LIMITATIONS IN THE UNDERSTANDING OF MATHEMATICAL LOGIC BY NOVICE COMPUTER SCIENCE STUDENTS

APPROVED BY
DISSERTATION COMMITTEE:

_____

_____

_____

_____

_____

Dedication

To the memory of two who loved to learn and loved to teach,

both taken from us before their time:

Lucille A. Almstrum

Louis E. Rosier

LIMITATIONS IN THE UNDERSTANDING OF MATHEMATICAL LOGIC BY NOVICE COMPUTER SCIENCE STUDENTS

by

VICKI LYNN ALMSTRUM, B.A. ED, M.A., M.S.

DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

May, 1994

Acknowledgements

The genealogy of research development is a fascinating phenomenon. Composing this section has allowed me to reflect on the many people who have influenced my research and the dissertation in diverse ways. I want to begin by expressing my gratitude to my Committee Members, Nell B. Dale, Ralph W. Cain, L. Ray Carry, David Gries, and Charles E. Lamb. Each of these gifted individuals has been a mentor, teacher, and advisor for me; I am grateful for their time, concern, and support throughout this endeavor.

My initial inspiration for this research can be traced to stimulating meetings with Jeff Brumfield and Lou Rosier and to interactions with the students in "my" sections of CS336. I am especially appreciative of the computer science educators who shared the precious resource of their time (some of them twice!) in serving as judges at various stages in the content analysis process. Without their assistance, this study would not have been possible. Listed in alphabetical order, the judges were Michael Barnett, Richard Bonney, Jeff Brumfield, Debra Burton, Ken Calvert, Rick Care, Charlotte Chell, Chiung-Hui Chiu, Michael Clancy, Ed Cohen, Ron Colby, Nell Dale, Ed Deaton, John DiElsi, Edsger W. Dijkstra, Ron Dirkse, Sally Dodge, Sarah Fix, Ann E. Fleury, Suzy Gallagher, David Gries, George C. Harrison, Helen Hays, Fran Hunt, Joe Klerlein, Edgar Knapp, Kathy Larson, David Levine, John McCormick, Nic McPhee, Pat Morris, J. Paul Myers, Jr., David Naumann, Barbara Boucher Owens, David C. Platt, K. S. Rajasethupathy, Charles Rice, Hamilton Richards, Lynden Rosier, Dale Shaffer,

Angela B. Shiflet, Angel Syang, Harriet Taylor, Henry M. Walker, J. Stanley Warford, and Cheng-Chih Wu.

Several people outside of my committee consented to review this document at various stages during its development. Special "thank you"s go to Hamilton Richards, Jean Rogers, Chris B. Walton, and Ronald P. Zinke for their thoughtful and thorough comments. I would like to thank Rick Morgan and Jeff Wadkins at Educational Testing Services for extensive help at several points during the course of this study. I am also indebted to Ed Barkmeyer and Brian Meeks for information about the Language-Independent Datatypes standard as well as for discussions of other points related to the research. Many others have contributed important ideas at key points in this process, among them Diane Schallert, who assigned a psycholinguistics project that shaped the way I approached the topic; Todd Gross, who brought to my attention a key article from *SIGPLAN Notices*; Barbara Dodd, who planted the idea for the content analysis methodology as well as ideas for other aspects of the study; Pat Kenney, who was a font of information about ETS; Pat Dickson, who asked many helpful and pointed statistical questions; and Michael Piburn, who shared information about the PLT and the use of logic as a predictor of success in science education research.

The Computer Science Education Seminar at the University of Texas at Austin has provided a vital forum for exploring ideas within and beyond this research; I am privileged to meet regularly with such a group. There are numerous professors and graduate students (former as well as current) at the University of Texas at Austin, particularly within the Department of Computer

Sciences, whom I thank collectively for their contributions to a stimulating environment. The experiences I have had through involvement in three NSF-sponsored workshops prove the value of such programs for fostering the exchange of ideas among educators and researchers; these workshops were held at SUNY Stony Brook in 1991 (USE-9054175; Principal Investigator Peter Henderson), at The University of Texas at Austin in 1992 (USE-9154220; Principal Investigator Nell Dale), and at Southwestern University in Georgetown, TX in 1993 (USE-9156008; Principal Investigator Walter M. Potter). The annual SIGCSE Technical Symposia have also provided important opportunities for professional growth.

In closing, I want to direct my thanks to family and friends who have supported and encouraged me through the trials, tribulations, and distractions of this process. In particular, I want to express my gratitude to my husband, Torgny Stadler, who has supported my work in countless ways. He is without question one of the primary reasons I was able to complete and enjoy this great adventure!

LIMITATIONS IN THE UNDERSTANDING OF MATHEMATICAL LOGIC BY NOVICE COMPUTER SCIENCE STUDENTS

Publication No._____

Vicki Lynn Almstrum, Ph.D.

The University of Texas at Austin, 1994

Supervisors: Ralph W. Cain and Nell B. Dale

This research explored the understanding that novice computer science students have of mathematical logic. Because concepts of logic are at the heart of many areas of computer science, it was hypothesized that a solid understanding of logic would help students grasp basic computer science concepts more quickly and would better prepare them for advanced topics such as formal verification of program correctness. This exploratory study lays the groundwork for further investigation of this hypothesis.

Data for the study were the publicly available versions of the Advanced Placement Examination in Computer Science (APCS examination) and files containing anonymous individual responses of students who took these

examinations. A content analysis procedure was developed to provide reliable and valid classification of multiple-choice items from the APCS examinations based on the relationship between concepts covered in each item and the concepts of logic. The concepts in the computer science subdomain of logic were clarified by means of a taxonomy developed for use in this study.

