Packet Broadcast Networks—A Performance

Analysis of the R-ALOHA Protocol*

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ABSTRACT

In packet broadcast networks, users are interconnected via a broadcast channel. The key problem is multiple access of the shared broadcast channel. The performance of the R-ALOHA protocol for multiple access is studied in this paper. Two user models with Poisson message arrivals are analyzed; each message consists of a group of packets with a general probability distribution for group size. In the first model, each user handles one message at a time. In the second model, each user has infinite buffering capacity for queueing. Analytic models are developed for characterizing message delay and channel utilization. Bounds on channel throughput are established for two slightly different protocols. Numerical results from both analysis and simulation are presented to illustrate the accuracy of the analytic models as well as performance characteristics of the R-ALOHA protocol.

1. INTRODUCTION

Packet broadcast networks may be defined to be packet switching networks in which the connectivity requirements of a population of distributed users are furnished by a broadcast medium. Two obvious examples of such broadcast media are satellite and ground radio channels [1,2]. However, they may also be multipoint cable networks [3,4]. In recent years, multipoint cable networks have been gaining increasing importance for local area network interconnection [3-7].

The basic operation of a packet broadcast network can be explained as follows. A single broadcast channel is shared among a population of distributed users. It is assumed that each user is capable of sending and receiving data at the channel transmission rate of C bits per second (bps). Data messages are segmented into fixed length packets for transmission. Each packet contains its destination address(es) as well as parity bits for error detection. A packet transmitted successfully by any user i.e. in the absence of errors due to noise or interference from another user, will arrive correctly at all users. The packet will be accepted by the intended receiver(s) and ignored by others. When packets transmitted by different users "collide" in the channel, it is assumed that (in the absence of some special coding technique) none of the packets involved in a collision will arrive correctly at the intended receivers; such collisions are detected as transmission errors.

The central problem of a packet broadcast network is conflict resolution among the population of users sharing use of the broadcast channel. The problem is nontrivial since the users are typically

geographically distributed. The distances involved range from thousands of miles for a satellite network to, perhaps, tens of feet for an in-house cable network.

Many multiple access protocols for conflict resolution have been proposed and studied [8]. They can be classified into three general categories: polling protocols, contention protocols and reservation protocols. Under polling protocols, a central controller is required and users are passive i.e. they normally keep quiet whether or not they desire access of the channel. They are queried from time to time by the central controller; a user can transmit data only when so queried.

Both contention and reservation protocols require users who have data to send ("ready" users) to actively seek channel access. Under contention protocols there is no attempt to coordinate the ready users to avoid collisions entirely. Instead, each user monitors the broadcast channel and tries to transmit his data packets the best he can without incurring a conflict. Collided packets are retransmitted by users according to control algorithms driven by local information as well as observable outcomes in the broadcast channel.

The objective of reservation protocols is to avoid collisions of data packets entirely. To do so, a queue global to all users needs to be maintained for channel access. Each user when he has data to send generates a request to reserve a place in the queue. A fraction of the channel capacity is used to accommodate the reservation request traffic. Since users are geographically distributed, the multiple access problem has not disappeared. It exists now in the access of the reservation channel. Synchronization of the distributed queue is also a nontrivial problem.

The three classes of protocols are suitable for different traffic environments. For some traffic environements which are a fixed or time-varying combination of the above, various "mixed" or adaptive protocols have also been proposed [8-12]. R-ALOHA is a protocol that contains both elements of contention and reservation. It was originally proposed by Crowther et al. [9] to improve the throughput of a satellite channel beyond that of slotted ALOHA which is a pure contention protocol [13-15]. Although it was invented for a satellite channel, R-ALOHA can be used for any of the other broadcast media. We shall next describe the R-ALOHA protocol. Assumptions and results of a performance analysis are then presented.

2. THE R-ALOHA PROTOCOL

The broadcast channel is assumed to be slotted in time, and the slots are organized into frames with M slots in each frame, just as in traditional TDMA (see Figure 1). Each time slot is long enough for the transmission of a packet of data. The duration T of a frame is assumed to be greater than the maximum channel propagation delay in the broadcast network. Consequently, each user is aware of the usage status of time slots one frame ago. The network operates without any central control but requires each user to obey the same set of rules for transmitting packets into time slots depending upon what happened in the previous frame. A time slot in the previous frame may be:

- unused, which means that either (a) it was empty, or (b) two or more packets were transmitted into it (a collision) and thus none could be received correctly;

- used, which means that exactly one packet was transmitted into it and the packet was successfully received. (It is assumed that the channel is error-free except for collisions.)

The transmission rules are:

1) If slot m (say) had a successful transmission by user X (say) in the previous frame, slot m is off limits to everyone except user X in the current frame. Slot m is said to be reserved by user X. (Note that user X has exclusive access to slot m as long as he continues to transmit a packet into it in every frame.)

2) Those slots in the last frame which were unused are available for contention by all users according to an adaptive algorithm (the details of which will be considered below).

Two protocols are differentiated depending upon whether an end-of-use flag is included in the last packet before a user gives up his reserved slot:

(P1) End-of-use flag not included, and

(P2) End-of-use flag included.

