A Generalized Hough-like Transformation for Shape Recognition

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ABSTRACT

A generalization of Hough transform techniques is introduced which can be used to recognize shapes independent of their position or orientation in an image. The transform technique is equivalent to certain conventional template matching procedures, but is, on the average, 10-20 times faster. An experimental study on synthetic images is presented, and applications of the technique to object tracking are discussed.

1. Introduction

An important problem in dynamic scene analysis is tracking objects from frame to frame after an initial "lock-in" on an object of interest. A variety of factors contribute to making this problem difficult. They include changing backgrounds as the object moves, say, from a homogeneous into a textured background, and partial obscuration when the object moves behind another object in the scene.

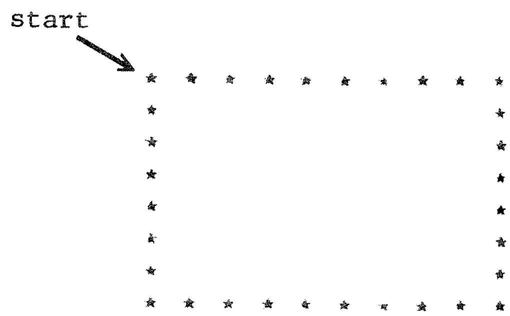
This paper introduces an approach to object tracking which is based on a generalization of Hough transform techniques [1,2] to the recognition of arbitrary planar shapes independent of picture position, orientation and limited scale changes. Section 2 of this paper discusses the technique for position invariant matching. The algorithm described in Section 2 was first introduced in [3]. Section 3 discusses the extension of this approach for orientation and scale invariant matching. Section 4 contains an experimental study which attempts to measure the robustness of the method. It should be noted that ideas similar to the ones presented in this paper have been recently reported in Ballard [4], who in addition discusses utilization of edge slope information (which our algorithms do not) and composite structures based on assemblies of shapes. This report contains some experimental results and a discussion of rotation and size invariant matching which complement the results reported by Ballard [4].

2. Position-Invariant Hough Shape Transforms

Let $B = \{(X_i, Y_i)\}_{i=0}^{n}$ be a list of boundary points for the shape to be tracked. Let P = (X,Y) be any point (in practice, a central point such as the centrord of B will be computationally convenient to use as p). Then the Hough-representation of B using p, H(B,p) is the sequence of vectors $\{\vec{d}_i\}_{i=0}^{n}$ where $dx_i = X-X_i$ and $dy_i = Y-Y_i$. Figure 1 contains a simple example for a rectangle shape.

Now, suppose we are given an image, f, which contains an instance of the shape whose boundary is described by B. A second array, h, which is an array of accumulators that is registered with f, will be used to compute the transform of f with respect to H(B,p). After the transform is computed, points in h with high values will correspond to hypothetical locations of p in f. Of course, once the location of p is known, B can be recovered from H(B,p). The array h will be larger than the array of f, since if the shape is only partly contained in f, the point p might be outside of f.

The transform, h, is computed by first applying an edge detector to f to produce an edge map, e, of f. Each edge, e_i, in e is a potential element of the set B. Although contrast and orientation information may limit the subset of B to which any e_i may correspond, there is, in general, no way to determine to which element of B any e_i corresponds without considering the position of all the other e_i.



DXI	DYI
3	4
3	3
3	2
3	1
3	0
3	-1
3	-2
3	-3
3	-4
3	-5
2	4
2	-5
1	4
1	-5
0	4
0	-5
-1	4
-1	-5
-2	4
-2	-5
-3	4
-3	-5
-4	4
-4	3
-4	2
-4	1
-4	0
-4	-1
-4	-2
-4	-3
-4	-4
-4	-5

Figure 1. Hough representation of a simple rectangle

Therefore, each edge element, e_i, is compared to each vector in H(B,p) to compute a possible location for p, and that location is incremented in the transform, h. That is, h is computed by the following simple algorithm, MATCH 1, originally reported in [3]:

For each $e_i = (X_i, Y_i)$ in e do

For each $d_j = (dx_j, dy_j)$ in H(B,p) do

$h(X_i + dX_j, Y_i + dY_j) :=$

$h(X_i + dX_j, Y_i + dY_i) + 1;$

Notice that the result of applying this algorithm is exactly the same as correllating a binary image representation of B with the binary edge map, e (this was originally pointed out by Sklansky [5]; see Ballard [4]). The correllation, however, is based on considering all points in h as potential location for p, and then for each location counting the number of appropriately positioned (according to H(B,p)) edges in e. The advantage of the transform algorithm is computational efficiency. If h is an rxs picture, then to compute h using a standard correllation algorithm requires 0(rxsxn) operations - i.e., for each of rxs potential location for p, we must check the n locations of possible edge points determined by H(B,p). Algorithm MATCH 1, on the other hand, requires $0(|e|xn)$ operations where $|e|$ is the number of edges detected. Since, in practice, edges account for no more than 5%-10% of any image, algorithm MATCH 1 will result in speed-ups of 10 to 20 over conventional correllation procedures.