Thirty-eight experts in computer science education were judges in the content analysis of the multiple-choice items. The judges' ratings provided criteria for grouping items into *strongly related* and *not strongly related* partitions. In general, the mean proportion of student respondents that correctly answered the items in a partition was lower for the *strongly related* than for the *not strongly related* partition, with a smaller standard deviation. The difficulty distributions for the two partitions were shown to be non-homogeneous ($p \leq .002$), with the difficulty distribution for the *strongly related* partition skewed more towards the "very difficult" end of the distribution.

The results of this study suggest that novice computer science students experience more difficulty with concepts involving mathematical logic than they do, in general, with other concepts in computer science. This indicates a need to improve the way in which novice computer science students learn the concepts of logic. In particular, pre-college preparation in mathematical logic and the content of discrete mathematics courses taken by computer science students need to be scrutinized.

Table of Contents

List of Figures

List of Tables

Chapter 1 Introduction

This research explored novice computer science students' understanding of *mathematical logic* (simply referred to as *logic* in this thesis). Logic restricted to two values is fundamental to many areas of computer science. Because logic pervades the field, the investigator hypothesized that a solid understanding of logic can help students grasp basic computing skills more quickly and can also prepare them to be more successful when studying advanced topics such as formal verification of program correctness. The findings and conclusions in this study establish the baseline for research investigating this hypothesis.

1.1 BACKGROUND

The inspiration for this research arose through the author's experience as a teaching assistant in an undergraduate course covering topics in discrete mathematics and formal verification of computer programs. Each semester, many students demonstrated a predictable set of misconceptions about and partial understandings of logic concepts. Because logic is the foundation for formal verification, these misunderstandings tended to sabotage students' ability to grasp the more advanced concepts.

The datatype boolean encompasses a fundamental subset of concepts in logic that belong to the requisite repertoire of most computer scientists. Essentially every modern programming language includes the notion of a boolean datatype and conditional control structures. For example, alternative statements (such as `if-then-else`) and repetitive statements (such as `while`) depend

upon a boolean expression that controls which part(s) of the structure will be executed and how often. Analogous control structures are used in creating algorithms and specifications.

There is an intricate relationship between the concepts of classic mathematics and the datatypes that have been included in programming languages. Because booleans and integers are among the fundamental building blocks of mathematics, they are included as simple datatypes in most programming languages. Moreover, these simple types are available on computers as basic, built-in datatypes complete with their operations. In the classic textbook *Algorithms + Data Structures = Programs*, Wirth (1976) explained: "Standard primitive types are those types that are available on most computers as built-in features. They include the whole numbers, the logical truth values, and a set of printable characters" (p. 8). In contrast, while other entities such as complex numbers and infinite sets are fundamental mathematical concepts, they are not included as built-in types in programming languages because there is no effective counterpart available on computers (N. Wirth, personal correspondence, January 21, 1994).

For many students, working with booleans while learning to program is their initial formal exposure to the concepts of logic. This research focused on datatype boolean as representative of the wider subdomain of logic.

1.2 PURPOSE OF THE STUDY

This study investigated the question: Do novice computer science students generally have more difficulty with the concepts of logic than they have with other areas in computer science? The goals of this study were: (a) to identify and define clearly concepts in the subdomain of logic; (b) to identify a method by which relevant material could be isolated from a larger source; and (c) to provide objective evidence as to whether novice computer science students had more difficulty understanding the concepts in this subdomain than they had with the concepts in other novice computer science areas.

1.3 SIGNIFICANCE OF THE STUDY

Many in the computing community have expressed the view that logic is an essential topic in the field (e.g., Galton, 1992; Gibbs & Tucker, 1986; Sperschneider & Antoniou, 1991). There has also been concern that the introduction of logic to computer science students has been and is being neglected (e.g., Dijkstra, 1989; Gries, 1990). In their article "A review of several programs for the teaching of logic", Goldson, Reeves and Bornat (1993) stated:

> There has been an explosion of interest in the use of logic in computer science in recent years. This is in part due to theoretical developments within academic computer science and in part due to the recent popularity of Formal Methods amongst software engineers. There is now a widespread and growing recognition that formal techniques are central to the subject and that a good grasp of them is essential for a practising computer scientist. (p. 373)

In his paper "The central role of mathematical logic in computer science", Myers (1990) provided an extensive list of topics that demonstrate

> … the importance of logic to many core areas in computer science:
>
> - "theoretical" computer science: automata, formal languages, computability, complexity, recursive function theory
> - artificial intelligence: deduction systems, expert systems, cognitive science, formalisms, automated proofs, natural language processing
> - programming languages and data structures: logic programming (PROLOG is but one such language), resolution, functional languages, semantics ([axiomatic], denotational, procedural, realizability), language design, computational completeness, data abstraction/operations, type theory, object-oriented approaches, parallel processing (optimality and equivalence to sequential algorithms)
> - database systems: alternatives for knowledge representation and data models (relational, entity-relationship, etc.), query processing languages, isolating effects of local inconsistencies, deductive databases and expert systems, dynamic/temporal modeling and temporal logics (for the dimension of time in databases), knowledge-based systems with incomplete and tentative information requiring modal and fuzzy reasoning, natural language interfaces
> - software engineering: program verification, including testing (path manipulation) and correctness, formal specifications and program design, executable specifications
> - hardware: circuit design/optimization, hardware design languages, processor verification, correctness of [operating system] kernel, language implementation on given processors
> - philosophical foundation for computer science: profound correspondences between reasoning and computation, formal systems, constructivity as a basis for computer science influencing language design, semantics, etc. (computer science as "applied constructivity") (pp. 23-24)

Myers warned that this listing is necessarily partial and that the items in the listing are not mutually exclusive. While many of the topics in Myers' list, for example type theory and constructivity, are more advanced than would be covered in the typical undergraduate program, the full list of topics covers much of the breadth and depth of the curriculum guidelines for computer science (Tucker, 1990). Because logic is fundamental to so much of the rest of computer science,

improving novice students' skills and understanding in this subdomain can affect their potential for success within the field as a whole.