With (P1), a time slot is always wasted whenever a user gives up his reserved slot. With (P2), such time slots are made available for contention according to the second transmission rule. (P2) therefore gives rise to a higher channel throughput than (P1) but (P1) is easier to implement; specifically, users do not have to examine the contents of each transmitted packet and look for the end-of-use flag.

3. THE ANALYSIS

A population of N users is considered with identical behavior and message arrival statistics. Messages arrive to each user according to a stationary Poisson process with rate λ messages/second. Each message consists of a group of h packets, with the first two moments $\bar{h}$ and $\overline{h^2}$ and probability generating function $H(z)$.

The analysis requires that each user can reserve at most one time slot in a frame at a time. With this requirement, the problem is interesting only if $N > M$. Another consequence is that the number of users who may access a

nonreserved slot is N - m, where m is the number of users holding reserved slots.

The following user models will be considered:

1) Single-message users - Each user handles one message at a time i.e. the Poisson source shuts itself off until all packets of the current message have been successfully transmitted.

2) Queued users - Each user has infinite buffering capacity; a queue is maintained with Poisson arrivals at the constant rate of λ messages/second.

Note that both user models are more general than user models previously considered for contention-based protocols [10,14,15].

The random variable v is defined to be the total number of packets that a user transmits before he gives up a reserved time slot. For the model of single-message users, v is just the number h of packets in a message. For the model of queued users, v is the number of packets that arrive within a busy period of the user queue; the mean value of v is denoted by $\bar{v}$.

It is well known that adaptive control algorithms are needed for proper operation of contention-based protocols. Such algorithms have been proposed and studied extensively in the past for slotted ALOHA channels based upon pure contention [12,15,16,17]. A mathematically tractable exact analysis of the R-ALOHA protocol using a realistic adaptive control algorithm for contention is not currently available. However, given an effective control algorithm the following assumption can be made for an approximate analysis.

Constant throughput assumption A successful packet transmission occurs in each nonreserved time slot with a constant probability, S.

The above assumption decouples the analysis of the R-ALOHA protocol from the specific details of the contention protocol as long as it has an effective control algorithm for stable operation; in the analysis, S is taken to be the steady-state slotted ALOHA throughput rate. The accuracy of the analytic results will be demonstrated below by comparing them with simulation results. Several practical control algorithms will also be discussed in conjunction with the simulation experiments.

We next define the equilibrium channel utilization probabilities

P_i = Prob [i slots in a frame are used].

<u>Proposition</u> For the model of single-message users and given the constant throughput assumption,

$$P_i = \binom{M}{i} U^i (1-U)^{M-i} \qquad\qquad i = 0,1,\ldots,M \qquad\qquad (1)$$

where

$$U = \frac{S}{S + (1/\bar{v})} \qquad\qquad\qquad (2)$$

under protocol (P1), and

$$U = \frac{S}{S + [(1-S)/\bar{v}]} \qquad\qquad\qquad (3)$$

under protocol (P2).

The key of the proof is to consider each of the M time division multi-plexed (TDM) subchannels in Figure 1 separately. (The ith subchannel is made up of the ith slot of each frame.) Given the constant throughput assumption and the assumption that users have independent identical arrival statistics, outcomes (used or unused time slots) in the M subchannels are statistically independent events. Also, each subchannel has alternating idle and busy periods which are statistically independent and constitute an alternating renewal process. (By definition, an idle period consists of unused time slots. A busy period consists of used time slots.) Let t_{idle} and t_{busy} be the idle and busy period duration respectively. The probability that a subchannel is busy is [18].

$$U = \frac{E[t_{busy}]}{E[t_{idle}] + E[t_{busy}]}$$

Since the M subchannels are statistically independent, Eq. (1) follows.

Given the constant throughput assumption, we have

$$Prob[t_{idle} = k \text{ slots}] = S(1-S)^{k-1} \qquad\qquad k = 1,2,\ldots$$

under (P1), and

$$Prob[t_{idle} = k \text{ slots}] = S(1-S)^{k} \qquad\qquad k = 0,1,\ldots$$

under (P2). Hence,

$$E[t_{idle}] = \begin{cases} 1/S & \text{under (P1)} \\ (1/S)-1 & \text{under (P2)} \end{cases}$$

Under both (P1) and (P2),

$$E[t_{busy}] = \bar{v}$$

Hence,

$$U = \frac{\bar{v}}{\frac{1}{S} + \bar{v}} = \frac{S}{S + (1/\bar{v})}$$

under (P1), and

$$U = \frac{\bar{v}}{[(1/S)-1] + \bar{v}} = \frac{S}{S + [(1-S)/(\bar{v})]}$$

under (P2).