To illustrate the procedure, an image was created in which the rectangle described in Figure 1 was inserted at an arbitrary location. Figure 2 contains the transform of that image. Notice that the peak of the transform occurs at the location of p.

SKILL: PLAIN
DETECTED POINT OF INTEREST IS (16, 16), MAXIMUM = 32

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	6	2	2	2	2	2	2	2	2	12	2	2	2	2	2	2	2	2	6	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 2. Hough transform for the shape in Figure 1 placed at position (16,16)

3. Rotation Invariant Matching

In the preceding section we assumed that the orientation of B in f was known. Suppose, on the contrary, that it is not known (this can occur, e.g., while tracking a vehicle, from above, which is moving along an unpredictable path). In this case, when we hypothesize that a particular e_i corresponds to some d_j, the strongest conclusion we can draw is that if e_i were indeed d_j, then p must lie somewhere on the circle of radius $R_j = \sqrt{dX_j^2 + dY_j^2}$ centered at e_i. The following algorithm, algorithm MATCH 2 , accomplishes rotation invariant matching.

```
For each e_i = (X_i,Y_i) in e do

    For each d_j in H(B,p) do
        R_j = sqrt(dX_j^2 + dY_j^2)
      For Θ = o, 2π, by dΘ do begin

          h_x = R_j *cos Θ + X_i;

          h_y = R_j *sin Θ + Y_i,

          H(h_x,h_y) = H(h_x,h_y) + 1

      end
```

Unlike algorithm MATCH 1 where the results were identical to what could have been obtained by correllating the binary image e with a binary image representation of B, the results of applying algorithm B are not identical to what would be obtained by individually correllating $m=2\pi/d\Theta$

rotated versions of H with e, and then choosing the maximum match amongst the m correlation planes. Instead, algorithm MATCH 2 adds the m correlation planes together to obtain a single plane (h). The position in this plane having maximum value is then interpreted as the location of B.

Notice that if prior information is available concerning the orientation of the object in the frame, then this information can be easily taken advantage of by the algorithm. One simply modifies the bounds on the inner FOR loop so that only a circular arc in h, rather than an entire circle, is incremented. In tracking vehicles moving along roads, e.g., one can ordinarily assume that between the successive frames the vehicle will not make a turn sharper than $\pi/2$, since roads do not bend that quickly. Therefore, the bounds on the inner FOR loop can, in this situation, be modified to $-\pi/2$ to $\pi/2$.

Although algorithm MATCH 2 can detect an arbitrarily oriented version of a shape, it does not compute the orientation of the shape. This could be done by maintaining m separate correllation plans and applying algorithm MATCH 1 to m rotated versions of H(B,p). In practice, however, this approach has unacceptable storage and time requirements.

Instead, it is possible to construct a second transform of B, but with respect to a different point, p'. If (i,j) is the point in the transform of H(B,p) having maximal value, and if (i',j') is the point in the transform of H(B,p) having maximal value (notice that these values must, in principle, be identical), then the direction from (i,j) to (i',j') gives the direction from p to p' in f. Points p and p' should be chosen to be sufficiently far apart so that small errors in the locations of the maxima in the transforms h of H(B,p) and h' of H(B,p') do not lead to large errors in the computed orientation of B.

One last modification to the matching algorithms worth mentioning is the incorporation of a "smoothing" step. The purpose of this is to overcome the slight mispositioning of edges by the edge detection algorithm; this will tend to distribute the contribution from the detected edges of the shape to a small neighborhood around the actual location of p. To overcome this, rather than simply incrementing a single location, (h_x,h_y) in h, one can increment a kxk neighborhood (with k ordinarly 3) of $h(h_x,h_y)$. Notice that this is equivalent to applying either of the match algorithms, and then replacing $h(h_x,h_y)$ by the sum of its kxk neighborhood. However, performing the neighborhood increment in the match algorithm requires only an additional $8|e|$ operation (for k=3), while applying the summation as a post-process to h requires $4n^2$ operations (using a fast square neighborhood summation algorithm). Since, as mentioned previously, $|e|$ is no more than $.05n^2 - .1n^2$, the former approach is computationally less costly.