1.4 NATURE OF THE STUDY

In this study, the concepts in the computer science subdomain of logic were described by means of a taxonomy. The taxonomy, in the form of a broad outline of the concepts in the subdomain of logic, served as the cornerstone of the rest of the research. By defining the set of concepts under study, the taxonomy served to focus the research effort.

The data for this research were based on test items and statistics from several publicly available computer science examinations. The multiple-choice items on these examinations were studied to identify those items that were strongly related to logic as well as those items that bore little or no relationship to logic. In order to accomplish this in a valid and reliable manner, all of the items from the examinations were classified using the research methodology *content analysis*. Experts in computer science education followed a well-defined procedure to rate each item for how strongly its content was related to the subdomain of logic. The results of the rating process provided the criteria for assigning items to partitions on the basis of whether or not they were strongly related to the subdomain. Comparative analysis of individual performance data for these items was carried out based on the composition of the partitions.

1.5 RESEARCH QUESTIONS

This study sought to provide objective evidence as to whether novice computer science students have more difficulty understanding concepts in the subdomain of logic than in understanding other novice computer science concepts. The first research question that was investigated was:

(a) Can a procedure be developed for reliable and valid classification of content-area test items according to their degree of relationship to a pre-defined set of logic concepts?

Given a positive answer to research question (a), the test items under consideration would be divided into sets of items according to the outcome of the classification process. The following research questions could then be considered:

(b) In considering student performance on the test items, was the distribution of performance different for items whose content was strongly related to logic than for items whose content was not strongly related to logic?

(c) Was there a relationship between individual performance on the set of items whose content was strongly related to logic and individual performance on the set of items whose content was not strongly related to logic?

1.6 OVERVIEW OF REMAINING CHAPTERS

Chapter 2 reviews related research. Mathematical logic is surveyed from a historical point of view, laying the groundwork for considering the role of logic in computer science. Curricular efforts that relate to logic as a subdomain of

computer science are reviewed and the use of logic in several areas of computer science is discussed. The role that logic plays in several psychological theories is described, followed by an overview of research that has investigated the connection between ability to use mathematical logic and success in courses in the natural sciences. Chapter 2 concludes with a brief background on the procedures of content analysis, including design of the procedures, gathering of the data, issues of reliability and validity, and a survey of ways in which content analysis has been used in recent research.

Chapter 3 describes the research design, including the process used in designing a taxonomy of concepts, motivation for choosing the examinations that were used as the source of data, the methodology of content analysis followed in analyzing the examination items, and the algorithms used in partitioning the examination items into *strongly related* and *not strongly related* groupings. The completed partitioning of items provided the basis for addressing the research questions posed in Chapter 1. Null hypotheses are developed and the statistical analyses to be used in considering these hypotheses is described.

Chapter 4 presents the findings of the study. The composition of the examinations used as data for the content analysis procedure is described and the overall performance of the large samples of students who took these examinations is given. The final outcome of the content analysis procedure is detailed, including the results of partitioning the items into strongly related and not strongly related groupings as well as reliability results for the item analysis procedure. The study showed that the items that were strongly related to logic tended to be more difficult than the items that were not strongly related. The

variability of individual responses to the strongly related items was shown to be only weakly explained by the variability in the responses to the not strongly related items. In numerical terms, on the scale 0.0 (no one answered correctly) to 1.0 (everyone answered correctly), the mean was .05–.18 lower for strongly related items than for not strongly related items, with the standard deviation being smaller by .05–.19. Finally, it was shown that, with respect to item difficulty, the distributions of items in the strongly related and not strongly related partitions were not homogeneous ($p \leq .002$).

Chapter 5 discusses the conclusions supported by the research findings, the generalizability of the findings, and recommendations for further research on this topic. The final section presents implications for computer science education, in particular the need for greater attention to pre-college preparation in mathematical logic and to the discrete mathematics courses taken by computer science students.

A glossary, the last section before the bibliography, defines important terms and acronyms. Several appendices are given between Chapter 5 and the glossary. The bibliography is the final section of the thesis.

Chapter 2 Literature Review

Chapter 2 begins with a brief historical perspective on the development of mathematical logic. After a discussion of the status of mathematical logic in curriculum guidelines in computer science and related fields, the use of mathematical logic in the age of computer science is explored from the point of view of programming languages and formal methods. Next, the relationship between logic and reasoning is considered. Theories about the role of logic and reasoning in psychology are discussed, followed by a survey of studies that have investigated the relationship between students' ability to correctly interpret propositional logic statements and their success in natural science courses. Chapter 2 concludes with a discussion of the research technique content analysis and a review of its use in recent studies.

2.1 Mathematical Logic — A Historical Perspective

The history of logic is closely related to the history of Western philosophy. As a form of systematic and scholarly inquiry, philosophy was used by the ancient Greeks (e.g., the pre-Socratics, Plato, and Aristotle) to develop a set of principles sufficiently comprehensive to account for their knowledge of both the natural and the human world. With time, the Greek thinkers understood that for each science there could be a corresponding philosophy of the science. The philosophy of a science would examine the fundamental principles of the discipline to see whether they were logical, consistent, and true. Eventually, philosophical aspects of scientific endeavors were recognized as being distinct

from attempts to delineate reality, leading to the establishment of the various branches of the natural sciences, such as astronomy, physics, chemistry, geology, biology, psychology, and computer science.[1]

In the Philosopher's quest for answers, the basic tools have been logical and speculative reasoning. In Western philosophy, the development of logic has generally been traced to Aristotle, whose aim was to construct valid arguments and, if true premises could be uncovered, true conclusions. As a tool, logic has played an important role in both ancient and modern philosophy by clarifying the reasoning process, providing standards for recognizing valid reasoning, and allowing analysis of basic concepts for consistency.