From Eq. (1), the probability generating function of P_i is

$$Q(z) = (1-U+Uz)^M \tag{4}$$

with mean

$$\bar{m} = MU \tag{5}$$

variance

$$\sigma_m^2 = MU(1-U)$$

and coefficient of variation

$$C_m = \sigma_m/\bar{m} = \sqrt{(1-U)/(MU)}$$

Note that in the above results, v can have a general probability distribution. However, if we restrict v to be geometrically distributed, i.e.

$$Prob[v=i] = r(1-r)^{i-1} \qquad i = 1,2,\ldots$$

for some parameter r, the following equations can be derived for $Q(z)$ using a Markov chain approach (see Appendix);

$$Q(z) = (1-S+Sz)^M \; Q(\frac{z+r(1-z)}{1-S+Sz}) \tag{6}$$

under (P1), and

$$Q(z) = (1-S+Sz)^M \; Q(\frac{(1-r)z}{1-S+Sz} + r) \tag{7}$$

under (P2). These equations are the result of a different solution approach
to our problem (and under the more restrictive assumption of v being geo-
metrically distributed). It can be easily verified that Q(z) given by Eq.(4)
with the appropriate expression for U from either Eq. (2) or (3) is a solution to
Eq. (6) or (7) above (respectively) as it should be.

Assuming that v is geometrically distributed, Kanehira [19] derived
an expression equivalent to Eq.(2) for U under protocol (P1). Our model and
results in this paper are more   general than his. In particular, v
can have an arbitrary probability distribution.

In the model of queued users, v corresponds to the number of packets
served in a busy period. In this case, the M subchannels are not statistically
independent. Nevertheless, simulation results indicated that Eq. (1) is
still an excellent approximation.

R-ALOHA channel throughput

At this point, let us investigate the maximum possible throughput of a
channel that employs the R-ALOHA protocol. The throughput of a channel is
defined to be the fraction of time slots in which data packets are success-
fully transmitted and is equal to U above for R-ALOHA. Let C_{RA} and C_{SA}
denote the maximum channel throughput of R-ALOHA and slotted ALOHA
respectively. With $\bar{v}$ fixed in Eq. (2) or (3), it is easy to see that U is
maximized when S is maximized. We then have under protocol (P1)

$$U \leq C_{RA} = \frac{C_{SA}}{C_{SA} + (1/\bar{v})} \qquad (8)$$

Hence,

$$\frac{C_{SA}}{1 + C_{SA}} \leq C_{RA} \leq 1 \qquad (9)$$

for $\bar{v}$ ranging from 1 to ∞.

Similarly, under protocol (P2)

$$U \leq C_{RA} = \frac{C_{SA}}{C_{SA} + [(1-C_{SA})/\bar{v}]} \qquad (10)$$

Hence,

$$C_{SA} \leq C_{RA} \leq 1 \qquad (11)$$

for $\bar{v}$ ranging from 1 to ∞.

For a large population of users, we know that[13,14]

$$C_{SA} = 1/e$$

The maximum channel throughput of R-ALOHA is shown in Figure 2 as a function of $\bar{v}$ for a large population of users for both protocols (P1) and (P2).

Eqs. (2) and (3) are useful analytic relationships between U, S and $\bar{v}$. We know that $\bar{v} = \bar{h}$ for the model of single-message users. However, $\bar{v}$ is still unknown for the model of queued users and S is unknown for both user models. Their derivations will be given below. The R-ALOHA channel throughput U, however, is easily obtained using the following argument without first determining S and $\bar{\bar{v}}$.

Since the system is assumed to be in equilibrium, the channel throughput rate must be equal to the channel input rate. We therefore have for the model of single-message users

U = channel input rate in packets/slot

$= (N-\bar{m}) \lambda \bar{h} \ T/M$

$= (N-MU) \lambda \bar{h} \ T/M$

from which we get

$$U = \frac{(N\lambda \bar{h} T)}{M(1+ \lambda \bar{h}T)} \qquad (12)$$

For the model of queued users, we simply have

$$U = N\lambda \bar{h}T/M \qquad (13)$$

Note that the channel utilization probabilities P_i depend only upon U and are independent of S and $\bar{v}$; they are thus independent of the specific contention protocol. The latter does, however, affect the message delay characteristic of R-ALOHA as shown below.

Message delay analysis

For the moment assume that the delay d_A incurred by a user to success-fully transmit a packet into a nonreserved time slot has a known probability density function (pdf) with the Laplace transform $D_A^*(s)$, mean value $\bar{d}_A$ and second moment $\overline{d_A^2}$.

Note, however, that although the delay incurred by a packet in a slotted ALOHA channel has known results [14,15], such results must be modified for our use here. The nonreserved time slots imbedded within a R-ALOHA channel belong to currently unused TDM subchannels and are thus not contiguous. The number of time slots in between nonreserved slots is a random variable J with

$$\text{Prob}[J=j] = U^j(1-U) \qquad j = 0,1,2,\ldots$$

under the constant throughput assumption.

For the model of single-message users, the overall delay d of a message consists of d_A and the transmission delay of the rest of the message (if more than 1 packet long). The pdf of message delay has the Laplace transform

$$D^*(s) = D_A^*(s)H(e^{-sT})e^{sT} \tag{14}$$

with mean

$$\bar{d} = \bar{d}_A + (\bar{h}-1)T \tag{15}$$

For the model of queued users, the overall delay d of a message can be obtained by considering each user queue as a generalized M|G|1 queue in which the first customer of each busy period receives exceptional service [20]. In that context, the service time pdf of customers who initiate busy periods has the Laplace transform

$$B_0^*(s) = D_A^*(s)H(e^{-sT})e^{sT} \tag{16}$$

with mean

$$\bar{x}_0 = \bar{d}_A + (\bar{h}-1)T$$

and second moment

$$\overline{x_0^2} = \overline{d_A^2} + 2\overline{d_A}(\bar{h}-1)M + (\overline{h^2} - 2\bar{h} + 1)M^2$$

