# UNBOUNDEDNESS DETECTION FOR A CLASS OF COMMUNICATING FINITE-STATE MACHINES

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# ABSTRACT

Let M and N be two communicating finite-state machines which exchange one type of message. We discuss an algorithm to decide whether or not the communication between M and N is bounded. The algorithm is based on constructing a finite representation of the reachability tree of M and N assuming that M and N progress in equal speeds.

# I. INTRODUCTION

Let M and N be two communicating finite state machines which exchange one type of message over two unbounded, one-directional, FIFO channels. Informally, the communication between M and N is said to be bounded iff there is a nonnegative integer K such that at each "reachable state" of M and N, the number of messages in each channel is less than K. (Formal definitions are given later.) Cunha and Maibaum [3] have discussed an algorithm to decide whether the communication between M and N is bounded. Their algorithm consists of (i) constructing a finite representation U of the "reachability tree" of M and N, then (ii) scanning U to detect unboundedness, if any. To ensure that all reachable states are represented in U, the algorithm takes into account all possible relative progress speeds of M and N. In this paper, we present a more efficient algorithm to solve the problem. In particular, our algorithm constructs a finite representation T of the reachability tree assuming that M and N progress in <u>equal</u> speeds. Thus, the total number of states generated by our algorithm is, in most instances, less than those generated by the Cunha-Maibaum algorithm.

There are two practical reasons to consider this problem:

i. Many self-timing VLSI arrays can be modeled as arrays of communicating machines where each two machines exchange, at most, one type of message. As shown in [4], if the communication between each pair of machines in such an array is bounded, then the communication within the array is bounded. Thus, our algorithm can be used to prove efficiently that the communication within a VLSI array is bounded.

ii. Many communication protocols can be modelled as two communicating finite state machines which exchange <u>many types</u> of messages [1, 2, 5, 6, 7]. Let $\overline{M}$ and $\overline{N}$ be two such machines. As shown in [8], if $\overline{M}$ and $\overline{N}$ are abstracted by two machines M and N which exchange one type of message and if the communication between M and N is shown to be bounded, then the communication between $\overline{M}$ and $\overline{N}$ is also bounded. Thus, our algorithm can be used to prove efficiently that the communication in a protocol is bounded.

# II. COMMUNICATING MACHINES

A communicating machine M is a directed labelled graph with two types of edges, namely sending and receiving edges. A sending (or receiving) edge is labelled "send" (or "receive" respectively). One of the nodes in M is identified as its initial node; and each node in M is reachable by a directed path from its initial node. For convenience, we assume that each node has at least one output edge.

Let M and N be two communicating finite state machines. A state of M and N is a four-tuple [v,w,x,y] where v and w are two nodes in M and N respectively and x and y are two non-negative integers. Informally, a state [v,w,x,y] means that the execution of M has reached node v, the execution of N has reached node w, and the input channels of M and N have x and y messages respectively.

The initial state of M and N is $[v_0,w_0,0,0]$ where $v_0$ and $w_0$ are the initial nodes of M and N respectively.

Let s=[v,w,x,y] be a state of M and N, and e be an output edge of node v or w. A state s' is said to follow s over e, denoted s--e-->s', iff the following four conditions are satisfied:

i. If e is a sending edge from v to v' in M, then s'=[v',w,x,y+1].

ii. If e is a sending edge from w to w' in N, then s'=[v,w',x+1,y].

iii. If e is a receiving edge from v to v' in M, then $x \geq 1$ and s'=[v',w,x-1,y].

iv. If e is a receiving edge from w to w' in N, then $y \geq 1$ and s'=[v,w',x,y-1].

If s--e-->s' for some edge e in M or N then s' is said to follow s.

Let s and s' be two states of M and N; and let $<e_1,\ldots,e_r>$ be a sequence of edges in M or N. s' is reachable from s over $<e_1,e_2,\ldots,e_r>$ iff there are states $s_0,s_1,\ldots,s_r$ of M and N such that $s_0$=s, $s_r$=s', and $s_i$--$e_{i+1}$-->$s_{i+1}$ for i=0,...,r-1.