The relationship between mathematics and philosophy was apparent almost from the beginning in ancient Greece. Because mathematics appeared to encompass a degree of certainty and rigor exceeding that observed in other subjects, some philosophers felt that mathematics was the key to understanding reality. Plato, for example, claimed that mathematics provided the "forms" out of which everything was made. In contrast, Aristotle maintained that mathematics dealt with ideal rather than real objects, so that mathematics could be absolute without informing about reality.

Modern logic began to arise during the middle of the 17th century, when G. W. Leibnitz theorized about constructing an ideal mathematical language in which to state and mathematically solve all philosophical problems (Popkin, 1993a). One of Leibniz's ideas was that of an *ars magna*, a machine able to

---

[1] The primary source for the first three paragraphs of this section is Popkin (1993a). The facts presented were reinforced by Church (1956), Hilbert and Ackerman (1950), Lewis and Langford (1959), and Popkin (1993b).

answer arbitrary questions about the world (Sperschneider & Antoniou, 1991). His attempts were the first in the history of science to represent logic in the form of an algebraic calculus (Stolyar, 1970).

Mathematical logic arose from the desire to establish systematic foundations for the practice of mathematics, for explaining the nature of numbers and the laws of arithmetic, and for replacing intuition with rigorous proof (Cumbee, 1993). The foundational crisis of mathematics in the late 19th and early 20th centuries greatly accounts for the existence of mathematical logic as a special branch of science (Sperschneider & Antoniou, 1991; Stolyar, 1970). Modern logic, developed from the 19th century onwards in the work of Boole, de Morgan, Frege, Jevons, Peano, Peirce, Schröder, Russell, Whitehead, and others, includes a body of proofs and modes of inference within which the work of Aristotle and other ancients falls naturally into place, but which in addition contains a comprehensive theory of relations. The primary difference between traditional logic and modern logic is that the latter is much more inclusive.

At the beginning of the 20th century, attempts were made to describe mathematics completely by means of formal systems. One goal was to mechanize mathematics; this task came to be known by the name *Hilbert's Programme*. Gödel's work, published in 1931, proved that this task was totally unrealistic by showing that, for every sufficiently rich formal system, a valid assertion could be constructed that could not be derived in the formal system. Another fundamental finding that showed the unfeasibility of Hilbert's Programme was the famous undecidability result of Turing and Church (Sperschneider & Antoniou, 1991).

The development of modern logic was made possible through the systematic use of symbolic notation as a medium for formulating even complex meanings in simple terms (Hasenjaeger, 1972; Stolyar, 1970). Even Aristotle used letter symbols in logic; however, since no symbolic language had been developed for mathematics at that time, his use of symbols was very limited. Formal logic became symbolic when it acquired its own technical language, essentially an extension of mathematical symbols (Copi, 1979; Stolyar, 1970). Alfred North Whitehead, an important contributor to the advance of symbolic logic, highlighted the significance of this progress in his observation that

> … by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. (Whitehead, 1911; cited in Copi, 1971, p. 7)

The extended use of symbolic procedures made the subject of logic broader in scope and brought logic into new relationships with other exact sciences, such as mathematics (Lewis & Langford, 1959). Mathematical logic has also been referred to as symbolic logic, exact logic, formal logic, logistic, and the algebra of logic (Hilbert & Ackermann, 1950; Lewis & Langford, 1959).

The following quote from Belnap & Grover (1973) concludes this brief historical perspective on logic by pointing out its widespread utility and the breadth of applications to which it is applied:

> Logic is many things: a science, an art, a toy, a joy. And sometimes a tool. One thing the logician can do is provide useful systems, systems which are both widely applicable and efficient: set theory has been developed for the mathematician, modal logic for the metaphysician, boolean logic for the computer scientist, syllogistics for the rhetorician; and the first order functional calculus for us all. (p. 17)

2.2 CURRICULUM GUIDELINES RELATED TO LOGIC IN COMPUTING

Three distinct sources of curricular guidelines address the issue of which topics of mathematical logic should be included in the education of computer science students. The first source is the field of computer science. In computer science-oriented guidelines, logic is not a major focus but is an integral part of many components of the curricular guidelines. The second source of guidelines considers logic in the context of the discipline of mathematics. Here, guidelines cover concepts of logic, but the agenda is broader than mathematical logic. In the context of this study, recommendations for the discrete mathematics course are of greatest interest. The third source of guidelines is the mathematical logic community. Guidelines from this source focus exclusively on the topics of logic that should be taught, when these topics should be covered, and the subsets of topics that are important for students in various fields. Throughout this section, emphasis is on post-secondary education, with pre-college issues discussed as appropriate.

2.2.1 Computer science curriculum guidelines

As academic disciplines go, computer science is a young field. The first widely accepted curriculum for academic programs in computer science was Curriculum '68 (Atchison, 1968), published by the Association for Computing Machinery; many alternatives and revisions have emerged since then.

In 1982, the Mathematical Association of America (MAA) published *Studies in Computer Science*, a book in the series titled *Studies in Mathematics*

(Pollack, 1982a). *Studies in Computer Science* included nine articles that provided snapshots of the still-emerging field of computer science. As Pollack pointed out in the introduction, "… the burgeoning of computer science programs cannot be equated with the maturation of computer science" (p. vii). The intention of the volume was to explore computer science as a field distinct from the various disciplines that *used* aspects of computing (e.g., numerical analysis). The first article, "The development of computer science", was authored by Pollack (1982b). As part of this historical perspective, Pollack explained how the very process of defining academic programs for computer science forced recognition of computing as a discipline separate from others such as mathematics and engineering. Curriculum '68 in particular acted as a catalyst, providing a basis for discussion as well as a developmental model for existing and budding computer science degree programs. However, Pollack (1982) explained that Curriculum '68

> … also had a dichotomizing aspect: Its basically mathematical orientation sharpened its contrast with more pragmatic alternatives. Most computer science educators agreed that the proposed core courses included issues crucial to computer science. However, the curriculum brought to the surface a strong division over the way in which these issues should be viewed. In defining the contents of the courses, Curriculum '68 established clearly its alignment with more traditional mathematical studies, giving primary emphasis to a search for beauty and elegance. (p. 41)

During the decade following the introduction of Curriculum '68, a number of alternative curricula appeared, each in response to objections to Curriculum '68. According to Pollack (1982), alternative curricula were defined for the areas of management information systems, software engineering, biomedical computer science, information science, computing center management, computer

engineering, and applied mathematics (with emphasis on the mathematics of computation).