The service time pdf of customers who arrive to find the queue busy has the Laplace transform

$$B^*(s) = H(e^{-sT}) \tag{17}$$

with mean

$$\bar{x} = \bar{h}T$$

and second moment

$$\overline{x^2} = \overline{h^2}M^2$$

The pdf of message delay has the Laplace transform [20]

$$D^*(s) = \frac{P_0[(\lambda-s)B_0^*(s) - \lambda B^*(s)]}{(\lambda-s) - \lambda B^*(s)} \tag{18}$$

where

$$P_0 = \frac{1-\lambda\bar{x}}{1- \lambda(\bar{x}-\bar{x}_0)}$$

$$= \frac{1-\lambda\bar{h}T}{1+ \lambda(\bar{d}_A-T)} \tag{19}$$

The average message delay is

$$\bar{d} = \frac{\bar{x}_0}{1- \lambda(\bar{x}-\bar{x}_0)} + \frac{\lambda(\overline{x_0^2} - \overline{x^2})}{2[1-\lambda(\bar{x}-\bar{x}_0)]} + \frac{\lambda\overline{x^2}}{2(1-\lambda\bar{x})} \tag{20}$$

Finally, we note that the message delay analysis here can be easily extended (following [21]) to a nonpreemptive priority queue discipline with a finite number of message priority classes.

Solution for S and $\bar{v}$

To solve for the average message delay $\bar{d}$, we need the first and second moments of d_A which depend upon S and $\bar{v}$. For the model of single-message users, we have

$$S = \frac{(N-\bar{m})\lambda T}{(1-U)M}$$

$$= \frac{\lambda T(N-MU)}{(1-U)M} \tag{21}$$

and

$$\bar{v} = \bar{h} \tag{22}$$

For the model of queued users, the Laplace transform $G^*(s)$ of the busy period pdf can be obtained using a delay cycle analysis [22]

$$G^*(s) = B_0^*(s+\lambda - \lambda Y^*(s)) \tag{23}$$

where

$$Y^*(s) = B^*(s+\lambda - \lambda Y^*(s)) \tag{24}$$

Let $\bar{g}$ and $\bar{y}$ be the mean values obtained from $G^*(s)$ and $Y^*(s)$ respectively. We have

$$\bar{g} = \bar{x}_0(1+\lambda\bar{y}) \tag{25}$$

where

$$\bar{y} = \frac{\bar{x}}{1-\lambda\bar{x}} = \frac{\bar{h}T}{1-\lambda\bar{h}T} \tag{26}$$

In this user model, $\bar{v}$ is the total number of packets that arrive within

a busy period (including those of the initial message). We then have

$$\bar{v} = (1+\lambda\bar{x}_0\bar{k})\bar{h} \tag{27}$$

where

$$\bar{k} = 1/(1-\lambda\bar{x}) \tag{28}$$

Thus

$$\bar{v} = (1 + \frac{\lambda\bar{x}_0}{1-\lambda\bar{x}})\ \bar{h}$$

$$= (1 + \frac{\lambda[\bar{d}_A+(h-1)T]}{1-\lambda\bar{h}T})\ \bar{h} \tag{29}$$

At this point, if we know the throughput-delay relationship for the nonreserved time slots (i.e., $\bar{d}_A$ as a function of S), $\bar{v}$ and S can be solved numerically using Eq.(29) together with the equation

$$U = \frac{S}{S+(1/\bar{v})} = N\lambda\bar{h}T/M \tag{30}$$

obtained from Eqs.(2) and (13) for protocol (P1), or

$$U = \frac{S}{S+[(1-S)/\bar{v}]} = N\lambda\bar{h}T/M \tag{31}$$

obtained from Eqs.(3) and (13) for protocol (P2).

4. NUMERICAL RESULTS

In this section, we compare the above analytic results with experimental results from simulation. In the simulation program, we let the number of slots in a frame M=10 and the number of users N=40. The model of queued users is more general than the model of single-message users and is the only one considered below. For simplicity, a Bernoulli process is used to approximate the Poisson arrival process of each user; in each time slot, a message arrives to each user with probability $\sigma = \lambda(T/M)$. Each message consists of a group of h packets with the following distribution

$$\text{Prob}[h=i] = \begin{cases} 0.2 & i = 1 \\ 0.1 & i = 2,3,4,5,6,7,8,16 \\ 0 & \text{otherwise} \end{cases}$$

which has a mean of 5.3 packets.

Recall that the R-ALOHA protocol is applicable as long as the channel propagation delay is less than the frame duration T. The channel propagation delay was assumed to be zero in both our simulation and analysis results presented below (without any loss of generality).

To obtain numerical results for either analysis or simulation, it is necessary to specify the contention protocol, in particular, the adaptive control algorithm. Many adaptive control algorithms have been proposed and studied in the past. Since this is not the primary concern of the present study, we have considered mainly algorithms which are easy to implement.