Let s and s' be two states of M and N. s' is reachable from s iff either s=s' or there is a sequence $<e_1,\ldots,e_r>$ of edges in M or N such that s' is reachable from s over $<e_1,\ldots,e_r>$.

A state s of M and N is reachable iff it is reachable from the initial state of M and N.

The communication between M and N is bounded iff there is a positive integer K such that for each reachable state [v,w,x,y] of M and N, $x \leq K$ and $y \leq K$. Otherwise, the communication between M and N is unbounded. A proof for the following lemma is in the appendix.

**Lemma 1:** The communication between M and N is unbounded iff there are two reachable states $s_0$=[v,w,x,y] and $s_1$=[v,w,x+i,y+j] of M and N such that $s_1$ is reachable from $s_0$ and either ($i \geq 0$ and $j>0$) or ($i>0$ and $j \geq 0$). []

Based on this lemma, an efficient algorithm to detect unboundedness for two communicating machines is presented in Section IV. This algorithm is also based on the concept of fair reachability discussed next.

# III. FAIR REACHABILITY

A state [v,w,x,y] of M and N is fair iff x=y. Obviously, the initial global state of M and N is fair.

Let s and s' be two fair states of M and N; and let e and f be two edges in M and N respectively. s' fairly follows s over e and f iff there exists a state s" such that either (s--e-->s" and s"--f-->s') or (s--f-->s" and s"--e-->s'). s' fairly follows s iff s' fairly follows s over some edges e and f in M and N respectively.

Let s and s' be two fair states of M and N; and let P be a directed path of edges $e_1,...,e_r$ in M, and Q be a directed path of edges $f_1,...,f_r$ in N. s' is fairly reachable from s over the edges of P and Q iff there exist fair states $s_0,s_1,...,s_r$ such that $s=s_0$, $s'=s_r$, and $s_{i+1}$ fairly follows $s_i$ over $e_{i+1}$ and $f_{i+1}$, i=0,...,r-1. s' is fairly reachable from s iff s' is fairly reachable from s over the edges of some two directed paths P and Q in M and N respectively.

A fair state of M and N is fairly reachable iff it is fairly reachable from the initial state of M and N.

Earlier, lemma 1 has stated a necessary and sufficient condition for unbounded communication between two machines. In the next lemma, whose proof is in the Appendix, we show that this condition can be equivalently stated in terms of two other conditions A and B.

**Lemma 2.** There are two reachable states $s_0$=[v,w,x,y] and $s_1$=[v,w,x+i,y+j] of M and N such that $s_1$ is reachable from $s_0$ and either ($i \geq 0$ and $j>0$) or ($i>0$ and $j \geq 0$) iff one of the following two conditions is satisfied:

A. There is a reachable state s'=[v',w',x',y'] of M and N such that either node v' is in a directed cycle of all sending nodes in M or node w' is in a directed cycle of all sending nodes in N.

B. There are two fairly reachable states $s'_0$=[v',w',x',x'] and $s'_1$=[v',w',x'+k,x'+k] of M and N such that $s'_1$ is fairly reachable from $s'_0$ and $k>0$. []

From Lemmas 1 and 2, the communication between M and N is unbounded iff either condition A or condition B is satisfied. Condition B is stated in terms of fair states and fair reachability; but condition A is not. In the next lemma, whose proof is in the Appendix, we show that condition A can be also stated in terms of fair states and fair reachability.

**Lemma 3.** A state s=[v,w,x,y] of M and N is reachable iff a fair state s'=[v,w',x',y'] of M and N where x'=y' is fairly reachable. (Similarly, a state s=[v,w,x,y] of M and N is reachable iff a fair state s'=[v',w,x',y'] of M and N where x'=y' is fairly reachable.) []

The next theorem follows immediately from lemmas 1,2, and 3.