In the mid-1970s, the ACM initiated a new curriculum effort, intended to answer the increased demand for professionally-focused computer science programs. This culminated in Curriculum '78 (Austing, 1979). Curriculum '78 was criticized by many for simply reflecting the status quo in computer science education, rather than providing a forward-looking model. Berztiss (1987) observed that, instead of successfully integrating the theoretical and practical developments that occurred between 1968 and 1978, Curriculum '78 stressed the practical side of the field and thus lent a vocational spirit to computer science education. Ralston and Shaw (1980) pointed out that the mathematics components in Curriculum '78 were essentially the same as those in Curriculum '68, only weaker: Curriculum '68 required a total of eight mathematics courses, while Curriculum '78 required only five. Ralston and Shaw predicted that, because the mathematics of central importance to computer science had changed drastically during the intervening decade, this would lessen the impact of the entire report.

A second professional organization for computer science with a deep interest in curricular issues is the Computer Society of the Institute of Electrical and Electronic Engineers (IEEE). In 1976 and 1983, the IEEE Computer Society published model programs in computer science and engineering (IEEE, 1976, 1983). These curricula were specified in the form of subject areas rather than courses and, for aspects of the curriculum outside of computer science and engineering, deferred to the standards of the Accreditation Board for Engineering

and Technology (ABET). The model program report (IEEE, 1983) described discrete mathematics as a subject area of mathematics that is crucial to computer science and engineering. The discrete mathematics course was to be a pre- or co-requisite of all 13 core subject areas except the first, Fundamentals of Computing, which had no pre-requisites. The description of the content of discrete mathematics consisted of detailed lists of topics for eight modules, the first of which was Introduction to Symbolic Logic. Theoretical concepts listed for this module were logical connectives, well-formed formulas, rules of inference, induction, proof by contradiction, predicates, and quantifiers; application concepts were computer logic and proofs of program correctness. In Shaw's opinion (1985), the IEEE program was strong mathematically but was disappointing because of a heavy bias toward hardware and its failure to expose basic connections between hardware and software.

An alternative model curriculum, one for a liberal arts degree in computer science, was described by Gibbs and Tucker (1986). This effort, carried out under the aegis of ACM, was the product of collaboration of computing educators at liberal arts colleges who had come to feel that

> … the standard set by 'Curriculum 78' has become obsolete as a guiding light for maintaining contemporary high-quality undergraduate degree programs and cannot serve as a basis for developing a new degree program in computer science within a liberal arts setting. (p. 203)

Given the liberal arts setting, the underlying agenda of the curriculum was to prepare students for a lifelong career of learning. In their description of the model curriculum, Gibbs and Tucker reaffirmed the view of computer science as a coherent body of scientific principles. They stressed the essential role of

mathematics, "not only in the particular knowledge that is required to understand computer science, but also in the reasoning skills associated with mathematical maturity" (p. 207). A discrete mathematics course was recommended as either a pre- or co-requisite for the second semester computer science course. One required topic area in the discrete mathematics course was

> introduction to logical reasoning, including such topics as truth tables and methods of proof; quantifiers should be included and proofs by induction should be emphasized; simple diagonalization proofs should be presented. (p. 207)

Other topics of mathematics were described and related to the remainder of the model curriculum. Mathematical topics that the liberal arts model curriculum included as "particularly relevant to computer science" were additional areas of discrete mathematics (e.g., recurrence relations, graph theory, matrices, partially ordered sets, lattices), calculus (e.g., limits, derivatives, max-min problems, simple integration), and linear algebra (e.g., vectors, matrix manipulation, eigenvalues, eigenvectors).

While the ACM and the IEEE curricula were widely used, they became quickly outdated due to the rate at which the computing field was changing. In 1988, a joint committee of the ACM and the IEEE Computer Society was charged with the task of defining the discipline of computing. The result of that committee effort was a document known as the Denning Report (Denning, 1989). This report became the foundation for an effort to develop computer science curriculum guidelines suitable for use into the 1990s. A task force with members from the ACM and the IEEE Computer Society was set up to produce new guidelines using the Denning Report as a basis. The final report, Computing

Curricula 1991 (Tucker, 1990), cited influences of the earlier ACM and IEEE guidelines as well as of other curricular recommendations produced during the previous 25 years.

The principles underlying Computing Curricula 1991 included nine *subject areas*, three key *processes* used by professionals in the computing field, a set of *recurring concepts* that permeate the topics of computing, and the *social and professional context* of the discipline. These principles provided the basis for defining *knowledge units*, smaller modules that specify the scope of topics that are essential for all computing students. Computing Curricula 1991 specifically avoided the definition of specific courses, recognizing that the wide variety of institutions and types of programs that existed implied a need for flexibility in how the subject matter would be mapped to courses. An overview of the subject areas, processes, recurring concepts, and knowledge units is given in Appendix A.

The mathematics and science requirements recommended by Computing Curricula 1991 are described in Table 2.1. In discussing the vital role of mathematics in the computing curriculum, the committee stated "Mathematical maturity, as commonly attained through logically rigorous mathematics courses, is essential to successful mastery of several fundamental topics in computing" (Tucker, 1990, p. 27). At least the equivalent of four or five semester-long courses were specified for all computer science students. The discrete mathematics recommended for all majors included many concepts of mathematical logic, with additional topics of logic to be covered in an optional logic or advanced discrete mathematics course.

