Goals and Methodology of Research
On Solving Physics Problems

Gordon S. Novak Jr.

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Computer Science Department
University of Texas at Austin
Austin, Texas 78712
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1.0 INTRODUCTION

This research project will investigate the cognitive processes and knowledge structures needed to solve physics problems by means of a computer program which can solve physics problems stated in English. The problem solving techniques of the program will then be compared with techniques inferred from protocols taken from expert and novice humans solving the same problems.

We have previously written a program which is able to solve physics problems stated in English in the limited area of rigid body statics; examples of problems solved by this program are shown in the figures below. This program uses a formulation of the laws of rigid body statics which is similar to the form in which these laws are presented in modern textbooks. However, the use of this form of the laws results in problem solving behavior which is very different from that of humans. For example, while a skilled human problem solver might generate only one equation for Problem 1, the program generates nine equations; it appears that people have specialized, redundant forms of physical laws which are more effective for problem solving than the general laws. We believe that a program would need similar abilities to recognize special situations and use the appropriate specialized laws in order to solve more difficult problems which involve several kinds of physical principles.

The writing of this computer program also uncovered a number of component skills which are required for problem solving, but which are taught only implicitly in physics textbooks. These skills include the specialized forms of laws and rules for their use, and rules for recognizing what "canonical object" or physical model should be used to model an actual object in a particular problem. We believe that explication of the component skills using the computer program will be valuable for teaching people to solve physics problems.

2.0 GOALS OF THE RESEARCH

Our major research goals include the following:

1. To identify the component skills (cognitive processes and their associated knowledge structures) required for effective problem solving. We believe that many of these component skills operate subconsciously in expert problem solvers, and are taught only implicitly in physics courses.

2. To determine how to organize the component skills for effective problem solving. In order to use knowledge effectively, it must be brought to bear at the proper time and place. Many students have difficulty recognizing how to proceed in solving problems.

3. To identify kinds of learning which can improve problem solving ability. Learning specialized forms of laws and conditions for their use, for example, can greatly improve problem solving ability, although it does not add any logical power beyond that provided by the general forms of the laws. Hopefully, these kinds of learning will also be valuable for human problem solvers.

3.0 METHODOLOGY

3.1 Advantages Of Using A Computer Program

While it is difficult to write a computer program which can solve physics problems, we believe that this methodology has several singular advantages. First, requiring that the program be able to solve complete physics problems helps to insure that no essential component skills have been left out or assumed to be trivial when in fact they are not, and that the component skills have been organized in a manner which is effective for problem solving. The computer serves as a thoroughly simple-minded student to whom everything must be explained in complete detail. Not only must all the component skills be included, but they must be specified in sufficient detail to be effective in solving real problems. For example, we cannot simply specify "write equations" as a component skill, but must specify what law is to be used, what form of the law is to be used, what object is involved, and at what time in the problem solving process the equation should be written. Thus, the running of the program serves as experimentation which disciplines the formation of a theory of the component skills and their interactions. (Of course, there is the danger that by bounding the set of problems with which the program deals, we will make the problems the computer faces much easier than the corresponding problems faced by humans; we hope that requiring the program to solve problems covering a variety of physical principles will help to minimize this danger.)

A second benefit of writing a program which can actually solve problems is that it can provide a testbed for different problem solving strategies. We hope that we will be able to make the program solve problems as experts do it, and also as novices do it; this might allow us to identify specific differences in the strategies of experts and novices and improve the teaching of the expert's strategies. (However, if the ways novices and experts represent problems are too different, it may be difficult to model both problem solving strategies without rewriting a great deal of the program.)

A third benefit of writing a program which can solve physics problems is that such a program could form the basis of an intelligent computer-assisted instruction (CAI) program which understands all the component skills required to solve problems. Such a program might be a better teacher of problem solving than an expert human for whom many of the component skills have become automatic subconscious processes.