**Theorem 1:** The communication between M and N is unbounded iff one of the following two conditions is satisfied:

A. There is a fairly reachable state $s=[v,w,x,x]$ of M and N such that either node v is in a directed cycle of all sending nodes in M or node w is in a directed cycle of all sending nodes in N.

B. There are two fairly reachable states $s_0=[v,w,x,x]$ and $s_1=[v,w,x+i,x+i]$ of M and N such that $s_1$ is fairly reachable from $s_0$ and $i>0$. []

# IV. UNBOUNDEDNESS DETECTION ALGORITHM

Based on Theorem 1, the following algorithm can be used to decide whether the communication between two machines is bounded.

**Algorithm 1**:

**Input:** Two communicating machines M and N which exchange one type of message.

**Output:** A decision of whether or not the communication between M and N is bounded.

**Variable:** A directed rooted tree T whose nodes are labelled with fair states of M and N, and whose directed edges correspond to the fairly-follow relation. Initially, T has exactly one node labelled with the initial state of M and N.

**steps:**
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i. while T has a leaf node n labelled with a state s such
     that there is at least one state which fairly follows s,
     and no non-leaf node in T is labelled with the same state s

     do if s=[v,w,x,x] is such that one of the
        following two conditions is satisfied:
        a. Either node v is in a directed cycle of all
           sending nodes in M, or node w is in a directed
           cycle of all sending nodes in N.
        b. Node n in T has an ancestor node labelled with a
           state [v,w,y,y] where y<x
           then stop: The communication between M and N
                      is unbounded
           else find all the states s_1,...,s_r which
                fairly follow s;
                add an equal number of nodes n_1,...,n_r
                to T; label each new node n_i with the state
                s_i, i=1..r; add a directed edge from node
                n to each new node n_i, i=1..r

ii. stop: The communication between M and N is bounded.
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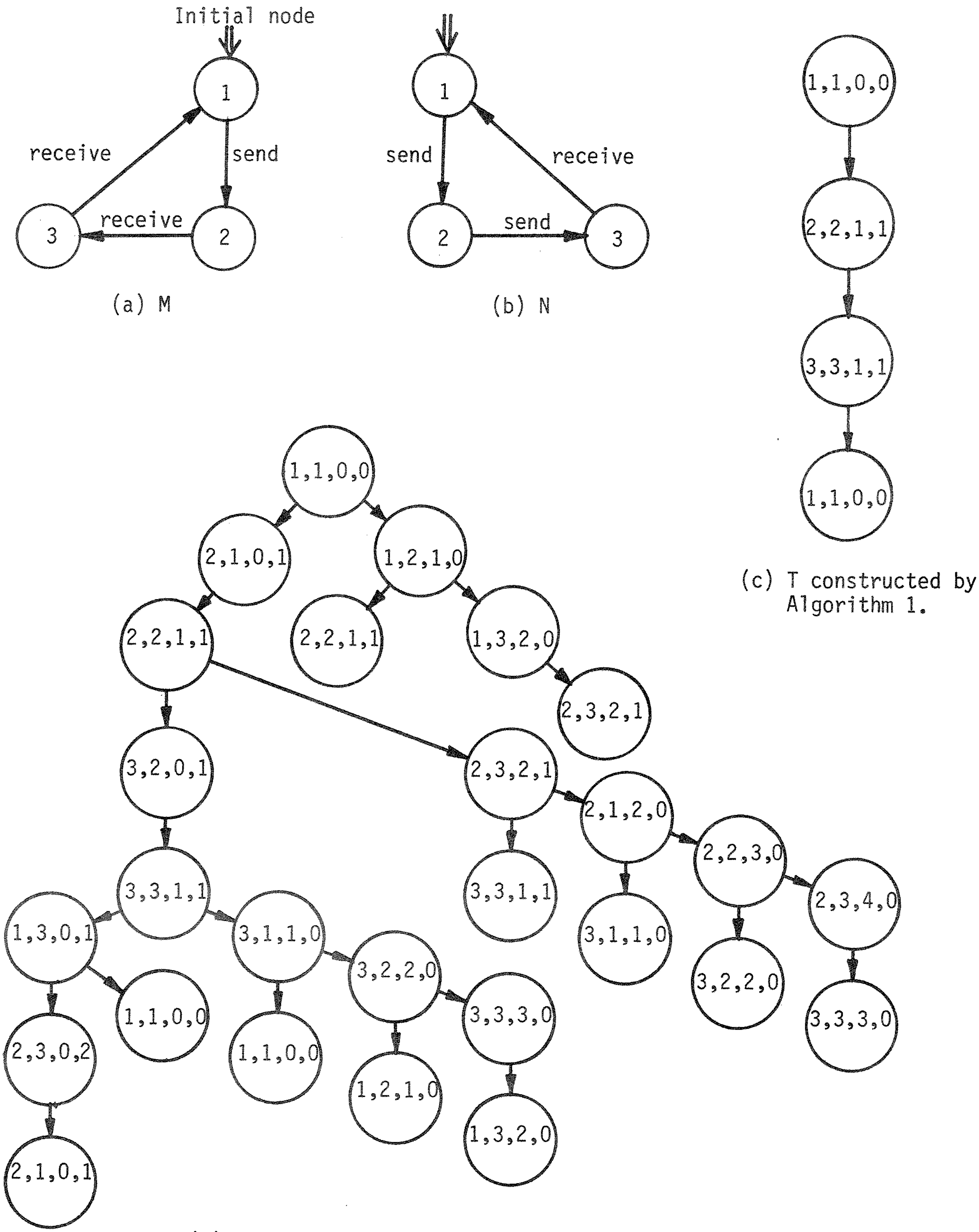
[]