Table 2.1 Mathematics Requirements in Computing Curricula 1991

Mathematics recommended for all computing majors: (minimum of 4 semester-long courses)

subject area	*topics covered*
discrete mathematics	• sets • functions • elementary propositional and predicate logic • boolean algebra • elementary graph theory • proof techniques (including induction and contradiction) • combinatorics • probability • random numbers
calculus	• differential and integral calculus • sequences and series • introduction to differential equations

It was recommended that additional mathematics include *at least one* of the following subjects:

subject area	*topics covered*
probability	• discrete and continuous probability • combinatorics • elementary statistics
linear algebra (elementary)	• vectors • linear transforms • matrices
advanced discrete mathematics	• additional advanced topics in discrete mathematics
mathematical logic	• propositional and functional calculi • completeness • validity • proof • decision problems

The Advanced Placement (AP) program, which offers high school students the opportunity to study college-level material, includes computer science as a subject area. The AP program, run by the College Board and administered by Educational Testing Services (ETS), targets three groups: "students who wish to pursue college-level studies while still in secondary school, schools that desire to offer these opportunities, and colleges that wish to

encourage and recognize such achievement" (College Board, 1990, p. i). The College Board has defined a topic outline for Advanced Placement in Computer Science courses, given in Appendix B. The Advanced Placement Examination in Computer Science is administered annually. Students who take the examination usually have taken one or more Advanced Placement (AP) courses in computer science. The items on each AP examination are designed to cover as closely as possible the topics recommended for the corresponding introductory college-level course(s). The APCS examination is designed to measure how well students have learned the requisite concepts of computer science. Students who do well may be granted placement, appropriate credit, or both by colleges and universities that participate in the program.

While the APCS program is targeted for college-bound high school students, the ACM has developed a model computer science curriculum (Merritt, 1993) to address the needs of all high school students. The model curriculum, which was developed to be consistent with the recommendations in Computing Curricula 1991, identified essential concepts in computing that every high school student should understand. The report outlines core, recommended, and optional topics as the basis for the model; several appendices at the end of the report describe a variety of possible implementations of the model. While the report made no specific recommendations for coverage of mathematical logic, one suggested implementation did address this area. Proulx and Wolf (1993) presented a set of 12 modules covering the model curriculum topics and, in a separate table, showed the relationship between the modules and the discrete mathematics topics given in Computing Curricula 1991 (refer to Table 2.1).

Proulx and Wolf explained that the modules covered all but one of the discrete mathematics topics: the topic of proof techniques was excluded because it was felt to be inappropriate for high school students.

2.2.2 Recommendations for discrete mathematics

It is generally agreed that students in undergraduate computer science programs should have a strong basis in mathematics, although there is no consensus as to what constitutes the appropriate mathematical background. In the evolution of undergraduate curricula, attempts to recommend which mathematics courses should be required, the number of mathematics courses, and when the courses should be taken have been the source of much controversy (e.g., Berztiss, 1987; Dijkstra, 1989; Gries, 1990; Ralston & Shaw, 1980; Saiedian, 1992). A central theme in the controversy within the computer science community has been the course called *discrete mathematics*. Among other topics, the discrete mathematics course often includes formal logic, the nature of proof, and set theory.

In 1989, the Mathematical Association of America (MAA) published a report about discrete mathematics at the undergraduate level (Ralston, 1989). This report related the experiences of six colleges and universities that were supported by the Alfred P. Sloan Foundation under a program to foster "the development of a new curriculum for the first two years of undergraduate mathematics in which discrete mathematics [would] play a role of equal importance to that of the calculus" (p. 1). The intention of the Sloan program was to make recommendations for revision of the first two years of the mathematics

curriculum for everyone — mathematics majors, physical science and engineering majors, social and management science majors as well as computer science majors. The recommendations put forward by the MAA Committee on Discrete Mathematics in the First Two Years were as follows:

1. Discrete mathematics should be part of the first two years of the standard mathematics curriculum at all colleges and universities.
2. Discrete mathematics should be taught at the intellectual level of calculus.
3. Discrete mathematics courses should be one year courses which may be taken independently of the calculus.
4. The primary themes of discrete mathematics courses should be the notions of proof, recursion, induction, modeling and algorithmic thinking.
5. The topics to be covered are less important than the acquisition of mathematical maturity and of skills in using abstraction and generalization.
6. Discrete mathematics should be distinguished from finite mathematics, which, as it is now most often taught, might be characterized as baby linear algebra and some other topics for students not in the "hard" sciences.
7. Discrete mathematics should be taught by mathematicians.
8. All students in the sciences and engineering should be required to take some discrete mathematics as undergraduates. Mathematics majors should be required to take at least one course in discrete mathematics.
9. Serious attention should be paid to the teaching of the calculus. Integration of discrete methods with the calculus and the use of symbolic manipulators should be considered.
10. Secondary schools should introduce many ideas of discrete mathematics into the curriculum to help students improve their problem-solving skills and prepare them for college mathematics. (Siegel, 1989b, p. 91)

With respect to the debate over "calculus vs. discrete mathematics", Ralston and Shaw (1980) have observed that

> … although we believe strongly that the values of a liberal education should infuse any undergraduate program, our focus

> here is on the professional needs of the computer scientist, not on the general education needs. Thus, it may be true that all educated men and women should be familiar with the essence of calculus but it does not *necessarily* follow that computer scientists have a significant professional need to know calculus. (p. 70)

The alternatives currently considered most viable are: (1) students should enroll in discrete mathematics and calculus courses simultaneously, (2) calculus should be delayed until the sophomore or junior year, at which time a more sophisticated course could be offered because of earlier training in discrete mathematics courses, and (3) offer a hybrid of calculus and discrete mathematics topics, with greater emphasis on problem solving and symbolic reasoning (Ralston, 1989; Myers, 1990).

