# DISTRIBUTED PROCESSING OF LOGIC PROGRAMS

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*ABSTRACT*

This paper is concerned with the issue of parallel evaluation of logic programs. To address this issue we define a new concept of *predicate decomposability*. If a predicate is decomposable, it means that:

- its evaluation can be conducted in parallel by several processors, without communication among them.
- the computation-time by $k$ processors is roughly equal to $1/k$ of the time required by one processor.

On both accounts decomposability is a stronger notion of amenability to parallel evaluation than the widely accepted membership-in-the-NC-complexity-class. The reason is that the latter notion may necessitate communication among the processors, and may not reduce computation time as drastically.

We completely characterize three classes of single rule programs (sirups) with respect to decomposability: nonrecursive, linear, and simple chain programs. All three classes were studied previously in various contexts. We establish that nonrecursive programs are decomposable, whereas for the other two classes we determine which ones are, and which ones are not decomposable. We also establish two sufficient conditions for sirup decomposability.

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## 1. Introduction

We propose a new method of evaluating logic programs in parallel. The method is suitable for sharing the computation load among an arbitrary number of processors, which have common memory or communicate by message passing. This makes it applicable to a large class of hardware architectures. Let us demonstrate the method using the classical example of the program computing the transitive closure of a graph. The arcs of the graph are given by the tuples of a database relation $A$. The program is written in DATALOG (see [MW]):

$$T(x,y) :- T(x,z), A(z,y).$$

$$T(x,y) :- A(x,y).$$

If the relation $A$ is replicated at two different processors, p1 and p2, we can partition the work of computing (the relation for) the predicate $T$ as follows. Processor p1 executes the program:

$$T(x,y) :- T(x,z), A(z,y).$$

$$T(x,y) :- A(x,y), even(x).$$

while processor p2 executes the program:

$$T(x,y) :- T(x,z), A(z,y).$$

$$T(x,y) :- A(x,y), odd(x).$$

In other words, p1 computes the tuples $(x,y)$ of the transitive closure, in which $x$ is even, and p2 computes those tuples in which $x$ is odd. A moment of reflection will reveal that this partitioning of the work has several nice properties. First, no processor computes a tuple which is also computed by the other processor. Second, if the relation computed by each processor is output to the same device, or stored in the same file, the result is always the complete transitive closure, regardless of the input graph. Third, no communication between the two processors is required during the computation. Therefore, for an "average" graph, the evaluation is completed in approximately half the time it takes one processor to do so. The last nice property we want to emphasize is that the work-partitioning was obtained only by adding evaluable predicates to the body of some rules of the original program.

If the parallelization method described above can be applied to the evaluation of a predicate in a program, then we say that the predicate is decomposable. The above example has demonstrated the advantages of decomposability for sharing the load between *two* processors. However, we shall argue in section 2 that in most cases if the computation load can be shared between two processors, then using the same method, it can be shared among an arbitrary number of processors, $k$. Moreover, similar considerations will indicate that the computation time is approximately $1/k$ of the time required by one processor.

Not every predicate is decomposable. Even for the same problem of computing the transitive closure, we will prove that the predicate $T'$ in the program:

$$T'(x,y) :- T'(x,z), T'(z,y).$$

$$T'(x,y) :- A(x,y).$$

is not decomposable. The proof of this fact will be given in section 5. Therefore, we feel that it is practically and theoretically important to first formally define decomposability and then characterize the decomposable predicates.

In this paper we completely characterize three subclasses of single rule programs (sirups) with respect to decomposability: nonrecursive, linear programs, and simple chain programs. Sirups were first studied as a syntactically restricted class of programs by Cosmodakis and Kanellakis ([CK]). They have only one output predicate, therefore we interchangeably use the term decomposability of a predicate or of a program. We also provide two sufficient conditions for any sirup to be decomposable. Linear programs and simple chain programs are important subclasses of sirups from the practical point of view. Simple chain programs were completely characterized with respect to membership in the complexity class NC by Afrati and Papadimitriou ([AP]). Linear sirups were studied as a distinct subclass in the context of bounded recursion ([I], [N1], [NS]) and one sided recursion ([N2]).