3.2 Modelling Of Psychological Processes

When it is claimed that a computer program is a model of human psychological processes, it is appropriate to question the level of detail of the processes modelled, the range of behavior covered by the model, and whether the claims made about the modelling of human psychological processes are justified. We take a rather conservative view toward making claims that a program models human thought processes. A computer model is just that: a model of certain aspects of a cognitive process, but certainly not a duplicate of that process. Our goal is to be able to break the process of solving physics problems down into a set of component processes at a finer level of detail than would be revealed by naive introspection, and to show that this set of component processes is sufficient to solve a variety of physics problems. At the same time, we wish to restrict our set of component processes to those which might plausibly be used by human problem solvers (both because of our interest in human problem solving and education and because human problem solving shows a power and richness which we would like to capture in the computer program). Below the level of detail of the component processes, we expect that the computer program will often perform the processes in a manner different from that of humans. For example, in solving a lever problem, one of the component processes is to make a geometric model of the objects in the problem and their spatial relationships. Humans surely do this differently than the program does (or will), probably making use of spatial pattern recognition abilities which are built into brain hardware and which cannot be simulated by computers given the current state of the art. However, it is reasonable to consider making a geometric model to be a basic component step in both problem solving processes and to compare the two models produced (directly or by inference). We found in our earlier program that the computer program must perform many more component processes than we initially realized; we believe that a great value of this methodology is that it forces us to examine processes which were so "obvious" that they were overlooked in a first introspective analysis of the problem solving process. We also believe that failure to understand these same "obvious" processes is a major cause of students' difficulties in learning to solve physics problems.

4.0 HYPOTHESES ABOUT PROBLEM SOLVING

In this section, we discuss several hypotheses about the nature of problem solving in an area such as physics, and relate these hypotheses to the computer program we plan to develop and to difficulties of humans in learning to solve physics problems.

4.1 Multiple Representations Of Knowledge

Much of the work on problem solving in the field of Artificial Intelligence has been based on the use of a single, "canonical" representation for the "atomic" concepts of the problem-solving area, the representation of the laws of the area as a set of logical axioms, and the use of a uniform proof procedure to solve problems by deducing the solution from the data of the problem and the axioms. We consider such methods to be very unlike any methods normally used by human problem solvers, and we believe that it would be very difficult to write a program which could effectively solve a large number of problems covering a variety of physical principles using such methods.

Our approach is to investigate the use of multiple representations of knowledge. We consider that there is no truly "atomic" level of representation, but that any level can in principle be broken down into a representation at a finer level of detail; at the same time, any level of detail may be considered to be "atomic" for purposes of a particular reasoning process. Likewise, there is no single "canonical" form for any particular type of information (although there will be forms of representation which are canonical for purposes of particular reasoning processes). In contrast to other approaches (especially those based on the use of predicate calculus) which consider different representations of the same thing to be logically equivalent (even when a process of reasoning is required to get from one representation to the other), we consider the selection and instantiation of new representations to be critical steps of the problem solving process (for both humans and programs).

4.2 Recognizing Canonical Objects

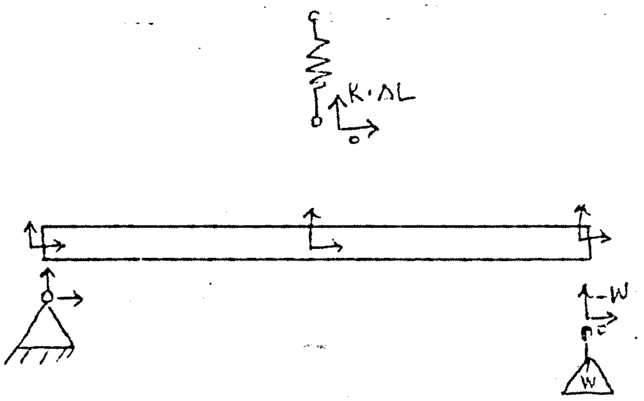
In order to solve physics problems, it is necessary for the problem solver to recognize what "canonical objects" (the objects dealt with by physical laws) should be used as alternate representations of the actual objects which occur in the problem. For example, in solving a rigid body statics problem involving a man standing on a ladder, the man would be considered to be a "point mass" and the ladder would be considered to be a "rigid body". The "canonical objects" are actually only approximate models of the actual objects in the problem; thus, they are not logically implied by the actual objects, but must be selected using "engineering judgment" as models appropriate to model particular objects in a particular context for a particular type of problem solving. For example, a ladder might be represented as a weightless and perfectly rigid "rigid body"; the selection of this canonical object reflects an engineering judgment that typical ladders weigh much less than the loads they typically carry, and are relatively rigid under those loads. Thus, the canonical objects used in solving physics problems must be considered to be approximate models of the actual objects for particular purposes. In some cases, the same object may be modelled by several canonical object models; for example, in the case of a block resting on an inclined plane, the block might be modelled both as a weight and as a participant in a frictional contact relationship.