**Example 1:** Consider the two communicating machines M and N in Figures 1a and 1b respectively. The tree T in Figure 1c is constructed by applying Algorithm 1 to M and N. From T, the communication between M and N is bounded. This same result can be obtained from the tree U in figure 1d, constructed by the Cunha-Maibaum Algorithm [3]. Clearly, T with 4 nodes is better than U with 27 nodes. []




(a) M

(b) N

(c) T constructed by Algorithm 1.

(d) U constructed by the Cunha-Maibaum Algorithm.

Figure 1. An example.

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# APPENDIX: PROOFS OF LEMMAS

Proof of Lemma 1:

If Part: Assume that there are two reachable states $s_0=[v,w,x,y]$ and $s_1=[v,w,x+i,y+j]$ of M and N such that $s_1$ is reachable from $s_0$ and $i \geq 0$ and $j>0$. (The proof for the other case where $i>0$ and $j \geq 0$ is similar.) Assume that $s_1$ is reachable from $s_0$ over a sequence of edges $e_1,e_2,...,e_r$; these edges form a directed cycle which starts and ends at node v in M, and possibly a directed cycle which starts and ends at node w in N. Therefore, the state $s_2=[v,w,x+2i,y+2i]$ of M and N is reachable from $s_1$ over the same sequence of edges $e_1,e_2,...,e_r$. In general, the state $s_k=[v,w,x+ki,y+kj]$, $k=2,3,4,...$, is reachable (from $s_1$). Since $i>0$, then $x+ki$ can be made larger than any given positive integer by selecting k large enough. Therefore, the communication between M and N is unbounded.

Only If Part: (Proof is by contradiction.) Assume that the communication between M and N is unbounded, and that for every two reachable states $s_0=[v,w,x,y]$ and $s_1=[v,w,x+i,y+j]$ of M and N, if $s_1$ is reachable from $s_0$ then either $i<0$ or $j<0$. Then, the number of reachable states is finite, contradicting the assumption of unbounded communication. []

Proof of Lemma 2:

If Part: There are two cases to consider.

i. Condition A is satisfied: Assume that there is a reachable state $s'=[v',w',x',y']$ of M and N where node v' is in some directed cycle of all sending nodes in M. Assume that this cycle has j sending edges; then the state $s'_1=[v',w',x'+0,y'+j]$ is reachable from $s'_0$. The two reachable states s' and $s'_1$ satisfy the required condition. (A similar argument can prove the case where node w' is in some directed cycle of all sending nodes in N.)

ii. Condition B is satisfied: The two (fairly) reachable states $s'_0$ and $s'_1$ satisfy the required condition.