Specifically, the following class of algorithms that depend only upon local information was considered. Each ready user who does not have a reserved slot transmits the packet at the head of his queue into each nonreserved slot with probability p_k where $k = 0,1,2,\ldots$ is the accumulated number of collisions incurred by the same packet. These algorithms were referred to as heuristic RCP policies in [16]. The following algorithms have been tested in our simulator.

(1)
$$p_k = \begin{cases} 1 & k = 0 \\ 0.2 & k = 1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

(2)
$$p_k = \begin{cases} 1 & k = 0 \\ 0.1 & k = 1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

(3) $p_k = (0.5)^k \quad k = 0,1,2,\ldots$

(4) $p_k = 1/(k+1) \quad k = 0,1,2,\ldots$

In both (1) and (2) above, users who have incurred more than 3 collisions for the same packet "lose" all their messages.

Both algorithms (1) and (2) were found to give rise to stable channel operation. Stability was achieved at the expense of some "lost" messages when the channel was heavily loaded. Of course, in a real system, messages are not actually lost but rather some users experience temporarily a busy condition and cannot generate new messages. Both algorithms (3) and (4) were found to be far inferior to (1) and (2). In particular, they failed to prevent excessive collisions in nonreserved slots when the channel is heavily loaded. They also give rise to much longer message delays than (1) and (2) when the channel is moderately loaded.

We have also considered the special case when global information is available to individual users. In particular, the instantanous number n of users competing for a nonreserved slot is known to each such user. The optimal (symmetric) strategy in this case is for each such user to transmit into the nonreserved slot with probability $1/n$. This particular algorithm is difficult to implement in practice. However, they give rise to throughput-delay results which are useful as performance bounds.

In Fig. 3 we have shown both experimental results and theoretical results of P_i given by Eq. (1) at four different values of channel throughput $U = 0.05$, 0.19, 0.60 and 0.86. Note that under both light load ($U = 0.05$ or 0.19) and heavy load ($U = 0.86$), experimental and theoretical results agree almost exactly. At $U = 0.60$, there is some minor discrepancy. The simulation results shown were obtained when the optimal control strategy was used. The good agreement between experimental and theoretical results in Fig. 3, however, was representative of all effective control algorithms considered.

To calculate $\bar{d}$ and $\bar{v}$ using Eqs. (20) and (29), the following formulas for the moments of d_A were used.

$$\bar{d}_A = (1 + \frac{1-q}{pq})/(1-U) \tag{32}$$

$$\overline{d_A^2} = \frac{2}{(1-U)^2}[1+ \frac{1-q}{pq} + \frac{1-q}{p^2 q^2}] - \frac{1}{1-U}(1 + \frac{1-q}{pq}) \tag{33}$$

where p is equal to 0.2, 0.1 respectively for control algorithms (1) and (2), and q is obtained as a function of S from

$$q = e^{-S/q} \tag{34}$$

Eqs. (32) – (34) were derived under very strong assumptions of independence using the approach in [14]. For more accurate results, the Markov chain technique in [15] may be used instead. Eqs. (32) – (34) were adopted mainly for their simplicity; despite their inaccuracies, the analytic results of $\bar{d}$, $\bar{v}$ and S for R-ALOHA compare very well with experimental results. These are illustrated in Figs. (4) and (5) for control algorithm (1) and in Figs. (6) and (7) for control algorithm (2).

5. CONCLUSIONS

Following a brief overview of 3 classes of multiple access protocols for packet broadcast networks, R-ALOHA was introduced as a protocol that contains both elements of contention and reservation. We found that R-ALOHA is a protocol that adapts itself to the nature of the input traffic. Figure 2 shows that the R-ALOHA maximum channel throughput ranges from that of slotted ALOHA at one extreme ($\bar{v}$ = 1) to that of fixed assigned TDMA channels at the other extreme ($\bar{v}$ = ∞).

Two user models with Poisson arrivals were considered. Each arrival is a message consisting of a group of packets. In the first model, each user handles one message at a time. In the second model. each user has infinite buffering capacity for queueing. We make two observations. First, our user models (which permit buffering and queueing) are more general than user models previously considered for

contention-based protocols. Second, our performance results on message delay and channel utilization depend upon the constant throughput assumption, which decouples the analysis of the R-ALOHA protocol from specific details of the contention protocol for nonreserved slots. We found that the approximations thus introduced in the analytic results are acceptable as long as an effective control algorithm is implemented for stable channel operation.

In this paper, both our R-ALOHA protocol and user models are such that a tractable analysis is possible. In a practical implementation, some other issues will have to be addressed and the protocol should probably be significantly enriched.

One issue is fairness. How do we prevent some users from being locked out of the channel for a long time? One idea is to allocate a minimum number of slots in each frame which cannot be reserved. (Our analysis can be trivially extended to include this.) Some fairness algorithm may also be designed into the contention protocol. The fairness algorithm will affect our analytic model only to the extent that it affects the accuracy of the constant throughput assumption.

Another issue is that of a nonhomogenerous user population. In addition, some users may be more important than others. It will be desirable to enrich the R-ALOHA protocol in several ways. A user may be permitted to reserve multiple slots in a frame. (What algorithm should be used to increase or decrease his reserved slots?) A user may use dummy packets to hold on to a reserved slot when his queue is empty. (This may be a fair and acceptable practice if users can be charged according to the number of slots used.) A user may also be forced to give up his reserved slots even if his queue is nonempty.