Notice that the algorithms can also be modified in a straightforward way to deal with a limited range of scale information. Suppose, e.g., it is known that the object in the image is S times the size of the model, with $S \in [S_1,S_2]$ (note $S_1 < S_2$ and $0 < S_1$). Then, in algorithm MATCH 1 rather than just incrementing a single point at distance $d=\sqrt{d_x^2+d_y^2}$ from an edge point, one marks all points in direction $\tan^{-1} d_y/d_x$ and with distances $d' \in [S_1 d, S_2 d]$. For rotation invariant matching, rather than incrementing a circle (or a circular arc if constraints on the orientation are available) one increments a ring of inner radius $S_1 d$ and outer radius $S_2 d$ (or the intersection of the ring with a wedge). Again, different correlation planes can be maintained for different values of the scale, but this increases the storage and computational requirements of the matching algorithms. Note that this idea was employed by Davis [6] to detect circles of various sizes using Hough transform techniques.

4. Experimental Study

A simple experiment using synthesized images was designed and performed to measure the robustness of the shape recognition procedures. In the first experiment orientation was controlled, while in the second experiment orientation was allowed to vary.

The data generated were binary images which are intended to represent the result of applying an edge detection procedure to a grey-scale image. The rectangle shown in Figure 1 was used as the shape to be detected. The images are generated by choosing a random angle for rotating the shape (for experiment 1 the angle was fixed at 0) and then choosing an arbitrary image position for point p and "painting" the shape around that position. The unreliability of the edge detector is modeled by then randomly reversing 100x p percent of the image points from 0 to 1 or 1 to 0. Figure 3 a-d shows representative images obtained for P=.40 and .50.

The transform at the degraded binary image is then computed and the shape was defined to be correctly matched if the location of the peak of the transform is within one pixel of the actual location of the shape. Figures 4 a-d contain the transforms for Figures 3 a-d. For each value of P, 15 images were constructed. Tables 1-2 list the probability of error as a function of p for fixed orientation (Table 1) and orientation invariant matching (Table 2).

