i}{c_i} \sin\gamma_i \right)^3 \Big[\operatorname{cosec}\alpha_i \cot\alpha_i + \operatorname{cosec}\beta_i \cot\beta_i + \log \Big| \big[(\operatorname{cosec}\alpha_i + \cot\alpha_i)\ (\operatorname{cosec}\beta_i + \cot\beta_i) \big] \Big| \Big]$$

See Figure 3 for the definitions of α_i, β_i, γ_i, a_i, b_i, c_i.

The first moment, u_1, represents the deviation from the mean of the perimeter distance to the origin, u_2 the variance, and u_3 the skewness of the perimeter distribution. Two simple measures of size, which are used in the experiments in Section 3, are area of the polarogram and perimeter.

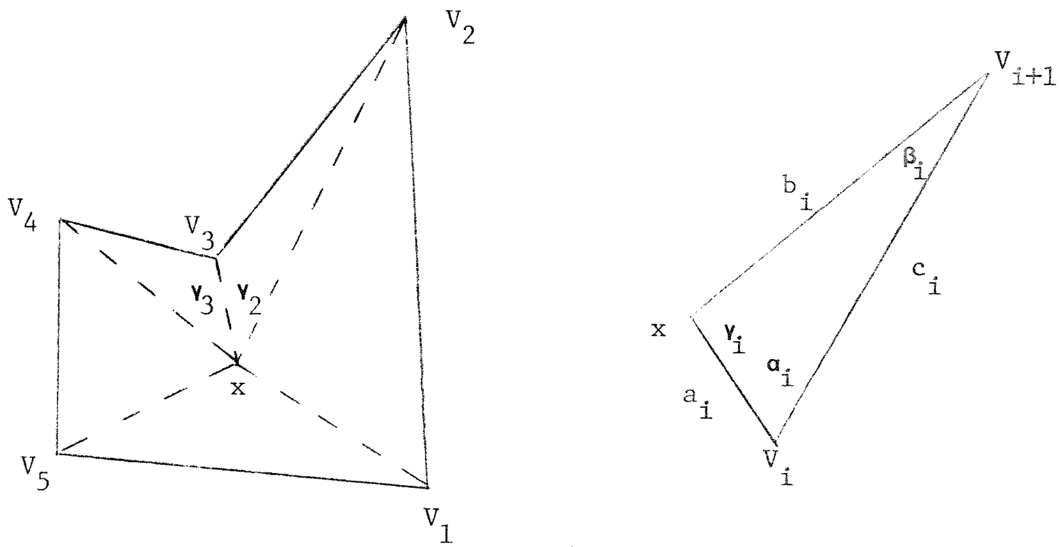


Figure 3. Computing moments

Notice that the statistics computed from the polarogram are invariant to the orientation of the polarogram - i.e., if $P'_f(a) = P_f(a+da)$, then P and P' will have the same statistics. Since rotating the original texture results in rotating the polarograms, these statistics are invariant to rotations of the image textures, yet can still be used to measure texture "directionality."

As an example, Figure 4 contains two texture samples, one of grating and the other of metal scrap. Figure 5 contains polarograms for the textures in Figure 4. These are based on the contrast statistic for cooccurrence matrices using D(a) with fixed magnitude 5. The symmetry in the polarograms is due to the symmetry in the cooccurence matrices - $C_{D(a)} = C_{D(a+pi)}$. Cooccurrence matrices were computed for directions which are multiples of 45 degrees. The larger size of the grating polarogram reflects the higher contrast of the grating (note that the first order grey level statistics of the two textures are identical). The relatively low value of the grating polarogram in the horizontal direction reflects the elongation of the grating texture elements, an aspect of texture directionality. The scrap metal, on the other hand, is more isotropic, so its polarogram is more circular.

