Edge Detection in Textures -- Maxima Selection

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Abstract

This paper continues the analysis contained in [3] concerning the design of edge detection procedures for textures. Edges are detected in textures by a combination of thresholding edge operator responses and then choosing local maxima of the surviving points. The thresholding step was analyzed in [3]. Here, we analyze the effects of neighborhood size on the computation of local maxima. The analysis indicates that only small neighborhoods are required to attain reliable local maxima selection. This result is consistent with experience with real images.

1. Introduction

It has been stressed in [1] and elsewhere [2] that an important first step in computer vision systems is to transform a digitized picture function into an edge position array and various edge property arrays. The success of subsequent processes based on these edge properties critically depends upon the quality of the individual assertions about the edge elements. An edge detection procedure designed specifically to detect edges in cellular textures was introduced in [3]. This procedure is based on a class of one dimensional contrast operators defined by

$$e_k(i) = LS(i) - RS(i) \quad \text{where}$$

$$LS(i) = \frac{1}{k} \sum_{j=1}^{k} f(i-j) \quad \text{and}$$

$$RS(i) = \frac{1}{k} \sum_{j=1}^{k} f(i+j)$$

In the above expressions f is the digitized picture function and i is the point at which the contrast is to be computed using a neighborhood of size k on each side of this point.

The edge detection procedure involved the following three steps:

a) Compute $e_k(i)$ for all points i.

b) Discard all points i for which $|e_k(i)| < t$ for some value of t. This <u>thresholding</u> step is intended to discriminate between edge points and deep interior points.

c) Discard all points i for which $|e_k(i)|$ is not a local maximum in a given interval of width r and centered at point i. This <u>non-maxima suppression</u> step is intended to discriminate between edge points and near edge points.

The analysis of step b) reported in [3] is briefly reviewed in Section 2 and the remainder of this report addresses the problem of non-maxima suppression.

2. Thresholding

Thresholding is analyzed by first synthesizing the image using region based one dimensional models. These models are generated by dropping cells on a line and coloring the cells by choosing one of m coloring processes according to a given probability vector $(p_1,...,p_m)$. For analytical simplicity, only two coloring processes C_1 and C_2 will be used. Each coloring process colors a cell by assigning the intensity to each point in that cell independently from a normally distributed population of intensities. If w is the random variable describing cell widths then examples of cell width models are

a) Constant cell width model

$$P_c(w) = \begin{cases} 1 \text{ if } w = b \\ 0 \text{ if } w \neq b \end{cases}$$

b) Uniform cell width model

$$P_u(w) = \begin{cases} 1/b \text{ if } 0 \leq w \leq b \\ 0 \text{ if } w > 0 \end{cases}$$

c) Exponential cell width model

$$P_e(w) = b\text{Exp}(-bw)$$

Notice that, given a one dimensional cell structure model, one can derive a component structure model where a component is a set of contiguous cells colored by the same coloring process.

Using these texture models it is possible to determine the distribution of e_k at edge points and deep interior points. This is achieved by computing the expected value and variance of e_k at edge points and deep interior points and assuming that e_k is normally distributed. The validity of this assumption is discussed in [3].

Finally, the prior probabilities of an edge point and a deep interior point are determined by deriving a component structure model from the given cell structure model and using general results on random incidence into a renewal process [4].

Knowing the distribution of e_k at edges and at interior points and the prior probabilities we can compute the minimum error threshold t corresponding to any given value of k and the plot of t versus k determines the overall best k for the model of interest.

Notice that the analysis described above could have been done by synthesizing the image with one dimensional component structure models directly. Instead, an ordered pair $\langle P,C \rangle$ where P is a cell width model and C is a set of coloring processes, has been used to allow generality for possible extention of the analysis to two dimensional models.

3. Non-maxima Suppression

To get a better insight into the problem of non-maxima suppression we must be able to answer the following key question - what is the probability that the value of $|e_k|$ at a point which is not an edge point is a local maximum? The answer to this question will give the probability of detecting an edge distance δ away from a true edge.

Let D be the interval of width $r = 2d + 1$ centered at a point of interest i, where non-maxima suppression is to be performed. We will restrict our attention to values of r such that $d < k$.