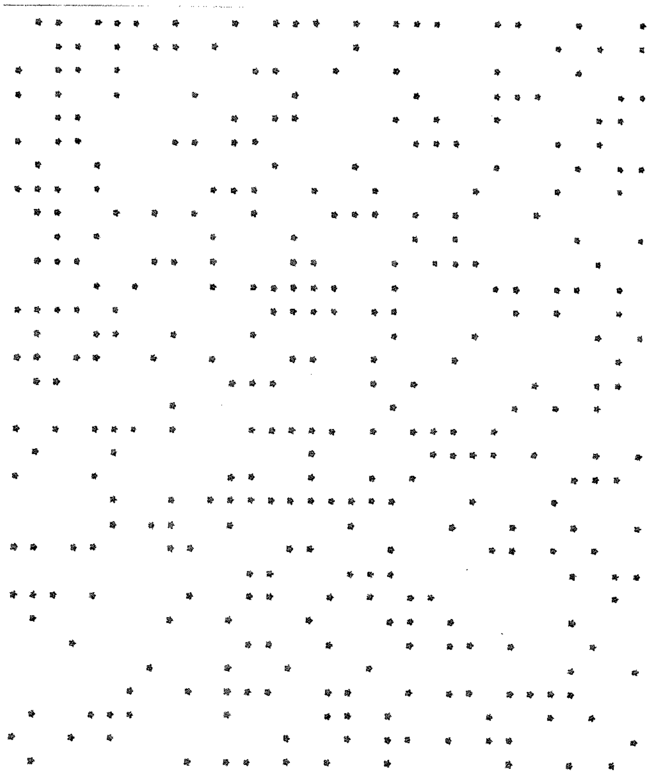

Figure 3a - p=40, orientation fixed

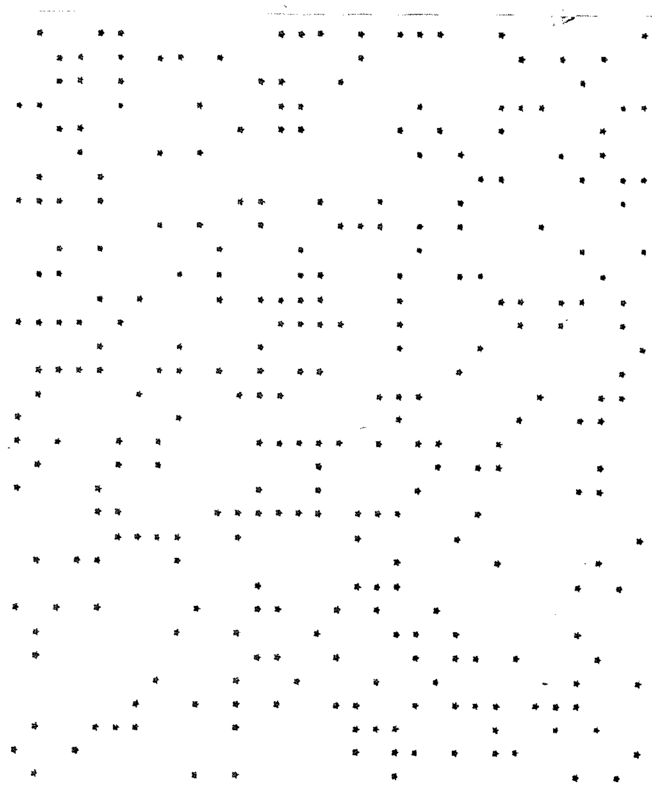

Figure 3b - p=40, orientation random

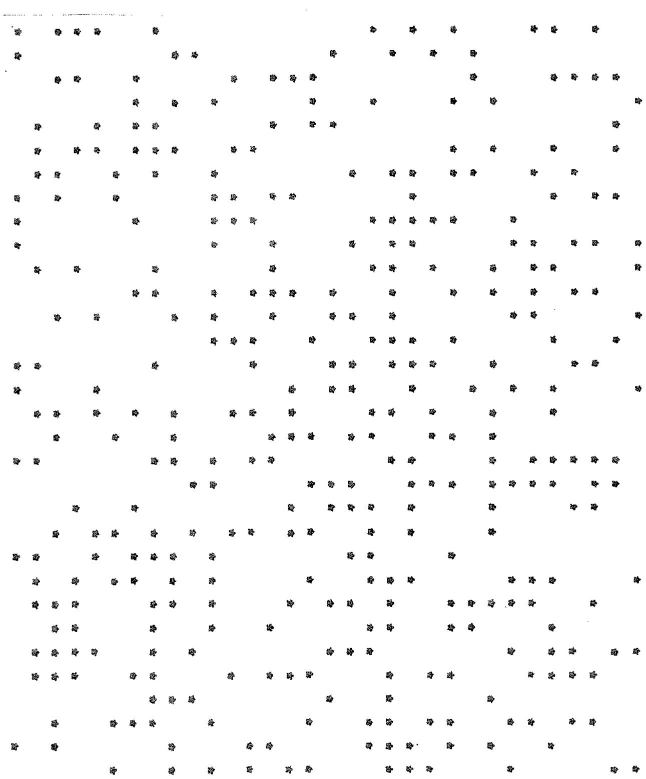

Figure 3c - p=50, orientation fixed

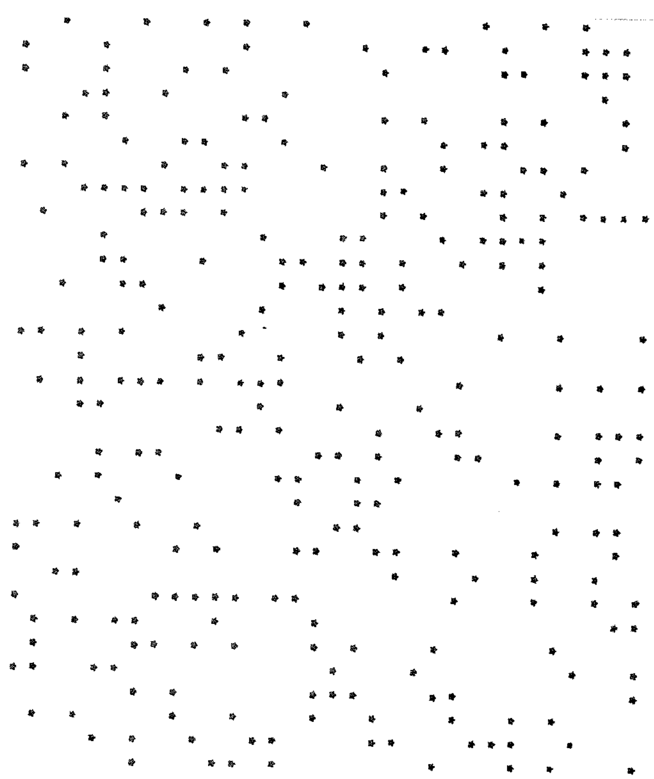

Figure 3d - p=50, orientation random

Figure 3. Noisy shapes in noise background

DETECTED POINT OF INTEREST IS (16, 16), MAXIMUM = 21

3	5	3	2	5	4	5	6	5	5	7	5	2	5	5	4	3	2	3	2	3	1	3	4	3	4	5	4	2	3	3	4
3	4	2	2	5	5	6	6	3	4	7	5	2	4	4	4	4	3	4	3	3	1	3	5	3	3	4	3	1	2	2	3
4	5	3	3	5	5	6	8	4	5	8	6	4	6	6	6	5	3	4	4	4	2	4	6	3	4	5	4	2	3	3	3
7	9	8	8	9	10	12	13	7	8	10	7	5	7	7	8	7	6	7	7	8	6	6	9	7	8	9	7	5	6	7	6
3	5	4	5	6	9	11	12	9	10	13	10	8	10	10	11	8	7	6	6	7	7	7	10	7	5	7	5	3	4	5	3
5	7	5	7	8	11	12	12	9	8	10	9	8	9	9	10	8	9	8	7	9	10	10	13	10	10	9	7	6	7	7	4
5	6	5	6	6	8	8	9	6	6	6	5	7	7	7	9	7	6	6	7	9	10	9	12	9	10	8	7	6	6	7	4
6	7	6	8	9	9	8	11	9	8	8	7	10	8	7	8	6	5	4	5	5	9	7	9	7	10	7	6	7	7	8	5
5	5	4	6	8	7	7	10	9	8	10	10	13	11	10	10	8	8	7	8	8	11	8	10	8	11	8	6	6	7	8	5
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7	5	7	9	8	11	8	6	7	11	9	9	11	10	11	15	15	12	13	13	12	11	11	10	11	9	7	4	4	4	2	3
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4	5	7	4	6	7	4	8	4	5	4	6	4	4	5	2	5	3	4	4	3	4	2	2	2	3	4	1	3	2	2	1

Figure 4a - Transform of Figure 3a.

SKILL: ORIENTATION INVARIANCE
DETECTED POINT OF INTEREST IS (16, 15) MAXIMUM = 398

61	62	77	111	142	146	155	140	136	132	133	137	162	166	126	87	96	104	119	146	162	168	171	156	127	117	120	107	114	110	91	78
71	78	91	132	180	172	171	171	165	172	169	180	189	170	125	83	86	109	117	133	149	181	180	177	133	130	123	124	119	118	96	100
86	100	134	181	202	214	224	209	202	190	189	204	223	202	142	112	93	124	123	153	164	174	177	159	125	127	151	146	137	134	118	112