Such implementation considerations will give rise to rather formidable analysis problems. One might be forced to rely more heavily upon simulation for performance evaluation.

APPENDIX

Given that v is geometrically distributed with parameter r, we derive

below Eqs. (6) and (7) using a Markov chain approach. Define

q_n = number of used slots in the n<u>th</u> frame

a_n = number of nonreserved slots used (successfully) in the n<u>th</u> frame

d_n = number of reserved slots given up in the n<u>th</u> frame.

In general, we have

$$q_{n+1} = q_n - d_{n+1} + a_{n+1}$$

and thus

$$Q_{n+1}(z) \hat{=} E[z^{q_{n+1}}] = E[z^{q_n - d_{n+1} + a_{n+1}}]$$

Under protocol (P1), d_{n+1} and a_{n+1} are dependent upon q_n but independent

of each other. Each has a binomial distribution. We then have

$$Q_{n+1}(z) = \sum_{i=0}^{M} P[q_n = i] \, z^i \, E[z^{-d_{n+1}}/q_n = i] \, E[z^{a_{n+1}}/q_n = i]$$

$$= \sum_{i=0}^{M} P[q_n = i] \, z^i \, (1 - r + \frac{r}{z})^i \, (1 - S + Sz)^{M-i}$$

$$= (1 - S + Sz)^M \sum_{i=0}^{M} P[q_n = i] \, (\frac{z+r(1-z)}{1-S+Sz})^i$$

$$= (1 - S + Sz)^M \, Q_n(\frac{z+r(1-z)}{1-S+Sz})$$

Eq. (6) is obtained in the limit as $n \to \infty$.

Under protocol (P2), d_{n+1} is dependent upon q_n while a_{n+1} is dependent upon both q_n and d_{n+1}. We then have

$$Q_{n+1}(z) = \sum_{i=0}^{M} P[q_n = i] \, z^i \, E[z^{-d_{n+1} + a_{n+1}}/q_n = i]$$

$$= \sum_{i=0}^{M} P[q_n = i] \, z^i \sum_{j=0}^{i} \binom{i}{j} r^j (1-r)^{i-j} z^{-j} \, E[z^{a_{n+1}}/q_n=i, \, d_{n+1}=j]$$

$$= \sum_{i=0}^{M} P[q_n=i] \, z^i \sum_{j=0}^{i} \binom{i}{j} r^j (1-r)^{i-j} z^{-j} (1 - S + Sz)^{M-i+j}$$

$$= (1 - S + Sz)^M \sum_{i=0}^{M} P[q_n = i] \, z^i \left(\frac{1-r}{1-S+Sz} + \frac{r}{z}\right)^i$$

$$= (1 - S + Sz)^M \, Q_n\!\left(\frac{(1-r)z}{1-S+Sz} + r\right)$$

Eq. (7) is obtained in the limit as $n \to \infty$.

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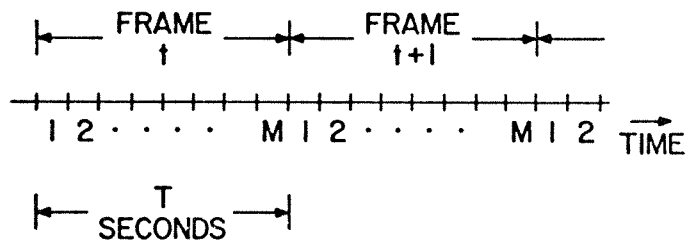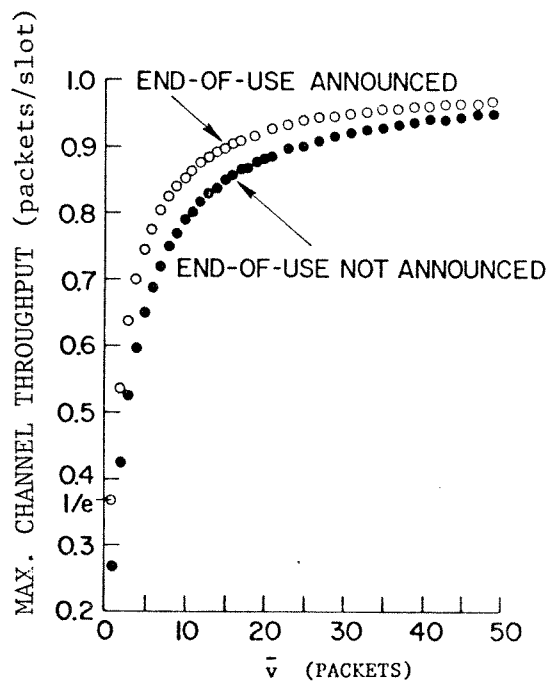
Fig. 1.   Frame structure.