Only If Part: Assume that there are two reachable states $s_0=[v,w,x,y]$ and $s_1=[v,w,x+i,y+j]$ of M and N such that $s_1$ is reachable from $s_0$ and $i \geq 0$ and $j>0$. (The proof for the case where $i>0$ and $j \geq 0$ is similar.) Assume also that $s_1$ is reachable from $s_0$ over a sequence of edges $e_1,e_2,...,e_r$. These edges form a directed cycle $C_M$ which starts and ends with node v in M and possibly a directed cycle $C_N$ which starts and ends with node w in N. There are two cases to consider.

i. If $C_M$ is a cycle of all sending nodes in M or $C_N$ is a cycle of all sending nodes in N, then condition A is satisfied.

ii. Otherwise, each of $C_M$ and $C_N$ is a directed cycle which contains at least one receiving edge. Let $s_M$ and $r_M$ be the numbers of sending and receiving edges (respectively) in $C_M$. Similarly, let $s_N$ and $r_N$ be the numbers of sending and receiving edges (respectively) in $C_N$. Since $i \geq 0$ and $j>0$, we have $s_N \geq r_M$ and $s_M > r_N$. Also, let $s'_0=[v',w',z,z]$ be a fairly reachable state of M and N where nodes v' and w' are in cycles $C_M$ and $C_N$ respectively. Define the directed cycle $D_M$ which starts and ends with node v' in M and which consists of cycle $C_M$ repeated $(s_N+r_N)$ times; similarly define the directed cycle $D_N$ which starts and ends with node node w' in N and which consists of cycle $C_N$ repeated $(s_M+r_M)$ times. Each of $D_M$ and $D_N$ has $(s_M+r_M)(s_N+r_N)$ directed edges; $D_M$ has $s_M(s_N+r_N)$ sending edges and $D_N$ has $r_N(s_M+r_M)$ receiving edges. Therefore, there is a fairly reachable state $s'_1=[v',w',z+k,z+k]$ of M and N which is fairly reachable from $s'_0$ over the edges in $D_M$ and $D_N$ where $k=s_M(s_N+r_N)-r_N(s_M+r_M)=s_Ms_N-r_Mr_N>0$. Thus, the two states $s'_0$ and $s'_1$ satisfy condition B. []

Proof of Lemma 3:

If Part: Since s' is fairly reachable, then it is reachable.

Only If Part: Assume that s=[v,w,x,y] is reachable; i.e., s is reachable from the initial state $s_0=[v_0,w_0,E,E]$ of M and N over a sequence of edges $e_1,e_2,\ldots,e_r$. These edges form a directed path P from $v_0$ to v in M and a directed path Q from $w_0$ to w in N. Let |P| and |Q| denote the numbers of edges in paths P and Q respectively. There are three cases to consider.

i. |P|=|Q|: In this case,

```
x = number of sending edges in Q
   -number of receiving edges in P
  =  |Q| - number of receiving edges in Q
   - |P| + number of sending edges in P
  =  number of sending edges in P
   - number of receiving edges in Q
  = y
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Thus, s is a fair state; the lemma is true.

ii. |P|<|Q|: Consider the proper prefix Q' of Q such that |P|=|Q'|. The state s'=[v,w',x',y'], reachable from the initial state of M and N over the edges of P and Q', is fair (i.e., x'=y'); and the lemma is true.

iii. |P|>|Q|: Extend the directed path Q in any possible way in N until the extended path Q' is such that |P|=|Q'|. The state s'=[v,w',x',y'], reachable from the initial state of M and N over the edges of P and Q', is fair (i.e., x'=y'); and the lemma is true. []