107	121	150	185	240	264	262	236	197	207	202	212	228	218	183	158	148	153	152	157	171	187	168	161	139	167	181	190	167	135	111	92
113	131	154	182	257	287	281	238	202	177	204	214	232	238	225	209	199	205	196	210	199	183	189	188	196	225	250	233	182	131	113	101
138	155	197	251	304	318	286	222	192	192	211	225	225	241	241	250	213	221	237	233	224	203	208	204	199	265	280	235	184	139	110	84
163	186	250	287	313	318	275	211	190	209	225	213	219	243	272	223	189	209	244	242	214	197	191	204	215	249	247	221	157	113	95	88
161	209	217	255	290	320	262	208	221	246	222	217	239	286	263	218	180	199	206	226	215	198	193	186	203	209	224	198	169	134	101	84
143	187	191	208	285	301	285	253	254	270	279	264	245	290	292	212	179	170	201	221	223	206	213	188	200	190	208	199	168	139	120	113
135	159	155	217	246	311	294	275	296	331	293	312	316	306	255	204	189	211	209	225	218	236	239	221	190	183	215	206	174	131	113	114
124	147	181	196	235	285	285	287	303	334	347	342	326	294	272	213	204	186	232	220	214	222	226	210	181	215	235	221	175	132	127	113
116	135	162	191	235	252	272	283	321	305	312	306	307	302	248	226	220	227	211	189	201	205	228	229	233	227	229	231	172	146	130	146
100	122	156	172	197	249	290	288	293	263	261	239	263	262	272	267	256	235	232	206	187	235	246	255	273	253	242	177	142	129	153	161
84	106	139	175	180	221	242	269	281	251	204	213	241	275	295	302	321	293	242	203	206	239	257	283	258	266	224	166	143	144	143	160
80	110	154	165	185	229	277	267	241	221	215	216	243	287	345	360	338	311	273	195	173	182	218	245	262	246	233	202	150	140	170	160
97	125	169	193	217	262	287	301	270	238	221	228	243	291	354	398	334	309	241	183	156	160	216	223	260	251	239	210	171	157	135	133
120	140	137	162	227	298	344	309	262	267	282	224	238	275	366	380	334	260	234	221	204	207	240	267	276	261	240	197	156	154	138	135
100	122	142	170	234	301	300	326	315	301	261	269	267	287	322	304	312	280	245	243	272	277	269	303	273	259	228	186	201	167	152	136