At the pre-college level, curricular recommendations for discrete mathematics have been issued by the National Council of Teachers of Mathematics (NCTM) as part of the Curriculum and Evaluation Standards for School Mathematics (Romberg, 1989). This document contains a set of individual standards for pre-college (grades K–12) mathematics curricula. One of the 14 curriculum standards for high school (grades 9–12) is for discrete mathematics. The discrete mathematics standard emphasizes that the topics of discrete mathematics would not necessarily constitute a separate course, but should instead be integrated throughout the high school curriculum.

Also at the pre-college level, the MAA Committee on Placement Examinations has attempted to identify skills needed by students taking discrete mathematics. Siegel (1989b) explained that the committee's intention was not necessarily to define an Advanced Placement examination for discrete

mathematics but rather to "… help to explain what might be the appropriate preparation for a successful experience in such a course" (p. 97).

2.2.3 Guidelines for logic education

Yet another view of the topics of logic that should be included in the educational experience of computing students comes from educators specifically interested in mathematical logic. The Association for Symbolic Logic (ASL), an international organization that has been devoted to the study of logic since 1936, formed an Ad Hoc Committee on Education in Logic in summer 1991. The committee was charged with making specific recommendations about logic education for both pre-college and undergraduate programs. Graduate programs were excluded from consideration because of the diversity of faculty research interests and of institutional traditions at the graduate level.

The committee's brief final report (ASL, in press) presents a general view of concepts in the field of logic with recommendations for the stages at which various concepts should be introduced. For pre-college students, the stated goal is to promote and facilitate logical and analytical reasoning at an early age. Nonspecific strategies are given for different age levels: informal incorporation of "good" and "bad" arguments for children aged 5–9; heuristic strategies for (logical) problem solving for children aged 10–13; and the explicit use of logical notions and techniques for students aged 14–17, probably as part of their mathematics courses. These recommendations can be contrasted with the NCTM Standards for School Mathematics (Romberg, 1989). While the NCTM Standards do not address mathematical or symbolic logic as a separate topic, key concepts of

logic are integral to several of the topics. For example, skills such as problem solving, symbolic manipulation, and reasoning are strongly related to three themes of the NCTM Standards: mathematics as problem solving, mathematics as communication, and mathematics as reasoning.

At the beginning post-secondary level, the ASL Guidelines recommend that all students should be encouraged to take at least one introductory course that teaches the basic notions of logic, including informal strategies, propositional calculus, and predicate calculus. The committee noted that such a course could be taught as a general service course in the philosophy department or as a more technical course in a mathematics or computer science department. At the advanced post-secondary level (e.g., at four-year institutions), the ASL Guidelines advocate an additional set of core topics that are relevant and applicable to many areas of science and scholarship.

The ASL Guidelines exclude specific course models because of the wide variety of academic programs and institutions to which the guidelines were addressed. The ASL report does not relate its recommendations to guidelines in related fields, such as Computing Curricula 1991 (Tucker, 1990) or the MAA recommendations for discrete mathematics (Ralston, 1989). The vagueness of the recommendations and the lack of specific connections to curriculum guidelines in mathematics and computer science reduce the potential impact of the ASL Guidelines for Logic Education.

2.3 MATHEMATICAL LOGIC IN THE AGE OF COMPUTER SCIENCE

Mathematical logic is pervasive in the field of computer science. Examples of the breadth and depth of the role of logic have been given by Galton (1992), Gries and Schneider (1993a), Myers (1990), and Sperschneider and Antoniou (1991), among others.

Because the uses of logic are so varied and opinions on the role of mathematical logic in computer science so diverse, this survey has been restricted to two areas that are closely related to the concepts considered in this study: programming languages and formal methods for proving program correctness.

2.3.1 Mathematical logic in programming languages

Use and understanding of mathematical logic in programming languages has centered on datatype boolean. Boolean is a primitive and important datatype in most computer programming languages.

A broad set of datatypes was defined in the Language-Independent Datatype (LID) project, which was carried out under the aegis of the International Standards Organisation (ISO, 1994). In the LID project, each datatype was defined independent of any particular programming language or implementation. A goal for the standard was to encourage commonality among and facilitate interchange of datatype notions between different programming languages and language-related entities. In the LID standard, each datatype is defined by a basic set of properties. The ultimate goal was to provide a single common reference model for all standards that use the concept “datatype”.

The formal LID definition of primitive datatype *boolean* is given in Figure 2.1. The definition gives three properties of datatype boolean: it is non-numeric, unordered, and discrete. A related datatype defined in the LID standard is datatype *bit*, defined as Modulo(2), the two-valued subtype of integer. Although datatypes boolean and bit resemble one another in many ways, they are distinct datatypes with different (if analogous) operations. Dijkstra and Feijen (1988) have admonished "The old-fashioned habit, still found in electrical engineering, of identifying the values *true* and *false* by the integers 1 and 0 respectively must not be imitated: it only leads to confusion" (p. 43). The separate definitions of boolean and bit in the LID standard help emphasize this distinction.

Datatype boolean has often been relegated to "second-class citizenship" in programming languages. Programmers, professionals as well as students, have tended to use datatype boolean differently than they have used, for example, datatype integer. For an object to be a first class citizen in a given language, it must be usable without restriction in whatever ways are appropriate for the language (D. Naumann, personal communication, November 24, 1993). First, the datatype should have as its basis a well-defined set of values. For example, datatype integer has as its basis the set of values {…-2,-1,0,1,2,… }; datatype boolean is based on the set of values { *true*, *false* }. Second, it must be possible both to evaluate expressions whose result is of that datatype and to assign the result of expression evaluation to variables of that type. Figure 2.2 gives two examples from the literature that contrast different ways of evaluating a boolean expression and assigning the result to a boolean variable. The third requirement

Boolean

Description: Boolean is the mathematical datatype associated with two-valued logic.