This work is related to the general subject of parallel evaluation of logic programs. The subject has recently emerged as a very important and active area of research ([K], [U]). However, as far as we know, existing research is concerned with membership in the complexity class NC. This class is a mathematical tool for analyzing parallel algorithms in general. Here we show that for analyzing parallel evaluation of logic programs, a different, and stronger tool can be used. Loosely speaking, if a logic program is in NC it does not guarantee that it has all the nice properties of a decomposable predicate. In particular, the processors executing an NC type algorithm usually have to communicate extensively, and therefore communication is assumed to take place through common memory. Also, the speedup achieved by such an algorithm is not always as good as achieved by decomposability ($k$ processors complete the computation in $1/k$ the time). The technique of program modification that we discuss here is also related to the magic sets technique ([BMSU]). Magic sets and decomposability, both aim at increasing the efficiency of query evaluation. However, the means of magic sets is selection propagation, whereas the means of decomposability is parallel evaluation.

In the next section we introduce the necessary definitions and notations, and prove that any nonrecursive sirup is decomposable. In section 3 we provide two sufficient conditions for a general sirup to be decomposable, and in section 4 we show that one of these conditions, called pivoting, is also necessary for decomposability of a linear sirup. In section 5 it is proven that a simple chain program is decomposable if and only if it is regular. In section 6 we discuss future work.

## 2. Preliminaries

An *atom* is a predicate symbol with a constant or a variable in each argument position. We assume that the constants are the natural numbers. An $R$-atom is an atom having $R$ as the predicate symbol. A *rule* consists of an atom, $Q$, designated as the *head,* and a conjunction of one or more atoms, denoted $Q^1, \ldots, Q^k$, designated as the *body.* Such a rule is denoted $Q :- Q^1, \ldots, Q^k$, which should be read "Q if $Q^1$ and $Q^2$, and, ...,and $Q^k$." A rule or an atom is an *entity.* If an entity has a constant in each argument position, then it is a *ground* entity. For a predicate $R$, a finite set of $R$-ground-atoms is a *relation* for $R$.

A DATALOG program, or a program for short, is a finite set of rules whose predicate symbols are divided into two disjoint subsets: the *base* predicates, and the *derived* predicates. The base predicates are distinguished by the fact that they do not appear in any head of a rule. An *input* to $P$ is a relation for each base predicate. An *output* of $P$ is a relation for each derived predicate of $P$. A *substitution* applied to an entity, or a sequence of entities, is the replacement of each variable in the entity by a variable or a constant. It is denoted $x1/y1, x2/y2, \ldots, xn/yn$ indicating that $xi$ is replaced by $yi$. A substitution is *ground* if the replacement of each variable is by a constant. A ground substitution applied to a rule is an *instantiation* of the rule.

A *database* for $P$ is a relation for each predicate of $P$. The output of $P$ given an arbitrary input $I$, is the set of relations for the derived predicates in the database, obtained by the following procedure, called *bottom up evaluation*.

(1) Start with an initial database consisting of the input relations.

(2) If there is an instantiation of a rule such that all the ground atoms in the body are in the database generated so far, and the one in the head is not, then: add to the database the ground atom in the head of the rule, and reexecute step (2).

(3) Stop.

This procedure is guaranteed to terminate ([VEK]). For simplicity we assume that the the rules of a program do not have constants, and are *range restricted,* i.e. every variable in the head also appears in the body.

An *evaluable predicate* is an arithmetic predicate (see [BR]). Examples of evaluable predicates are sum, greater than, modulo, etc. A rule *re* is a *restricted version* of some rule $r$, if $r$ and *re* have exactly the same variables, and $r$ can be obtained by omitting zero or more evaluable predicates from the body of *re*. In other words, *re* is $r$ with some evaluable predicates added to the body, and the arguments of these evaluable predicates are variables of $r$. For example, if $r$ is:

$$S(x,y,z) :- S(w,x,y), A(w,z)$$

then one possible rule *re* is:

$$S(x,y,z) :- S(w,x,y), A(w,z), x-y=5$$

A program $P_i$ is a *restricted version* of program $P$ if each one of its rules is a restricted version of some rule of $P$. Note that $P_i$ may have more than one restricted version of a rule $r$ of $P$. To continue the above example, if $P$ has the rule $r$, then $P_i$ may have the rule $re$ as well as the rule $re'$:

$$S(x,y,z) :- S(w,x,y), A(w,z), x-y=6$$

Throughout this paper, only restricted versions of a program may have evaluable predicates. The input of a program with evaluable predicates, i.e. a restricted version, is defined as before. The output is also defined as before, except that step (2) also verifies that the substitution satisfies the evaluable predicates in the ground rule; only then the atom in the head is added to the database and step (2) is reexecuted. For example, the substitution x/14,y/8 satisfies the evaluable predicate x-y=6, whereas the substitution x/13,y/9 does not. A predicate $Q$ in a program $P$ *derives* a predicate $R$ if it occurs in the body of a rule whose head is a $R$-atom. $Q$ is *recursive* if $(Q,Q)$ is in the nonreflexive transitive closure of the "derives" relation. A program is recursive if it has a recursive predicate. A rule is recursive if the predicate in its head transitively derives some predicate in its body.

Let $P$ be a program, $T$ a derived predicate in $P$, and $P_1, \ldots, P_r$ restricted copies of P. For a derived predicate T of P, denote by $T_i$ the relation output by $P_i$ for T; the relation output by P is denoted T. Observe that this is a somewhat unconventional notation, since for $P_i$ the relation name is different than the predicate name. Predicate T is *decomposable* in P with respect to $P_1, \ldots, P_r$ if the following two conditions hold:

1. For each input I to $P, P_1, \ldots, P_r$
   i. $\bigcup_i T_i = T$ (completeness)
   ii. $T_i$ and $T_j$ are disjoint for each $i \neq j$; furthermore, if some derived predicate Q derives T in P, then $Q_i$ and $Q_j$ are disjoint (lack-of-duplication).
2. For some input I to $P_1, \ldots, P_r$ each $T_i$ is nonempty (nontriviality).

The above definition is central to this paper, and we shall discuss it next.

Requirement 1(i) states that no output is lost by evaluating the relation for $T$ in each $P_i$ rather than the relation for $T$ in $P$; the fact that no additional output is generated is explicit here, but also implied by the fact that each $P_i$ is a restricted version of $P$. Requirement 1(ii) states that no ground atom in the output is computed by two different processors. Therefore, loosely speaking, requirement 1 says the following: For every input (i.e. set of base relations replicated at each processor), computing the relation for $T$ by $r$ processors using the $P_i$'s is equivalent to computing it by one processor using $P$; furthermore, the sum of all computation times at the $r$ processors is not higher than the computation time at one processor using $P$ (up to some initialization and termination constant).

The strength of requirement 1 enables the relaxed form of requirement 2. It is enough that for "some" inputs each $T_i$ is nonempty, since for those inputs completing the computation of $T$ happens sooner in the distributed case. In other words, since there is nothing to lose by distributing the computation, it is enough that we gain only in some cases to make the scheme worthwhile. However, for the decomposable predicates that we discuss in this paper, nontriviality holds for more than an isolated case input.

For instance, in the transitive closure example nontriviality holds for any input graph in which arcs exit both, even and odd nodes. Specifically, for the class of predicates that we prove decomposable in this paper, decomposability is shown using the *odd−even* predicates alone. This has two implications. First, the work performed by each processor for an arbitrary input, is roughly equal (e.g. for an arbitrary graph, the number of odd and even nodes is roughly equal). Second, note that the *odd* and *even* predicates are a special case of the *i mod r* predicates for $r=2$. When we show that $T$ is decomposable in $P$ with respect to $P_1$ and $P_2$, then it will be easy for the reader to convince itself that for any $r$ there are restricted copies $P_1, \ldots, P_r$ such that $T$ is decomposable in $P$ with respect to $P_1, \ldots, P_r$. This means that the work can be divided among any number of processors. For instance, in the transitive closure example, in order to do so processor $i$ evaluates $T_i$ where:

$$P_i.\quad T(x,y) :- T(x,z), A(z,y).$$

$$T(x,y) :- A(x,y), x = i \bmod r.$$

These facts stress the robustness of the decomposability definition.