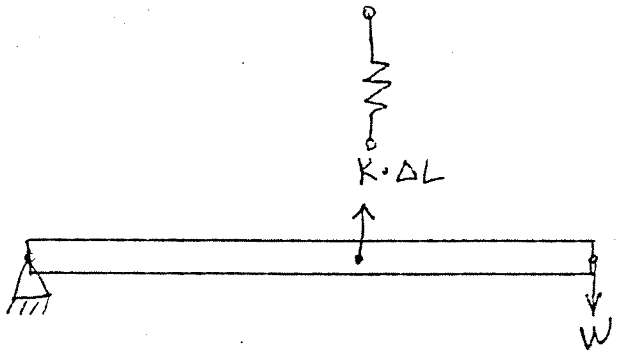
Selection of the correct canonical object models for a particular problem is a crucial step in problem solving, and one which is taught only implicitly in physics courses and textbooks. The teacher does not tell the students the rules which allow the use of a "weightless rigid body" model for a ladder, but instead works an example problem using this model. The student must absorb the rules for selecting canonical models by acculturation rather than by explicit teaching. The student who is unable to infer the rules from the teacher's examples will be unable to solve problems in that area correctly.

4.3 Composite Canonical Objects

Modern physics textbooks tend to present physical laws in a very compact, elegant, general form; for example, the laws of rigid body statics are presented by stating that the three-dimensional vector sums of the forces and moments on the body must be zero. Such a formulation of the laws has clear advantages because of their compactness and generality; however, we believe that other forms of the laws are better for solving certain kinds of problems. In a text of thirty years ago, three laws for "class 1, class 2, and class 3 levers" are presented instead; these laws do not cover the general case of rigid body statics, but are easier to use in solving problems which involve these kinds of levers. We believe that an expert knows all of these formulations of the laws (in the sense of being able to use all of them in solving problems); however, in modern physics courses it may be very difficult for some students to acquire the problem-solving forms when only the general forms are presented explicitly.

We consider that the specialized forms of physical laws represent composite canonical objects, i.e., new types of canonical objects which have specialized laws associated with them and which are composed of several more elementary canonical objects or represent specializations of more general canonical objects. For example, our example Problem 1 can be represented as a rigid body attached to several other objects (as our original program did it) or as a "3rd class lever" (as a human expert might do it):





Clearly, the second representation in terms of the composite canonical object is conceptually simpler (at least to humans), and results in a much simpler set of equations. An example of a specialization of a more general canonical object is the use of two-dimensional or one-dimensional vectors rather than the general three-dimensional vector when the problem permits; the simplification gained is so significant that it is often worthwhile to construct a local geometric model to reduce the number of dimensions in the local model.

4.4 "Recognize / Map / Compute / Map Back" Paradigm

We hypothesize that a great deal of problem solving power can be gained by using relatively large, structured pieces of problem solving knowledge ("frames" or "schemas") as opposed to a collection of many small rules or pieces of knowledge, and by invoking these larger collections of knowledge using the following four steps:

1. Recognize that part of the current problem representation is an instance of the type of canonical object dealt with by the specialist knowledge.

2. Map the existing representation of the objects in question into the form required by the specialist. This may require assumptions or inferences of features of the objects which

are implied but not explicitly represented; it could also involve more detailed tests to insure that the object is really appropriate for the specialist.

3. Compute the desired features from the model created for the specialist.

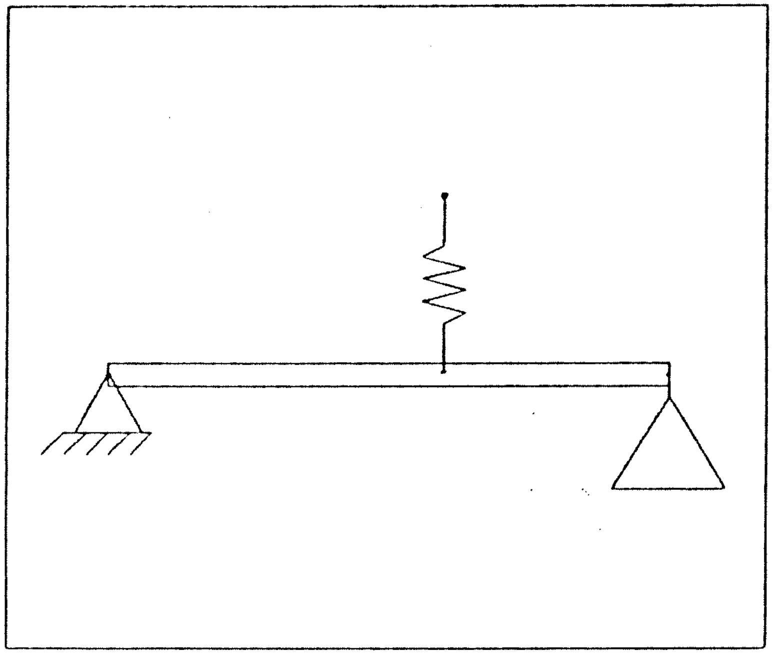
4. Map the results found by the specialist back into the form of the original representation.