If $|e_k(i)|$ is a local maximum then $|e_k(i)| > |e_k(j)|$ for each j in D. To evaluate Prob[$|e_k(i)|$ is a local maximum] we cannot simply evaluate Prob[$|e_k(i)| > |e_k(j)|$] individually for every point j in interval D. This is because the expressions for e_k at points i and j may involve evaluating the picture function at common points. This means that for points j_1 and j_2 in interval D Prob[$|e_k(i)| > |e_k(j_1)|$, $|e_k(i)| > |e_k(j_2)|$] is not equal to Prob[$|e_k(i)| > |e_k(j_1)|$] Prob[$|e_k(i)| > |e_k(j_2)|$].

To determine Prob[$|e_k(i)|$ is a local maximum] in interval D we will instead model the pattern of responses as a multivariate normal distribution of 2d random variables, each one of which corresponds to a particular point $j \neq i$ in D. More specifically we will consider

$$\text{Prob}[|e_k(i)| - |e_k(j_1)|, \ldots, |e_k(i)| - |e_k(j_{2d})|] = N(m, c)$$

where m is the mean vector and c is the covariance matrix

$$m = (m_1,\ldots,m_{2d}) \quad m_j = E[|e_k(i)| - |e_k(j)|] \text{ and } j \neq i$$

$$c(j_1,j_2) = \mathrm{Cov}(|e_k(i)| - |e_k(j_1)|,\ |e_k(i)| - |e_k(j_2)|)$$

and $j_1,\ldots,j_{2d}$ are the points in D other than point i.

3.1 The mean vector

Let i and j be two points in D, $i \neq j$ and let i be distance δ_1 away from the nearest edge E. Notice that $\delta_1 < k$. For the purpose of analysis we will assume without loss of generality that this edge is located to the left of point i as illustrated in Figure 1. Also let μ_1 and μ_2 be the means of the coloring processes that colored the cells c_ℓ and c_r on the left and right of E respectively and P_ℓ and P_r be the probability density functions that describe the width w_ℓ of c_ℓ and the width w_r of c_r respectively. Finally let a be the average $a = p\mu_1 + (1-p)\mu_2$ where p is the probability of choosing the coloring process with mean μ_1.

We will derive expressions for the expected value of LS(i) and RS(i). There are two cases to consider for E[RS(i)].

a) $w_r > k + \delta_1$

b) $2\delta_1 \leq w_r \leq k + \delta_1$

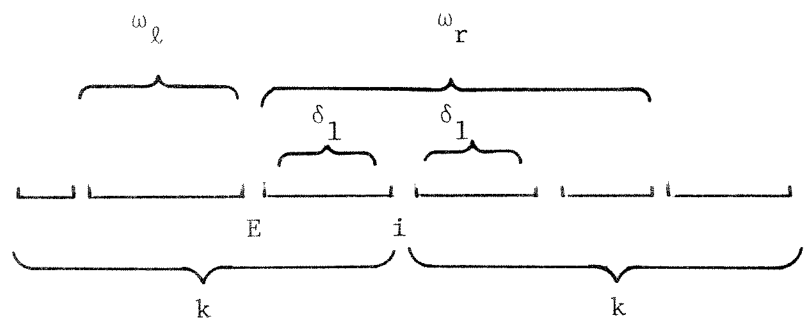

Figure 1. Composition of the neighborhood of point i.

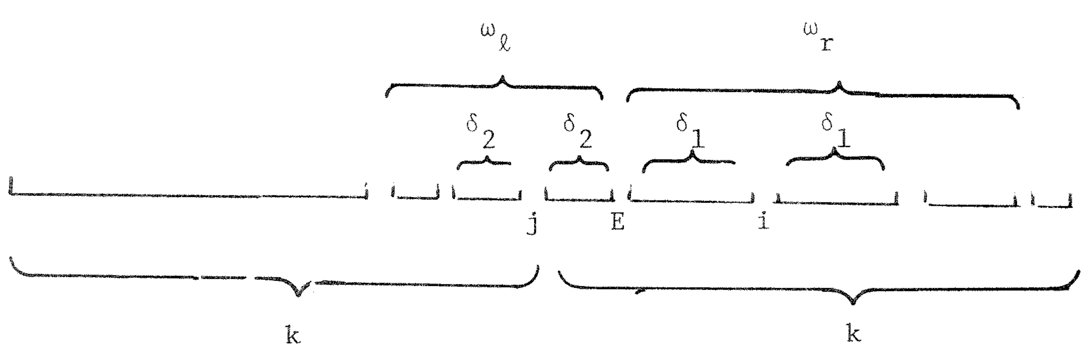

Figure 2. Composition of the neighborhoods of points i and j.

If $w_r > k + \delta_1$ then $E[RS(i)]$ is $\delta_1\mu_2 + (k - \delta_1)\mu_2$ or $k\mu_2$.