We say that predicate T is *decomposable* in P if it is decomposable with respect to some restricted copies $P_1,\ldots,P_r$ such that r > 1.
A *single rule program* (see [CK]) is a DATALOG program which has a single derived predicate, denoted $S$ in our paper, a nonrecursive rule,

$$S(x1,\ldots,xn) :- B(x1,\ldots,xn).$$

where the xi's are distinct variables, and one other, possibly recursive, rule in which the predicate symbol $B$ does not appear.

**Theorem 1**: If a sirup P is nonrecursive, then its derived predicate is decomposable.
**Proof**: Assume that P is:

$$S(x1,\ldots,xn) :- Q^1(\ldots.), \ldots, Q^k(\ldots.)$$

$$S(x1,\ldots,xn) :- B(x1,\ldots,xn).$$

where $B$ and each $Q^i$ are base predicates. Consider the following restricted copies of P:

$$P_1.\quad S(x1,\ldots,xn) :- Q^1(\ldots.), \ldots, Q^k(\ldots.), even(x1).$$

$$S(x1,\ldots,xn) :- B(x1,\ldots,xn), even(x1).$$

$$P_2.\quad S(x1,...,xn) :- Q^1(....), \ldots, Q^k(....), odd(x1).$$

$$S(x1,...,xn) :- B(x1,...,xn), odd(x1).$$

It is easy to see that $S$ is decomposable in $P$ with respect to $P_1$ and $P_2$. []

## 3. Sufficient Conditions for Decomposability

In this section we provide two sufficient conditions for decomposability of a general sirup. The first one is motivated by the next example, which also merits attention for the following reason. From the preceding discussion one might suspect that our notion of decomposability is equivalent to "naive" propagation of variable bindings (see introduction of [BKBR]). The latter notion means simply substituting a constant for a variable in some rules. The constant is usually taken from a query. For example, in order to find all the arcs exiting the node 2 in the transitive closure of a graph, the constant can be naively propagated into the program as follows:

$$T(2,y) :- T(2,z), A(z,y).$$

$$T(2,y) :- A(2,y).$$

It is quite clear that if a sirup is amenable to naive propagation of variable bindings, then it is decomposable. However, the reverse is not true. For example, consider the program:

$$S(x,y) :- S(y,x).$$

$$S(x,y) :- A(x,y).$$

which outputs an arc in both directions for every arc of an input graph. It is easy to see that a binding cannot be naively propagated into this program, but the sirup is decomposable; one restricted copy has the nonrecursive rule:

$$S(x,y) :- A(x,y), even(x+y).$$

and the other:

$$S(x,y) :- A(x,y), odd(x+y).$$

Our first sufficient condition for decomposability is based on the preceding observation. Next we formally define it. Assume that $R$ is a set of atoms with each atom having a variable in each argument position. The set $R$ is *pivoting* if there is a subset $d$ of argument positions, such that in the positions of $d$:

1. the same variables appear (possibly in a different order) in all atoms of $R$, and
2. each variable appears the same number of times in all atoms of $R$. A member of $d$ is called a *pivot*.

The recursive rule of a sirup is *pivoting* if all the occurrences of the recursive predicate in the rule constitute a pivoting set. For example, the rule:

$$S(w,x,x,y,z) :- S(u,y,x,x,w), S(v,x,y,x,w), A(u,v,z)$$

is pivoting, with argument positions 2, 3 and 4 of $S$ being the pivots.