An example of this kind of computation in our original program is the solving of the triangle in Problem 8. First, the program must recognize that three objects are connected so that they form a triangle. Next, the known line segments and angles of the triangle are abstracted from the representations of the objects to form a canonical triangle model with some sides and/or angles known and others unknown. A triangle specialist can then solve the triangle, producing a fully instantiated triangle model. Finally, the newly calculated line segments and angles must be transferred back to the original model; for example, one of the line segments might define the length of the ladder.

The most difficult step in this process is the first: to recognize in a complex problem representation an instance of the kind of object dealt with by a piece of specialist knowledge. Clearly, it is easier for the computer to recognize that the lever in Problem 1 is a rigid body than it will be to recognize the lever and pivot together, in the context of the other forces on the lever, as a Class 3 lever. Determining how such a recognition process can be organized, and how newly learned patterns can be incorporated into the organization, is a major goal of this research.

4.5 Additive Knowledge

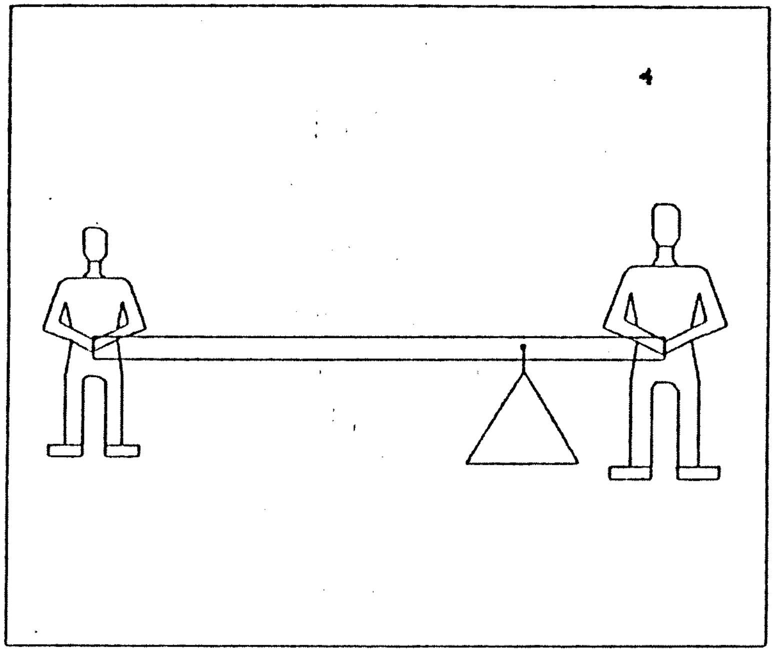
Much research on learning in the field of Artificial Intelligence has centered on the notion of "debugging", i.e., the notion that a novice learns in large part by debugging incorrect procedures. Our notion of composite canonical objects suggests an additive form of learning: the novice who learns to use a specialized composite canonical object is adding knowledge in the form of a new recognition procedure and a new procedure to be used once the pattern is recognized. However, the new knowledge is not simply "added"; in particular, we believe that the recognition procedure must be added in the form of discriminating tests at an appropriate place in a discrimination network to cause the new procedure to be executed instead of the more general one (which will still be applicable). We hypothesize that adding new knowledge so that it fits into the framework of the old knowledge properly is another area where students have difficulty, and where the solution to this problem in the computer program may be helpful in improving education.



P1

(A LEVER 10 FT LONG IS PINNED AT ITS LEFT END)(THE LEVER IS SUPPORTED BY A SPRING WITH A CONSTANT OF 40 LB/FT)(THE SPRING IS ATTACHED 6 FT FROM THE LEFT END OF THE LEVER)(A WEIGHT OF 20 LB IS ATTACHED AT THE OTHER END OF THE LEVER)(THE WEIGHT OF THE LEVER IS 8 LB)(HOW MUCH IS THE SPRING STRETCHED)