112	126	156	173	233	261	286	274	291	314	335	324	257	271	275	282	240	232	262	288	324	304	279	253	263	249	255	248	209	195	172	142
95	127	151	183	198	220	230	250	283	308	340	333	303	267	236	227	209	234	271	307	310	297	249	222	233	256	288	278	250	222	163	135
113	122	153	166	171	218	221	223	251	314	360	340	296	254	227	219	203	207	255	333	311	279	226	218	213	289	322	290	267	219	156	104
133	148	169	197	202	187	202	229	269	308	331	352	314	270	209	186	191	229	270	298	325	283	234	221	215	245	270	292	254	211	159	114
116	178	234	250	257	226	226	223	271	272	341	378	347	293	247	206	225	253	283	296	307	273	256	188	192	211	228	222	245	213	173	129
123	200	245	306	299	253	240	234	227	280	332	357	362	317	266	261	276	271	280	285	277	251	235	215	180	185	221	219	216	216	198	138
143	180	246	278	295	276	285	253	215	247	302	324	312	287	308	283	267	245	239	244	259	241	252	222	222	239	224	191	209	192	185	141
138	162	216	263	263	272	289	275	234	244	262	316	293	258	274	261	225	229	212	242	247	244	273	294	292	257	255	226	194	182	152	149
132	153	175	235	254	244	246	269	265	235	258	297	256	220	239	236	226	188	187	240	265	266	289	290	293	288	269	224	180	184	159	137
108	129	174	212	213	235	232	247	230	233	231	259	222	189	191	204	204	179	183	219	227	260	267	294	266	247	227	188	179	166	155	128
100	94	123	180	188	211	216	209	212	227	235	226	183	150	158	197	178	197	192	194	210	209	219	229	225	232	186	171	132	157	148	133
80	86	108	113	143	176	203	200	220	208	213	208	175	145	167	161	160	208	194	183	172	159	176	195	177	190	169	150	146	148	120	118
76	72	103	99	123	152	171	181	184	197	198	190	174	145	127	130	153	190	183	183	158	141	118	147	141	163	152	148	139	117	112	92
77	92	112	110	129	130	139	151	158	170	167	159	136	113	98	123	143	181	184	160	129	113	109	105	103	125	129	124	110	95	89	81