Next suppose that $2\delta_1 \le w_r \le k + \delta_1$. If we assume that the $k + \delta_1 - w_r$ pixels not in c_r are colored independently by processes C_1 and C_2 with probability p and 1-p, then the expected value of RS(i) is $\delta_1\mu_2 + x\mu_2 + (k-\delta_1-x)a$ where x is $w_r - 2\delta_1$.

Remembering that w_r is at least $2\delta_1$ since i is distance δ_1 away from the nearest edge, we combine the two cases a) and b) above and obtain

$$E[RS(i)] = \delta_1\mu_2 + \int_0^{k-\delta_1} [x\mu_2 + (k-\delta_1-x)a]P_r[2\delta_1 + x|2\delta_1]dx$$

$$+ \int_{k+\delta_1}^{\infty} (k-\delta_1)\mu_2 P_r[x|2\delta_1]\, dx$$

Now consider E[LS(i)]. Distinguishing the two cases

a) $w_\ell \le k - \delta_1$

b) $w_\ell > k - \delta_1$

one can derive the following expression for E[LS(i)]

$$E[LS(i) = \delta_1\mu_2 + \int_0^{k-\mu_1} [x\mu_1 + (k-\delta_1-x)a]P_\ell\, [x]dx$$

$$+ \int_{k-\mu_1}^{\infty} (k-\delta_1)\mu_1 P_\ell[x]dx$$

For the models considered in [3] and in this report, the cells on the left and on the right of an edge have identical width distributions. Let δ_2 be the distance from point j to edge E. Two cases need be considered.

a) j is to the right of E.

b) j is to the left of E.

If point j is to the right of E then the expressions for the expected values of RS(j) and LS(j) are similar to the ones at point i if we assume that E is also the nearest edge to j (i.e., non-maxima suppression is considered only for points not too far from edges, thus avoiding interference from other edges).

$$E[RS(j)] = \delta_2\mu_2 + \int_{o}^{k-\delta_2} [x\mu_2 + (k-\delta_2-x)a]P_r\ [2\delta_2+x|\ 2\delta_2]dx + \int_{k+\delta_2}^{\infty} (k-\delta_2)\mu_2 P_r[x|2\delta_2]dx$$

$$E[LS(j)] = \delta_2\mu_2 + \int_{o}^{k-\delta_2} [x\mu_1 + (k-\delta_2-x)a]P_\ell\ [x]dx + \int_{k-\delta_2}^{\infty} (k-\delta_2)\mu_1 P_\ell\ [x]dx$$

Now suppose that point j is to the left of E as illustrated in Figure 2. Reasoning as for points to the right of E one can derive the following expressions for the expected values of RS(j) and LS(j).

$$E[RS(j)] = 2\delta_1\mu_2 + \int_o^{\delta_2} [x\mu_1 + (\delta_2 - x)a]P_\ell\ [x]dx$$

$$+ \int_{\delta_2}^{\infty} \delta_2\mu_1 P_\ell [x]dx$$

$$+ \int_o^{k-2\delta_1-\delta_2} [x\mu_2 + (k-2\delta_1 - \delta_2 - x)a]P_r\ [2\delta_1 + x \mid 2\delta_1]dx$$

$$+ \int_{k-\delta_2}^{\infty} (k-2\delta_1 - \delta_2)\ \mu_2 P_r\ [x]dx$$

$$E[LS(j)] = \int_o^{\delta_2} kaP_\ell\ [x]dx$$

$$+ \int_o^{k} [x\mu_1 + (k-x)a]P_\ell\ [\delta_2 + x]dx$$

$$+ \int_k^{\infty} k\mu_1 P_\ell\ [x + \delta_2]dx$$

From the above expressions we obtain

$$E[\ |e_k(i)| - |e_k(j)|\ ] = |E[LS(i)] - E[RS(i)]| - |E[LS(j)] - E[RS(j)]|$$

The expressions for the expected values for LS and RS at points to the left of E are rather complicated. In actual computations we will instead assume that E is the nearest edge to j and symmetry of e_k with respect to the position of E. Thus, only the simpler expressions for points to the right of E will be used.