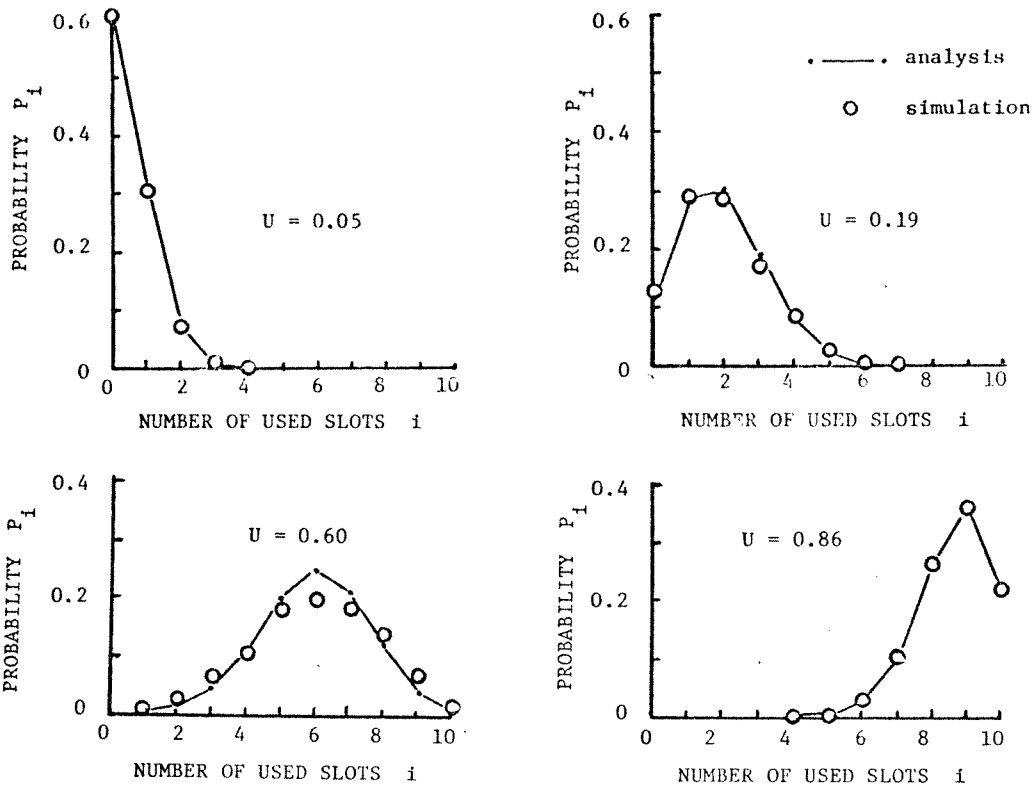Fig. 2.   Maximum channel throughput
versus v̄ .

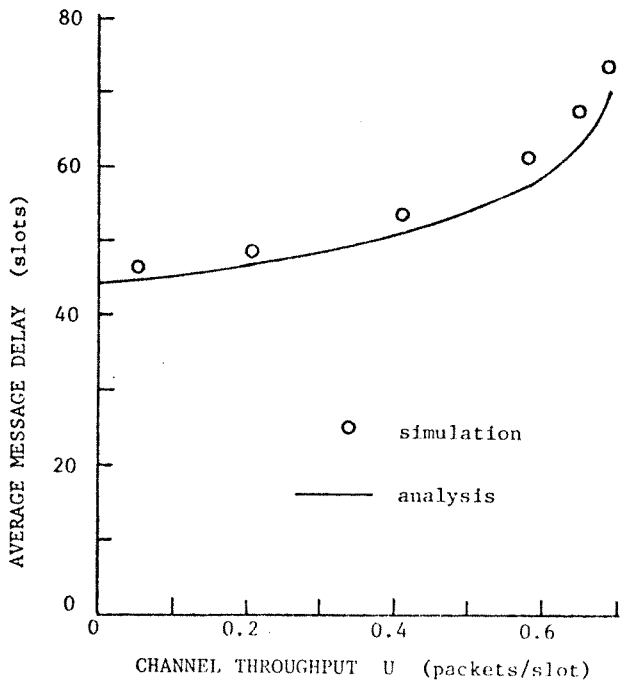Fig. 3. Analysis and simulation results for P_i .

Fig. 4. Ave. message delay vs. channel throughput for control algorithm (1).
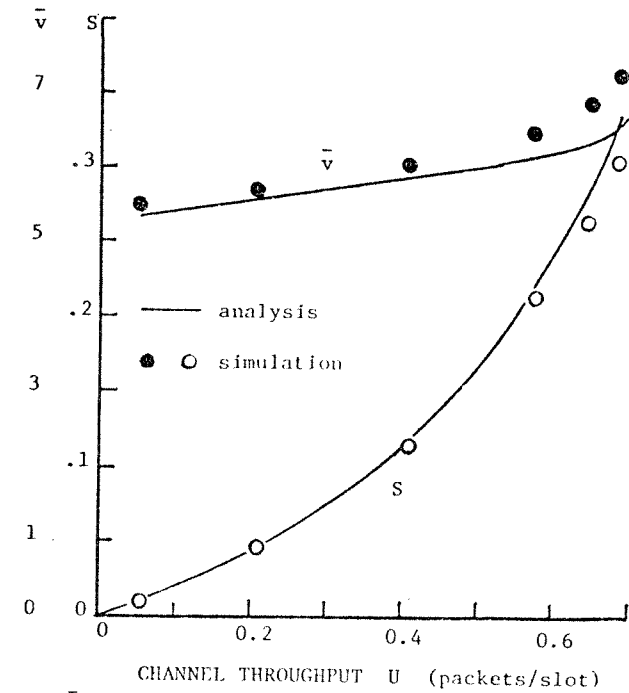