Figure 4b - Transform of Figure 3b.

SKILL: PLAIN
DETECTED POINT OF INTEREST IS (16, 16), MAXIMUM = 21

3	5	4	3	7	6	6	7	8	6	8	6	3	7	7	6	6	6	7	6	5	3	6	7	5	6	6	5	3	3	3	4
3	4	3	2	6	6	6	6	5	4	7	6	2	6	6	6	6	6	6	5	4	2	5	6	4	4	5	4	2	2	2	3
4	5	4	3	6	5	6	8	6	5	8	7	4	8	7	7	7	7	7	7	6	4	8	10	7	8	9	7	5	5	5	5
9	11	11	10	13	11	13	13	9	8	10	8	5	9	9	10	10	10	9	10	10	8	9	11	8	10	12	7	5	5	6	5
4	5	6	6	9	9	13	13	11	10	13	12	8	11	11	13	11	11	7	8	9	9	10	12	9	8	11	6	4	4	5	3
5	6	6	6	10	10	13	12	10	9	10	12	11	14	12	14	13	14	10	9	11	12	12	13	11	12	12	7	6	7	7	4
7	7	8	8	10	9	11	11	8	8	6	7	8	10	7	9	8	8	6	7	10	12	11	12	10	11	10	6	5	5	6	3
7	6	7	9	12	10	10	12	10	10	8	9	12	12	9	10	9	9	7	9	9	13	11	12	10	13	11	7	8	7	8	4
7	5	5	7	11	8	9	10	9	9	8	9	13	13	11	10	9	9	7	9	9	13	9	10	8	12	10	6	7	7	8	4
5	3	4	6	10	7	8	9	10	10	9	10	14	13	13	13	11	12	11	12	12	15	12	13	12	13	11	8	8	8	8	4
9	8	10	12	14	11	9	9	9	9	6	5	10	8	9	8	7	8	10	11	10	13	9	10	11	12	9	8	8	8	8	6
4	2	3	7	9	8	7	8	9	11	9	8	14	10	11	11	10	7	9	10	9	13	9	12	13	12	9	10	10	9	8	5
4	3	4	8	10	9	11	12	11	13	12	11	18	13	15	16	14	12	14	14	13	17	11	13	14	11	9	8	9	7	5	3
7	7	8	10	12	10	13	14	10	10	9	9	16	11	13	15	11	9	11	11	10	12	7	10	12	8	9	7	7	6	5	4
6	8	9	11	14	11	14	14	10	9	9	9	15	9	12	18	13	10	13	12	11	15	11	14	18	13	15	12	10	8	6	5
5	7	8	10	12	10	15	14	12	12	13	12	18	11	14	21	15	11	15	15	11	17	12	15	18	12	15	13	11	8	8	7
4	5	7	9	11	8	13	11	9	9	11	9	14	7	11	20	15	12	15	15	10	17	12	17	18	11	13	11	10	7	8	6
5	5	8	8	12	9	14	11	10	11	11	9	12	6	8	15	9	7	9	10	6	12	7	12	14	7	10	6	6	5	6	5
5	4	7	6	11	9	13	10	9	10	8	8	10	6	9	15	7	6	8	10	7	12	8	13	15	9	14	10	10	8	8	7
7	5	8	7	10	8	11	10	11	13	11	11	14	12	15	20	14	15	15	16	13	16	11	14	13	7	10	4	5	3	2	2
8	5	7	7	10	11	12	10	11	13	12	12	14	9	11	14	9	10	9	10	7	9	6	11	11	7	10	6	7	6	6	5
8	5	8	8	8	9	8	6	8	10	7	8	11	7	11	13	8	8	8	10	8	9	6	11	12	9	11	7	7	6	5	5
9	6	9	11	13	14	10	8	10	13	9	11	13	9	13	16	14	13	11	12	12	12	10	12	13	10	11	7	7	7	5	6
6	5	8	10	12	13	10	10	10	12	8	9	12	8	10	13	12	13	10	9	10	11	10	11	14	12	11	7	8	7	5	6
7	6	7	9	12	14	11	11	11	13	9	10	14	9	9	9	8	10	6	4	4	7	5	6	8	7	7	2	4	3	2	2
10	9	10	13	17	19	15	14	13	13	8	9	13	7	8	7	8	9	6	6	6	9	5	6	9	9	8	4	5	3	3	3
6	6	8	10	13	14	11	12	13	15	12	12	17	13	14	12	15	15	12	12	10	14	10	9	12	11	9	5	7	4	3	3
6	6	8	9	13	13	11	12	10	12	10	9	13	9	10	8	11	12	9	10	6	10	7	7	9	8	8	3	6	3	3	2
4	3	5	4	7	6	6	9	6	8	5	5	9	6	7	4	7	8	5	7	2	6	2	3	6	6	7	2	6	3	3	2
7	6	8	7	10	9	7	9	7	10	7	7	9	7	9	6	10	11	8	10	5	8	4	4	5	5	7	2	6	3	3	2
4	3	6	5	7	8	7	10	8	12	9	8	9	7	9	5	8	9	7	9	5	7	4	4	4	5	7	2	5	2	2	1
6	5	7	6	7	8	6	9	6	8	6	6	7	5	6	2	5	5	4	6	2	4	3	3	3	4	6	2	5	3	2	1