**Theorem 2**: If the recursive rule of a sirup is pivoting, then the sirup is decomposable.

**Proof Sketch:** Assume that argument positions $i_1,...,i_k$ of $S$ are the pivots. Consider restricted copy $P_1$ of $P$ which has the same recursive rule as $P$, and a nonrecursive rule:

$$S(x1,...,xn) :- B(x1,...,xn), even(xi_1+xi_2+,...,+xi_k)$$

Restricted copy $P_2$ of $P$ is the same, except that the nonrecursive rule is:

$$S(x1,...,xn) :- B(x1,...,xn), odd(xi_1+xi_2+,...,+xi_k)$$

Assume that for input $I$, the ground atom $a=S(c_1,...,c_n)$ is in the relation $S$ output by $P$. Assume further that $c=c_{i_1}+,...,+c_{i_k}$ is even. Denote by $t$ the necessary and sufficient number of iterations of step (2) of bottom-up-evaluation. for adding $a$ to the database, in evaluating $P$. It can be shown by induction on $t$, that $t$ iterations are necessary and sufficient to add $a$ to $S_1$. It is also quite easy to prove that $a$ is not in $S_2$. Therefore completeness and lack-of-duplication hold. Finally, note that for an input consisting of two atoms, $B(c_1,...,c_k)$ and $B(d_1,...,d_k)$, for which the sum of constants in the pivots is even and odd respectively, nontriviality holds. []

Theorem 2 can be extended to general programs, not necessarily sirups, provided that we extend the pivoting definition properly. Since in this paper we concentrate on sirups, we shall be informal about general programs. A rule in a program is pivoting, if all its derived-predicate-atoms (in the head or the body) constitute a pivoting set. A program is pivoting if each one of its rules is pivoting, with the same argument positions being the pivots in all the rules; additionally it is required that the heads of rule do not have repeated variables. For example, the program:

$$S(x,y,z):- R(y,x,w), A(w,z).$$

$$R(x,y,z):- R(x,y,w), B(w,z).$$

$$R(x,y,z):- C(x,y,z).$$

is pivoting, with positions 1 and 2 being the pivots. It can be shown that a predicate in a general program is decomposable if the rules which derive the predicate constitute a pivoting program. For example, predicate $S$ in the program above is decomposable ( add $odd-even(x+y)$ to the body of the third rule).

The condition of theorem 2 is not necessary for decomposability. For example, the sirup:

$$S(x,x):- S(y,y), A(x,y).$$

$$S(x,y):- B(x,y).$$

is obviously not pivoting, but it is decomposable. Again, $odd-even(x+y)$ is added to the body of the nonrecursive rule. The intuition indicates that in this example the

computation load for an arbitrary input is not evenly divided between the processors executing the two restricted versions of the program (because only the processor executing the copy with the *even* evaluable predicate can output an atom as a result of instantiation of the recursive rule). The example is unique (throughout the paper) in this respect. Expectedly, the last example motivates our next sufficient condition for decomposability of a sirup. It is defined as follows. Assume that $R$ is a set of atoms with each atom having the same predicate symbol, $Q$, and a variable in each argument position. The set $R$ is *repeating* if there are at least two argument positions of $Q$, $i$ and $j$, such that the same variable appears in position $i$ and position $j$, and this is true for each member of $R$ (note that the variable of one member of $R$ may be different than the variable of another). The recursive rule of a sirup is *repeating* if all the occurrences of the recursive predicate in the rule constitute a repeating set. For example, the rule:

$$S(x,z,x) :- S(z,z,z), S(x,x,x).$$

is repeating because of argument positions 1 and 3.

**Theorem 3**: If the recursive rule of a sirup is repeating, then the sirup is decomposable.

## 4. Linear Sirups

In this section we completely characterize the class of linear sirups with respect to decomposability. A sirup is *linear* if it is recursive, and in the body of the recursive rule there is exactly one occurrence of the recursive predicate. We also require that a linear sirup does not have repeated variables in an occurrence of the recursive predicate. The characterization of linear sirups with respect to decomposability is done by proving that the sufficient condition of theorem 2 is also necessary. We assume that the recursive rule is:

$$S(x1,...,xn) := S(Y1,...,Yn), A_1(.....), \ldots, A_k(....).$$

where the $A_i$'s are base predicates. Observe the notation used in this section to distinguish between two types of variables. The ones starting with a lowercase letter are logic program variables, or variables for short, as before. The ones starting with an upper case letter, e.g. $Y1$, are *notation-variables*. They denote program variables. For example, $Y1$ may denote the variable $xn$. If the predicate $S(x1,...,xn)$ in sirup $P$ is decomposable with respect to $P_1,...,P_r$, then we define the *home-site* of a sequence of $n$ constants, $\bar{c} = c_1,...,c_n$. It is the $S_i$ to which the output atom $S(\bar{c})$ belongs, if each $P_i$ is given the input consisting of a unique atom, $B(\bar{c})$. Note that the home-site of a sequence is unique (lack-of-duplication), every sequence of $n$ constants has a home-site (completeness), and each $S_i$, $1 \le i \le r$, has a sequence of constants for which $S_i$ is the home-site. Let $\bar{c} = c_1,...,c_n$ and $\bar{d} = d_1,...,d_n$ be two sequences of constants. The ordered pair of ground atoms $<S(\bar{d}), S(\bar{c})>$ is a *one-step-derivation* if there is an instantiation of the recursive rule of $P$, in which the first atom is in the head and the second is in the body.

**Theorem 4**: A linear sirup is decomposable if and only if its recursive rule is pivoting.

**Proof Sketch**: (if) Special case of Theorem 2.
(only if) The proof is based on four lemmas which we will state without proof.

**Lemma 1**: If the derived predicate, $S$, of a linear sirup $P$ is decomposable, and there are two sequences of constants $\bar{d} = d_1,...,d_n$ and $\bar{c} = c_1,...,c_n$ such that $<S(\bar{d}),S(\bar{c})>$ is a one-step-derivation, then the home-site of $\bar{d}$ and $\bar{c}$ is identical. []

We now present a procedure, called *Derive–New–Variables* ($P$) which it is used to state the next three lemmas.

*Derive–New–Variables* ($P$).

1. *Last–Rec–Rule* := The recursive rule of $P$ with all the variables given the subscript of one.
2. *Last–Deriv* := $S(x1_0,...,xn_0)$
3. Do until none of the variables of the atom in *Last–Deriv* is equal to one of the variables $x1_0,...,xn_0$.
4. Assume that *Last–Rec–Rule* $= S(x1_i,...,xn_i) :- S(Y1,...,Yn),A_1(...), . . . , A_k(...)$ and *Last–Deriv* $= S(Z1,...,Zn)$.
   Let *Last–Deriv* := The atom in the head of the rule obtained by applying the substitution $Y1/Z1,...,Yn/Zn$ to *Last–Rec–Rule*.
5. Let *Last–Rec–Rule* := *Last–Rec–Rule* with the subscript of the variables increased by one.
6. END;

*Derive–New–Variables* takes as a parameter a linear sirup $P$, and iteratively substitutes for the variables in the recursive rule of $P$. It starts by subscripting all variables by 1, and then at each iteration it increases the subscript of the variables and unifies the $S$-atom in the body with the atom in the head of the previous iteration.

| Iteration | Last-Rec-Rule | Last-Deriv |
|---|---|---|
| 1. | $S(x_1,y_1,z_1):-S(w_1,x_1,y_1),A(w_1,z_1)$ | $S(x_0,y_0,z_0)$ |
| 2. | $S(x_2,y_2,z_2):-S(w_2,x_2,y_2),A(w_2,z_2)$ | $S(y_0,z_0,z_1)$ |
| 3. | $S(x_3,y_3,z_3):-S(w_3,x_3,y_3),A(w_3,z_3)$ | $S(z_0,z_1,z_2)$ |
| 4. | $S(x_4,y_4,z_4):-S(w_4,x_4,y_4),A(w_4,z_4)$ | $S(z_1,z_2,z_3)$ |

Figure 1: Example of an execution of the procedure Derive-New-Variables

In Figure 1 we demonstrate the procedure *Derive–New–Variables* for the recursive rule: $S(x,y,z) :- S(w,x,y),A(w,z)$.