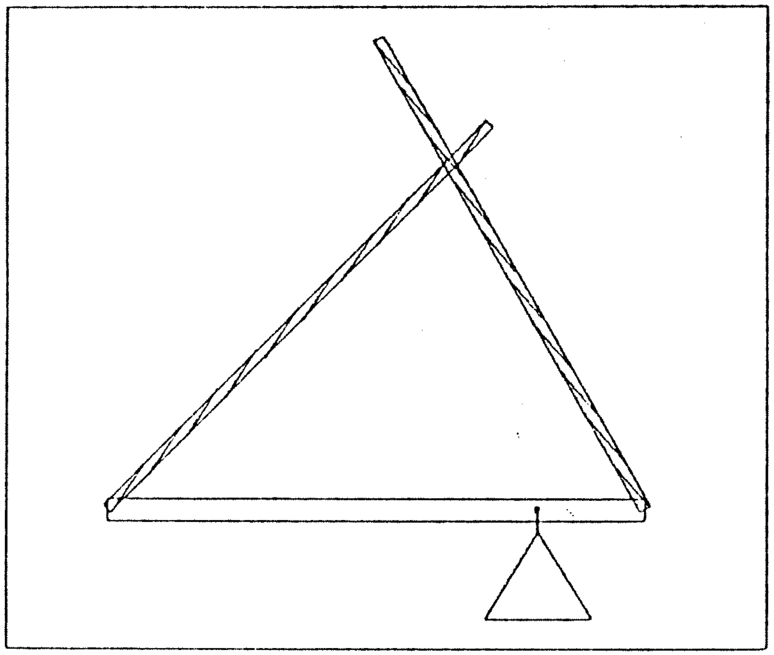
ANSWER: 1.00000 FT



P2 SCHAUM PAGE 12 NUMBER 4

(WHERE MUST A WEIGHT BE HUNG ON A POLE . OF NEGLIGIBLE WEIGHT . SO THAT THE BOY AT ONE END SUPPORTS 1/3 AS MUCH AS THE MAN AT THE OTHER END)

ANSWER: (TIMES LENGTH76 7.50000E-1) FROM THE BOY . WHERE LENGTH76 IS THE LENGTH OF THE POLE



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P4

(A HORIZONTAL UNIFORM BAR 10 M LONG IS
SUPPORTED BY TWO ROPES ATTACHED AT ITS ENDS)(
THE ROPE ON THE LEFT END MAKES AN ANGLE OF 45
 DEGREES WITH THE HORIZONTAL . WHILE THE ROPE
 ON THE RIGHT END MAKES AN ANGLE OF 60
DEGREES WITH THE HORIZONTAL)(A WEIGHT OF 100
NT IS ATTACHED 2 M FROM THE RIGHT END OF THE
BAR)(WHAT IS THE WEIGHT OF THE BAR)

ANSWER:  123.92305 NT
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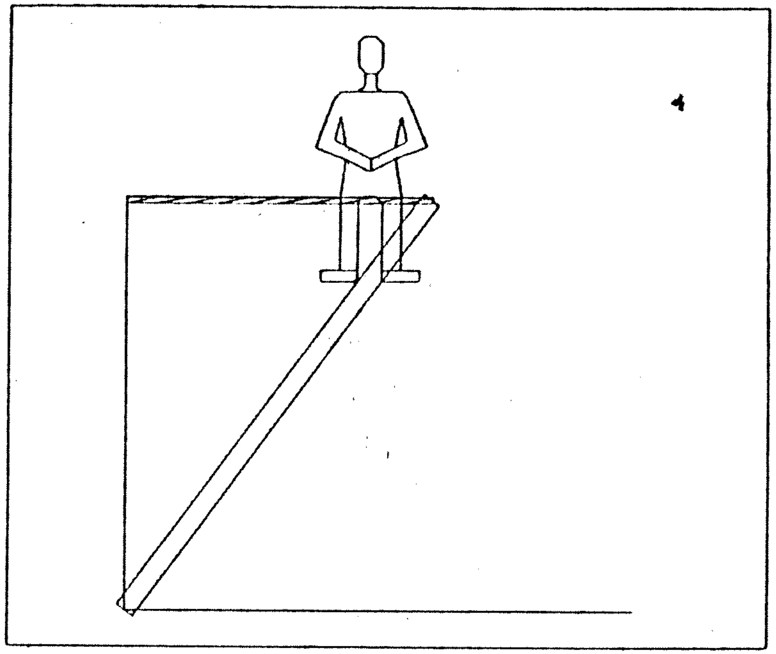

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P8  SCHAUM  PAGE 25  NUMBER 19

(THE FOOT OF A LADDER RESTS AGAINST A
VERTICAL WALL AND ON A HORIZONTAL FLOOR)(THE
TOP OF THE LADDER IS SUPPORTED FROM THE WALL
BY A HORIZONTAL ROPE 30 FT LONG)(THE LADDER
IS 50 FT LONG . WEIGHS 100 LB WITH ITS CENTER
 OF GRAVITY 20 FT FROM THE FOOT . AND A 150
LB MAN IS 10 FT FROM THE TOP)(DETERMINE THE
TENSION IN THE ROPE)

ANSWER:  120.00000 LB
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