Figure 4c - Transform of Figure 3c.

SKILL: ORIENTATION INVARIANCE
DETECTED POINT OF INTEREST IS (16, 15) MAXIMUM = 438

73	78	95	110	132	145	162	180	182	188	175	170	157	182	206	197	186	153	144	149	132	149	203	202	188	176	154	156	124	121	117	102
64	78	87	110	128	151	199	198	185	209	226	209	175	220	234	223	185	163	170	172	168	201	257	242	216	185	159	164	142	132	142	133
68	95	105	129	135	203	212	223	219	210	214	210	210	242	302	285	218	180	208	218	224	240	298	289	220	187	186	172	159	128	167	151
102	118	139	156	207	234	258	233	212	205	215	220	219	276	336	311	247	214	224	235	233	248	323	330	258	197	188	197	180	154	150	155
119	154	186	202	250	276	292	271	229	237	252	248	239	297	362	361	287	226	216	259	248	263	339	358	319	254	213	229	218	185	162	175
123	173	184	220	272	313	336	303	264	261	297	275	285	316	381	371	331	263	273	312	293	311	346	349	310	286	230	237	252	241	211	198
140	169	205	216	261	310	332	318	300	313	343	350	301	329	378	397	350	328	316	355	386	354	340	343	299	267	246	261	300	274	254	196
126	167	182	212	270	304	315	338	333	349	373	376	376	393	414	409	374	331	340	371	406	373	379	352	327	319	325	324	324	311	257	209
123	162	180	219	264	291	303	308	317	353	399	407	419	413	418	427	374	334	352	393	397	382	371	390	411	377	384	383	362	287	225	201
118	148	178	204	255	283	275	296	290	358	401	418	426	426	411	411	374	360	374	407	420	419	385	417	405	423	419	363	342	256	184	150
92	140	182	202	247	267	288	265	277	329	365	384	413	433	412	368	362	377	401	407	433	417	396	403	376	404	401	351	285	243	181	154
107	159	199	236	296	294	254	229	244	275	332	352	393	391	377	384	357	419	424	411	412	436	387	367	363	395	355	348	274	254	191	173
122	176	219	269	315	316	250	234	250	293	308	297	359	362	389	376	390	407	433	423	377	373	401	365	378	359	365	319	260	238	205	177
139	148	201	236	321	322	293	279	300	323	345	335	340	352	374	381	414	401	409	402	370	329	335	350	378	362	353	322	288	231	222	182
112	135	164	209	261	304	323	337	360	358	370	374	359	378	382	406	435	411	422	395	373	329	313	317	340	326	336	323	309	251	228	197
116	96	127	182	223	296	354	389	396	359	365	378	401	395	393	438	427	390	391	388	386	377	334	299	310	321	297	284	277	259	249	219
123	123	148	154	210	289	346	405	408	397	396	412	377	402	381	416	413	388	338	361	384	392	378	356	340	341	317	312	283	251	226	208
142	130	138	191	193	222	315	346	367	376	369	373	363	350	329	364	398	363	332	354	353	386	392	413	371	366	347	294	262	240	219	200
150	157	164	204	192	222	270	291	300	299	291	320	337	308	324	363	371	371	346	334	355	388	398	392	357	371	352	316	261	209	167	173
140	171	219	248	260	227	265	292	274	245	236	298	352	336	324	345	381	371	314	321	368	367	393	376	369	361	374	330	262	231	181	168
154	191	241	276	275	282	306	298	263	258	230	248	316	346	351	308	309	339	320	305	348	373	380	351	336	365	370	326	279	232	209	169
155	214	221	257	274	286	282	311	309	284	241	244	283	309	308	313	298	327	314	334	333	355	333	345	299	333	337	315	257	234	195	174
161	195	216	230	240	268	286	311	308	327	289	263	277	304	303	282	291	340	353	323	307	278	312	320	321	305	318	270	243	207	187	163
168	193	206	222	248	253	272	314	318	299	290	285	292	283	276	293	303	307	330	285	295	272	299	306	337	334	320	312	252	199	168	147
167	194	208	214	241	267	279	286	303	305	299	269	258	244	263	271	292	290	303	306	275	291	318	348	363	356	378	342	289	204	167	150
191	193	188	210	240	263	288	326	337	310	290	251	255	233	244	253	286	301	302	302	325	349	358	361	372	394	388	344	287	243	203	173
186	210	203	200	229	260	280	291	294	320	290	252	222	230	226	268	287	285	280	326	375	380	360	334	337	371	352	344	278	242	208	194
183	214	218	211	207	185	233	262	264	252	234	252	236	208	186	221	251	278	276	319	333	362	317	286	293	336	303	278	244	229	232	208
154	201	199	208	214	219	223	192	201	215	245	261	229	187	178	171	192	219	267	302	311	316	287	233	239	297	305	270	233	203	197	175
117	149	188	197	236	223	198	161	185	194	222	225	209	172	140	109	160	195	218	258	288	261	255	227	201	224	260	222	193	174	182	152
96	128	139	160	186	201	189	147	139	159	184	160	151	140	134	109	109	138	211	215	242	234	237	185	164	174	166	155	162	159	154	131
70	104	132	146	168	167	128	116	118	125	135	141	144	131	122	102	104	123	151	190	201	217	221	187	145	147	157	135	138	135	120	101

Figure 4d - Transform of Figure 3d.

P	Prob[error]
.05	0
.10	0
.15	0
.20	0
.25	0
.30	0
.35	0
.40	0
.45	.07
.50	.13
.55	.24
.60	.54

Table 1 - Probability of error for fixed orientation shape matching.

P	P(error)
.05	0
.10	0
.15	.07
.20	0
.25	.07
.30	.07
.35	.13
.40	.24
.45	.42
.50	.87
.55	.62
.60	.87

Table 2 - Probability of error for rotation invariant matching.

5. Conclusion

We have presented a generalization of Hough transform techniques to allow recognition of arbitrary shapes independent of position, orientation and limited scale. The algorithm is an extension of the one presented in [3] and the Hough representation used is similar to, but less general than, the one discussed in [4]. We are currently applying the ideas in this paper to image registration.

References

1. S. Shapiro, "Transformation for the computer detection of curves in noisy pictures," Comp. Graphics and Image Processing, 4, pp. 325-335, 1975.

2. A. Rosenfeld and A. Kak, Digital Picture Processing, Academic Press, N.Y., 1976.

3. P. Merlin and D. Farber, "A parallel mechanism for detecting curves in pictures," IEEET-Computers, 24, pp. 94-95, 1975.

4. D. Ballard, "Generalizing the Hough Transform to detect arbitrary shapes", Univ. of Rochester Comp. Science, TR-55, Oct., 1977.

5. J. Sklansky, "On the Hough technique for curve detection," IEEET-Computers, 1977.

6. L. Davis, "Computing the spatial structure of cellular textures," Computer Graphics and Image Processing, II, pp. 111-122, 1979.