**Lemma 2**: Let $P$ be a linear sirup, and assume that $S(Y1, \ldots, Yn)$ and $S(Z1, \ldots, Zn)$ are two consecutive values of *Last–Deriv* in the execution of the procedure *Derive–New–Variables* ($P$). Furthermore, assume that there is a ground substitution $\rho$ of the program variables in the sequence $S(Y1, \ldots, Yn)$, $S(Z1, \ldots, Zn)$, resulting in the sequence of ground atoms $S(c_1, \ldots, c_n), S(d_1, \ldots, d_n)$. Then the pair $<S(d_1, \ldots, d_n), S(c_1, \ldots, c_n)>$ is a one-step-derivation. []

**Lemma 3**: If for a linear sirup, $P$, the procedure *Derive–New–Variables* ($P$) halts (see step 3), then $P$ is not decomposable. []

**Lemma 4**: If the recursive rule of a linear sirup $P$ is not pivoting, then *Derive–New–Variables* ($P$) halts. []

## 5. Simple Chain Programs

A simple chain program is a recursive sirup in which the following three conditions must hold:

(a) all the predicates are binary.

(b) the argument positions in the left hand side of the recursive rule have distinct variables, and these variables appear in the first argument position of the first atom in the body, and in the last argument position of the last atom, respectively.

(c) all the argument positions in the body of the recursive rule have distinct variables, except that the first argument position of the second atom has the same variable as the last argument position of the first atom, the first argument position of the third atom has the same variable as the last argument position of the second atom, etc.

For example, the following is a simple chain program:

$$S(x,y):- A(x,z_1),S(z_1,z_2),S(z_2,z_3),C(z_3,z_4),D(z_4,y)$$

$$S(x,y):- B(x,y).$$

where the $A,B,C,D$ are base relations. A simple chain program is *regular* if in its recursive rule there is one occurrence of the predicate S and this occurrence is the first or the last in the body of the recursive rule. Note that a simple chain program is pivoting if and only if it is regular.

**Theorem 5**: A simple chain program $P$ is decomposable if and only if it is regular.

**Proof**: (if) Immediate, based on Theorem 2.
(only if) Assume that $P$ is decomposable with respect to restricted copies $P_1, P_2, \ldots, P_r$ of $P$, for $r>1$. Denote the recursive rule of $P$ by:

$$S(x,y):- Q^1(x,z_1), \ldots, Q^t(z_{t-1},y)$$

where some of the $Q^i$'s are $S$'s, and $t>1$. Using the regular notation, the nonrecursive rule is:

$$S(x,y):-B(x,y).$$

By nontriviality there are two sequences of constants, $j_1,k_1$ and $j_2,k_2$ with home sites $S_1$ and $S_2$ respectively. Since the recursive rule of P is not regular, there are two cases to analyze:

**Case 1**: There is a subsequence in the body of the recursive rule, of the following form:

$$Q^{i-1}(z_{i-2},z_{i-1}),S(z_{i-1},z_i),Q^{i+1}(z_i,z_{i+1}).$$

Let $I^2$ consist of the set of ground atoms:

$$Q^1(c_1,c_2),Q^2(c_2,c_3),\ldots,Q^{i-1}(c_{i-1},j_1),B(j_1,k_1),$$
$$Q^{i+1}(k_1,c_{i+1}),Q^{i+2}(c_{i+1},c_{i+2}),\ldots,Q^t(c_{t-1},c_t)$$

where:

1) each predicate $S(m,n)$ in the list is a notation for $B(m,n)$;

2) each pair of different $c$'s represents different constants;

3) none of the $c$'s is in the set $\{j_1,k_1,j_2,k_2\}$.