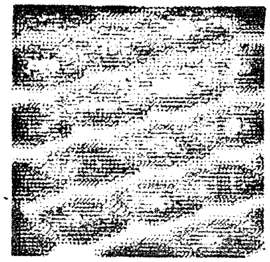


Figure 4a. A 64x64 grating texture

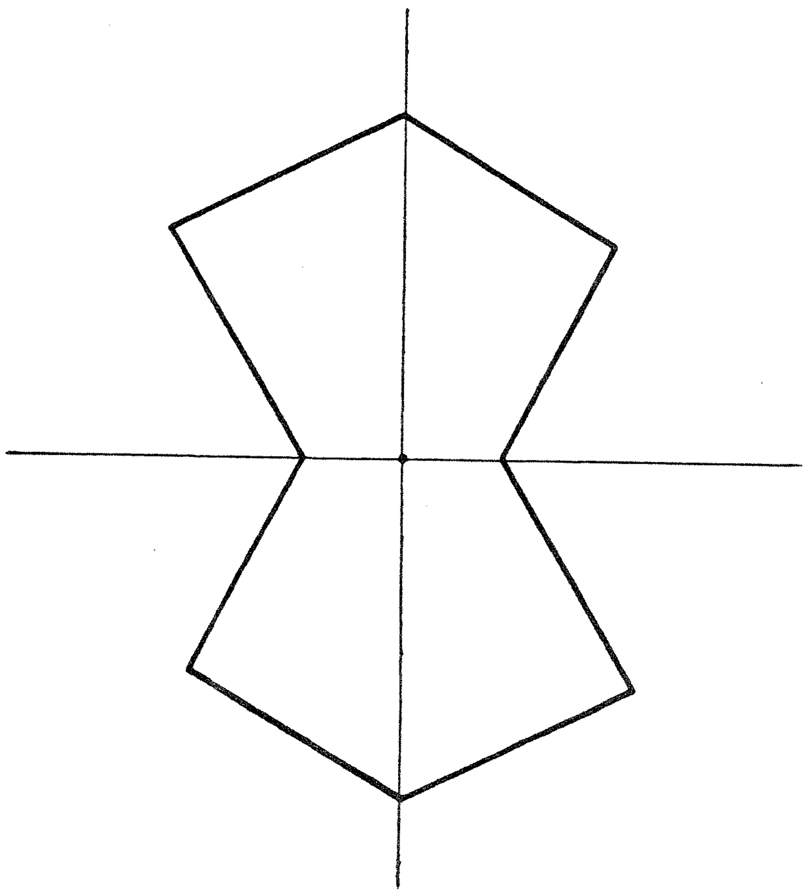


Figure 5a - Polarogram of Figure 4a for distance 5 cooccurrence matrices and the contrast descriptor.

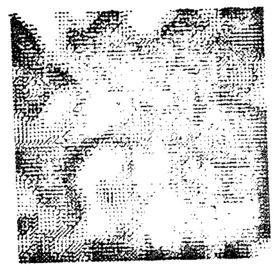


Figure 4b. A 64x64 metal texture

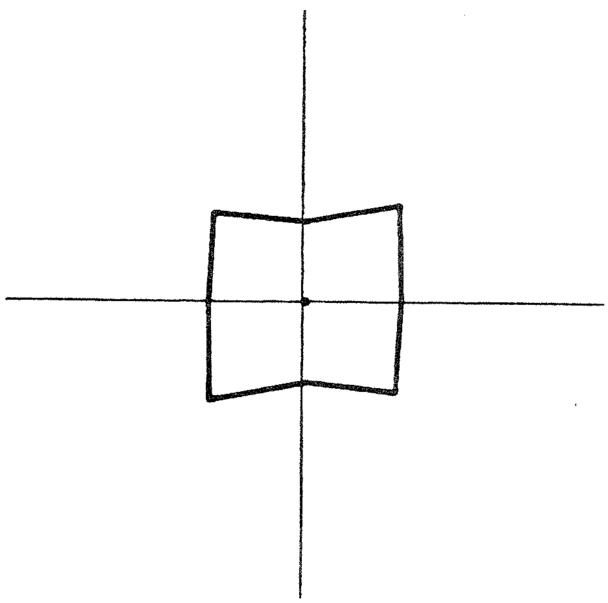


Figure 5b. Polarogram for Figure 4b for distance 5 cooccurrence matrices and the contrast descriptor.

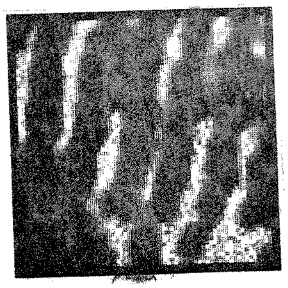
3. EXPERIMENTS

An experiment was performed to illustrate the use of polarogram statistics for texture classification. A database of five texture classes was chosen, and ten 64x64 samples were included in each class. The textures in the database include grating (G), tree bark (T), metal scrap (M), pebbles (P), and concrete (E). These samples form part of a larger database described in experiments reported in [3]. Figure 6 contains one 64x64 sample from each of the five classes. All of the textures have been subjected to a grey scale normalization procedures which "flattens" their histograms and guarantees that all of the samples have identical first-order grey level statistics.