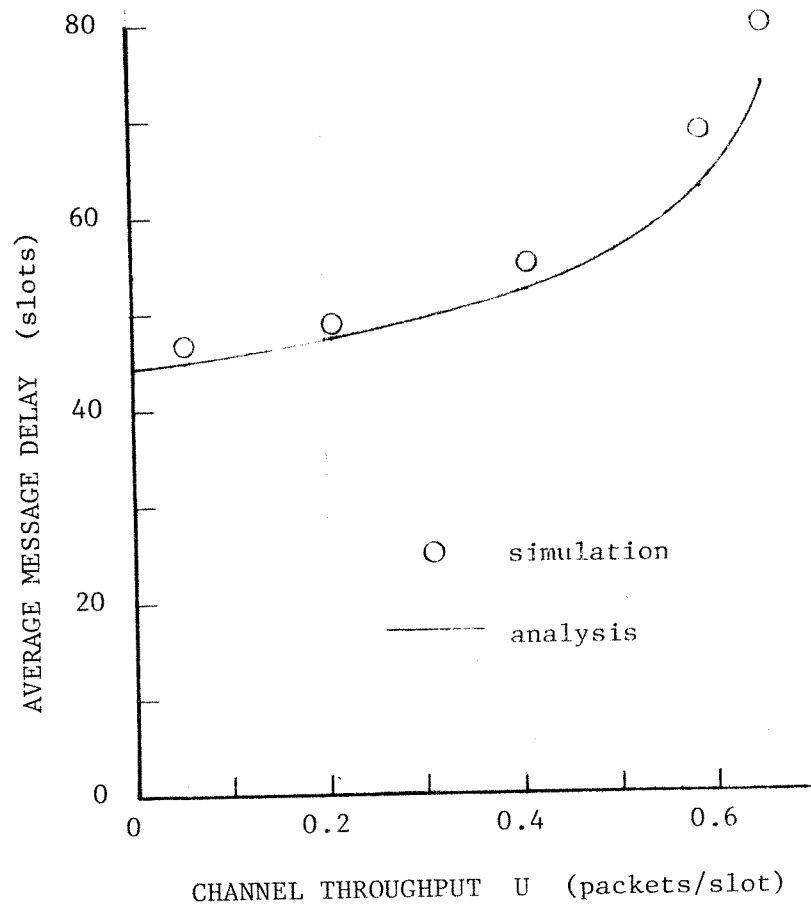Fig. 5. $\bar{v}$ and S versus channel throughput for control algorithm (1).

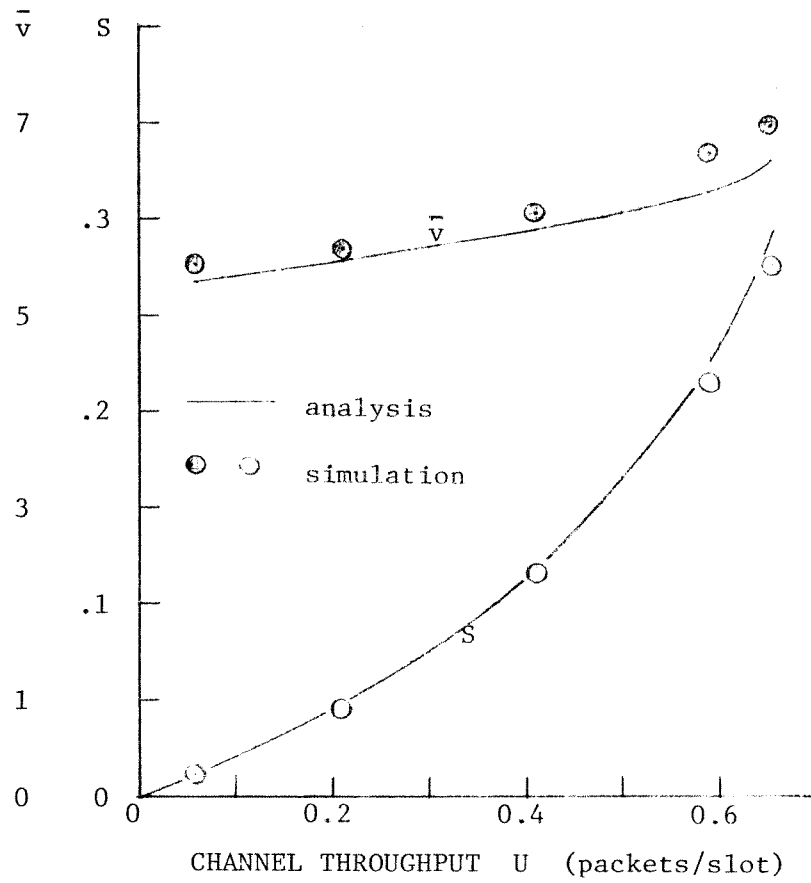Fig. 6. Ave. message delay vs. channel throughput
for control algorithm (2).

Fig. 7. $\bar{v}$ and S versus channel throughput for control algorithm (2).