3.2 The covariance matrix.

For notational convenience let

$$X = |e_k(i)|$$

$$Y = |e_k(j_1)|$$

$$Z = |e_k(j_2)|$$

where i, j_1 and j_2 are in D, $j_1 \neq i$ and $j_2 \neq i$. Also let

$$d_1 = X - Y$$

$$d_2 = X - Z$$

Then we can write the following

$$Cov(d_1, d_2) = E[d_1 d_2] - E[d_1]E[d_2]$$

$$E[d_1 d_2] = E[(X - Y)(X - Z)]$$

$$= E[X^2] - E[XZ] - E[XY] + E[YZ]$$

If m_x, m_y, and m_z are the expected values of X, Y and Z respectively, then,

$$E[d_1] = E[X] - E[Y]$$

$$= m_x - m_y$$

$$E[d_2] = E[X] - E[Z]$$

$$= m_x - m_z$$

$$E[d_1]E[d_2] = (m_x - m_y)(m_x - m_z)$$

$$= m^2_x - m_xm_y - m_xm_z + m_ym_z$$

$$Cov(d_1,d_2) = E[x^2] - E[XZ] - E[XY] + E[YZ]$$

$$- m^2_x + m_xm_z + m_xm_y - m_ym_z$$

$$= (E[X^2] - m^2_x) - (E[XZ] - m_xm_z)$$

$$- (E[XY] - m_xm_y) + (E[YZ] - m_ym_z)$$

$$= Var[X] - Cov[X,Z] - Cov[X,Y] + Cov[Y,Z]$$

Assuming equal variance v for the regions on either side of an edge we can write

$$Var[X] = 2kv/k^2 = 2v/k$$

We make the additional mild assumption that $e_k(i)$, $e_k(j_1)$ and $e_k(j_2)$ are all of the same sign. This assumption is reasonable since we are assuming no interference between edges. Then,

$$\mathrm{Cov}[X,\ Z] = (n_1 - n_2)v/k^2$$

where n_1 is the number of terms with the same sign and n_2 the number of terms with opposite sign common to the expressions of $e_k(i)$ and $e_k(j_2)$.

If $|i - j_2| = \Delta$ then

$$n_1 = 2(k - \Delta)$$

$$n_2 = \begin{cases} \Delta - 1 \text{ if } \Delta > 0 \\ 0 \text{ if } \Delta = 0 \end{cases}$$

Similar simple expressions can be derived for Cov(X,Y) and Cov(Y,Z). Finally, we have the following expressions for the covariance terms, with two cases being distinguished

a) $j_1 = j_2$

In this case $d_1 = d_2 = d$ and

$$\begin{aligned} \mathrm{Cov}[d_1,\ d_2] &= \mathrm{Var}[d] \\ &= 2v/k - v(2k - 3|i - j_1| + 1)/k^2 \\ &\quad - v(2k - 3|i - j_1| + 1)/k^2 + 2kv/k^2 \\ &= 6v|i - j_1|/k^2 \end{aligned}$$

b) $j_1 \neq j_2$

$$\mathrm{Cov}(d_1,d_2) = v(3|i - j_1| + 3|i - j_2| - 3|j_1 - j_2| - 1)/k^2$$

And, of course, the covariance matrix elements are specified by

$$c(j_1,j_2) = \mathrm{Cov}(d_1,d_2)$$

4. Interval Width Selection

The above analysis has been applied to image models with exponentially distributed cell widths. The curves in Figure 3 show Prob[$|e_k(i)|$ is a local maximum] as a function of the distance δ from point i to the nearest edge and for several values of the width r of the interval D in which non-maxima suppression is performed. These curves have been computed using an IMSL (International Mathematical and Statistical Libraries) routine for generating samples from a multivariate normal distribution. For each value of r, a total of 1000 (r-1)-dimensional points are sampled in order to compute Prob[$e_k(i)$ is a local maximum] for each value of the distance δ from i to the nearest edge.

Prob[$|e_k(i)|$ is a local maximum] is highest at the edge and decreases when we go farther from the edge, assuming there is no interference from other edges. Also, it can be noticed that the curves for different values of r are not only similar but close for small values of δ. This means that, at least for points not too far from edges, no sensible increase in the performance of the non-maxima suppression step should be expected when the width of the interval in which it is performed is increased. In other words, the error rate around edges will remain approximately the same over a range of values of r. Since non-maxima suppression is a process intended to discriminate between edge points and near edge points (which are points with small δ values) this suggests that non-maxima suppression need only be performed in small intervals. This also enhances the computational efficiency of non-maxima suppression.