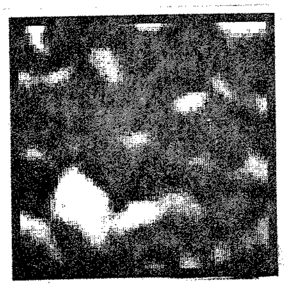
Polarograms were computed for the contrast descriptor of the grey level cooccurrence matrix. Cooccurrence matrices were computed for each of the eight principle directions on the digital grid, and for distances between pairs of points of 1, 3 and 5. Thus, three polarograms were computed for each texture sample. For each of the polarograms, the five statistics discussed in Section 2 were computed. Three classification experiments were performed; one based on the distance 1 statistics, one based on the distance 3 statistics and one based on the distance 5 statistics.

The textures were classified using a multivariate linear discriminant function [4], computed based on the assumptions that the probability density functions for the statistics of

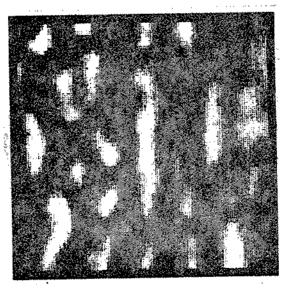
a) grating (G)



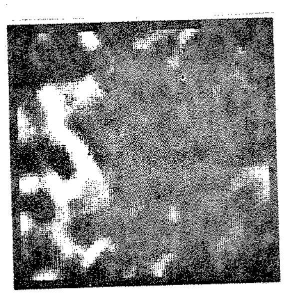
b) concrete (E)



c) pebbles (P)



d) tree bark (T)



e) metal (M)

Figure 6. Texture samples.

all classes were multivariate normal, with a common covariance matrix, but different mean vectors, and that the prior probability of each class is known. Subroutine ODNORM of IMSLIB was use to compute the discriminant function and to perform the classification.

The results of the experiments are summarized on Tables 1-3. Both the distance 5 and the distance 3 polarograms gave classification rates of 88% on this database of 50 textures.

	G	E	P	T	M
G	1.0	0	0	0	0
E	0	1.0	0	0	0
P	0	0	.9	0	.1
T	.1	0	.2	.6	.1
M	0	0	.3	0	.7

Percent error = 16%

Table 1. Confusion matrix, D=1

	G	E	P	T	M
G	1.0	0	0	0	0
E	0	1.0	0	0	0
P	0	0	.9	0	.1
T	.1	0	.1	.7	.1
M	0	0	.2	0	.8

Percent error = 12%

Table 2. Confusion matrix for, D=3

	G	E	P	T	M
G	1.0	0	0	0	0
E	0	1.0	0	0	0
P	0	0	.8	.1	.1
T	0	0	.1	.8	.1
M	0	0	.2	0	.8

Percent error = 12%

Table 3. Confusion matrix for, D=5

4. SUMMARY

We have presented a new computational tool for image texture analysis called a polarogram, and illustrated its use by applying it to a texture classification problem. Polarograms were designed to yield texture descriptors which are sensitive to both texture coarseness and directionality. An important aspect of directionally sensitive polarogram statistics is that they are invariant to rotations of the image texture. We are currently applying the polarogram to the unsupervised segmentation of scenes containing many differently textured areas.

REFERENCES

1. R. Haralick, "Statistical and structural approaches to texture," Proc. IEEE,67, 1979, pp. 786-805.

2. J. Weszka, C. Dyer and A. Rosenfeld, "A comparative study of texture measures for terrain classification," IEEE Trans. Systems, Man and Cybernetics,4,1976, pp. 269-285.

3. L. Davis, S. Johns and J. K. Aggarwal, "Texture analysis using generalized cooccurrence matrices," IEEE Trans. Pattern Analysis and Machine Intelligence,1, 1979, pp. 251-258.

4. International Mathematical and Statistical Libraries, IMSL Lib-0007, Subroutine ODNORM.