For the input $I^2$ the ground atom $S(c_1,c_t)$ is in the output $S$ of $P$. By completeness, for this input, $S(c_1,c_t)$ is in some $S_j$. We will show that $S(c_1,c_t)$ is in $S_1$. Assume otherwise, i.e. $S(c_1,c_t)$ is in $S_b$ for $b\neq 1$. The atom $B(c_1,c_t)$ is not in $I^2$ because $t>1$, therefore $S(c_1,c_t)$ must be added to the database by instantiating a recursive rule of $P_b$ in step 2. Consider the first such instantiation. Note that the sequence $s$ of ground atoms constituting the body of the instantiated rule cannot have the same atom twice; if it does, then there is at least one ground atom in $s$, for which both argument positions have the same constant. However, it is easy to realize from the definition of $I^2$, that in the database generated by *bottom−up−evaluation* so far, all atoms must have different constants in the two argument positions. (Note that even though $j_1$ may be equal to $k_1$, the ground atom $S(j_1,k_1)$ is not in $S_b$, since $S_b\neq S_1$). Therefore, in the first instantiation of a recursive rule, the body of the instantiated rule has $t$ distinct atoms, each of which is not a $B$-ground-atom (because by sirup definition the predicate symbol $B$ is not in the recursive rule). However, it is easy to realize that in the database generated so far there can be at most $t-1$ such atoms. The contradiction results from our assumption that $S(c_1,c_t)$ is in $S_b$. Therefore, for input $I^2$ to $P_1,\ldots,P_r$, $S(c_1,c_t)$ is in $S_1$. Now consider the input $I^3$, which is defined identically to $I^2$, except that the constants $j_1$, $k_1$ are replaced by $j_2$, $k_2$ respectively. Similar arguments as before will reveal that $S(c_1,c_t)$ is in $S_2$. The proof of this case is completed by noticing that for the input $I^2\cup I^3$, the ground atom $S(c_1,c_t)$ is in both, $S_1$ and $S_2$, contradicting lack-of-duplication.

**Case 2**: The body of the recursive rule of $P$ is of the following form:

$$S(x,z),S(z,y)$$

Consider the input $I^4$ consisting of the ground atom $B(j_3,k_3)$, where $j_3$ and $k_3$ are distinct, and none of them is in the set $\{j_1,j_2,k_1,k_2\}$. Assume without loss of generality that the home site of $j_3,k_3$ is $S_d$, for $d\neq 1$ (otherwise the analysis below can be carried out by replacing $j_1,k_1$ by $j_2,k_2$ respectively).

**Subcase 2.1**: Assume that $j_1\neq k_1$. Let input $I^5 = \{ B(j_1,k_1), B(k_1,j_3), B(j_3,k_3)\}$. This input relation can be regarded as a graph consisting of a path, therefore $S(j_1,j_3)$ and $S(k_1,k_3)$ are in $S$. Assume that the home site of $k_1,j_3$ is $S_i$ for $i\neq 1$. But then it is easy to see that $S(j_1,j_3)$ is not in any $S_i$; contradicting completeness. If the home site of $k_1,j_3$ is $S_1$, then it is easy to see that for input $I^5$ the atom $S(k_1,k_3)$ is not in any $S_i$; again contradicting completeness.

**Subcase 2.2**: Assume that $j_1=k_1$. In other words, the home site of $j_1,j_1$, is $S_1$. Let input $I^6 = \{ B(j_1,j_3), B(j_3,k_3), B(k_3,j_1)\}$. This input relation can be regarded as a graph consisting of a cycle, therefore $S(j_1,j_1)$ is in $S$. Since $S(j_3,k_3)$ is in $S_d$ only, for the input $I^6$, $S(j_1,j_1)$ cannot be in any $S_i$ other than $S_d$. But then, for the input $I^6 \cup \{B(j_1,j_1)\}$ the ground atom $S(j_1,j_1)$ is in both, $S_1$ and $S_d$; this contradicts lack of duplication. []

## 6. Future Work

We shall continue the work on decomposability in several directions. One of them is to extend the characterization of decomposable predicates to other sirups first, e.g. typed (see [K]), and then to general logic programs. Another direction is to determine whether decomposition implies that the work can be evenly divided among the processors, as we have seen that can be done using the *mod* predicate. For this purpose a notion of *fair* decomposition should be defined. Another topic which merits attention is minimizing communication when evaluating nondecomposable predicates in a distributed environment. We feel that the work on decomposability should also be helpful in this area. More specifically, observe that the method proposed in this paper to partition the load in evaluating decomposable predicates, can be applied to nondecomposable ones as well; however in that case communication among the processors is necessary. The question is, how does the amount of necessary communication compare in different partitioning schemes. Finally, we shall mention that we intend to study the relationship between the class of decomposable programs and the programs in the complexity class NC.

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