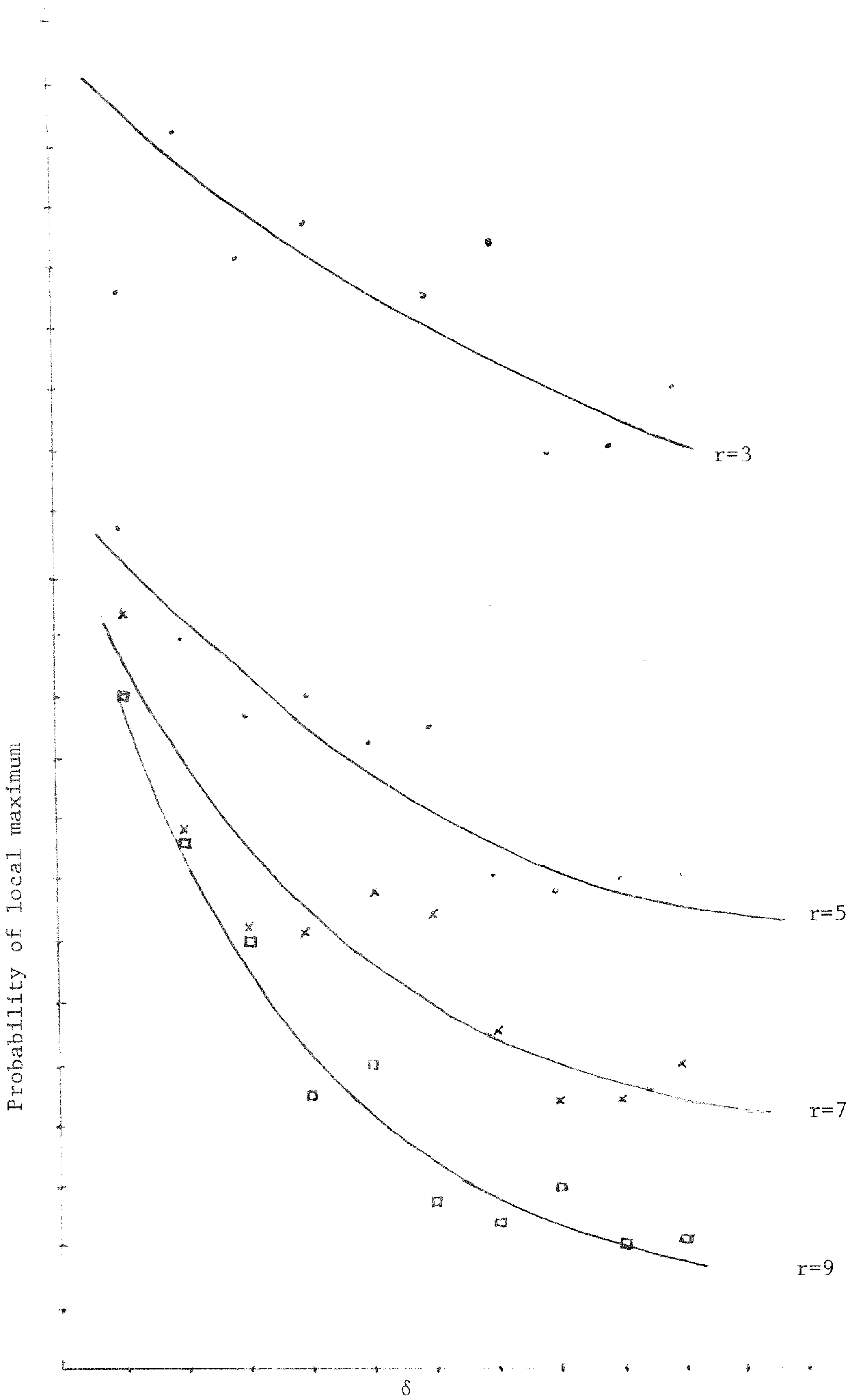

Figure 3. Probability of a point being a local maximum as a function of its distance from the nearest edge and for different values of the non maxima suppression interval width, r.

5. Conclusions

Our ultimate goal for designing edge detectors for textures is to produce as reliable a "cartoon" of a texture as we can using computationally simple techniques. Various first- and second-order statistics of the distributions of edges in the texture can then be used to discriminate between different classes of textures, as in [5-7]. The effectiveness of such an approach clearly depends on the reliability with which edges can be detected in textures.

The analysis presented in this paper, in conjunction with that presented in [3] prescribes how the parameter of a minimum error edge detection procedure can be chosen given parametric information about the distributions of size and colors of the texture elements. Given an unknown texture, one must estimate the latter parameters; or, more generally, given an image which contains many different textures, the image must be segmented into regions where these parameters are constant. The design of procedures to estimate these parameters as well as their application to a variety of natural textures will be discussed in a subsequent paper.

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