Syntax: Boolean = "boolean"
boolean-value = "true" | "false"

Parameters: none

Values: "true", "false", such that true ≠ false

Properties: non-numeric, unordered, discrete

Operations: Equal, Not, And, Or

Equal (x, y: boolean): boolean is defined by tabulation:

x	y	Equal (x, y)
true	true	true
true	false	false
false	true	false
false	false	true

Not (x: boolean): boolean is defined by tabulation:

x	Not (x)
true	false
false	true

Or (x, y: boolean): boolean is defined by tabulation:

x	y	Or (x, y)
true	true	true
true	false	true
false	true	true
false	false	false

And (x,y: boolean) = Not (Or (Not(x), Not(y)))

Note: Either And or Or is sufficient to characterize the boolean datatype, and given one, the other can be defined in terms of it. They are both defined here because both of them are used in the definitions of operations on other datatypes.

From: *Information technology — Language-independent datatypes*, International Organization for Standardization, 1994, ISO/IEC draft International Standard 11404, Geneva, Section 7.1.1.

Figure 2.1 Datatype Boolean Definition

Example 1

The following program statement (2.1) appeared in an algorithm published in *Communications of the ACM* (Irons, 1961):

(2.1) `SW := if INPUT[j] = STAB[i] then true else false`

In this statement, a variable of type boolean is being assigned the value of the expression on the right. Statement (2.1) could be verbalized as "If expression `INPUT[j]` has the same value as expression `STAB[i]`, then store the constant value `true` in variable `SW` ; otherwise store the constant value `false` in variable `SW` ". This treatment disregards the fact that the expression "`INPUT[j] = STAB[i]`" has a boolean value — that is, the result of evaluating this boolean expression is either true or false, depending on the program state. Thus, statement (2.1) can be rewritten as:

(2.2) `SW := INPUT[j] = STAB[i]`

Example 2

Another example of variations in use of datatype boolean is the following warning given in Jensen and Wirth's *Pascal User's Manual* (1974, p. 27):

> If `found` is a variable of type Boolean, another frequent abuse of the `if` statement can be illustrated by:
>
> (2.3) `if a = b then found := true else found := false`
>
> A more parsimonious statement is:
>
> (2.4) `found := a = b`

In this example, statement (2.3) uses an `if-then-else` control structure to assign one of two constants to boolean variable `found` , while statement (2.4) uses an assignment statement to assign the value of a boolean expression to `found`. In statement (2.3), the outcome is described via the "control" decisions needed to determine the final value of `found` ; as the number of conditions increases, the decision structure becomes more complex. In statement (2.4), the value of the boolean expression is evaluated and that result assigned directly to the boolean variable. As the number of conditions increases, such an expression can be expressed more succinctly than can the corresponding multi-part control structure —and thus the advantage grows.

Note: Program segments are given in Courier font. Reserved keywords such as `if`, `then`, and `else` are, by convention, underlined. The statement "`x := e`" assigns the result of evaluating expression `e` to variable `x`. The two-character symbol "`:=`" is pronounced "becomes", "receives the value", or "is assigned the value of". The single-character symbol "=" is the infix relational operator for equality.

Figure 2.2 Two Examples that Contrast Approaches to Assigning Boolean Values to a Variable

for first-class citizenship is that it must be possible to pass arguments and return function values of the type. (N. McPhee, personal communication, November 22, 1993; D. Gries, personal communication, December 10, 1993).

In their 1988 text *A Method of Programming*, Dijkstra and Feijen observed:

> For the first 15 years, program execution was understood as a combination of 'the computation of numbers' and 'the testing of conditions'. While the result of such a (numerical) computation was formed and stored for later use in the register or memory, the result of the test of a condition was used immediately (as in an alternative statement) to influence the further execution of the computation. One merit of Algol 60 was that by introducing variables of the type *Boolean*, it was made clear that the testing of a condition could be better understood as a computation — not as the computation of a number but as the computation of a 'truth value'. This generalization of the idea of computation is a very important contribution: the proof of a theorem can now be regarded as the demonstration that the computation of a proposition yields the value *true*. Although we shall only come across a modest number of variables of the type *Boolean* in our programs, the type *Boolean* should not be missing from any introduction to programming. (p. 43)

Whether student or professional, programmers' understanding of fundamental computer science concepts will be influenced by the features of the programming language(s) they use. As a result, many programmers fail to benefit from a full understanding of all of the characteristics of datatype boolean. Algol 60 brought datatype boolean into first-class citizenship. The programming language C, on the other hand, allows the programmer to treat boolean as a special case of the type integer. The confusion caused by such conventions may interfere with students' understanding of mathematical logic.

A series of articles and letters published in the professional journal *SIGPLAN Notices* illustrates the lack of consensus among practicing professionals

regarding the use of logic in computer science (Boute, 1990, 1991; Meeks, 1990, 1991; Nocolescu, 1991; Sakkinen, 1990). *SIGPLAN Notices*, published by the ACM Special Interest Group in Programming LANguages, has a readership of professionals specifically interested in issues surrounding programming language use, design, and standardization. In 1990, a heated discussion was launched when Boute (1990) presented his self-proclaimed "heretical" view of datatype boolean as a subtype of the numeric type *natural*. Boute's position was based on a mathematical argument, which received limited acceptance but was eloquently rebutted by Sakkinen (1990) and Meeks (1990). Both Sakkinen and Meeks accepted Boute's restriction for appropriate situations, but demonstrated that there were, in fact, many instances where a less restricted view of logic was more natural and useful. Meeks presented the full range of datatypes related to logic that were defined as part of the LID standardization effort (ISO, 1994), discussed earlier. The conclusion drawn by both Sakkinen and Meeks was that, while Boute's formulation was valid in its own context and when needed, trying to view logic purely as a restricted subset of the natural numbers would limit the ability to express meaning accurately. In other words, if `true` and `false` are the notions being expressed, it is unnecessary, confusing, and restrictive to coerce humans into translating them into `1` and `0` or some other representation.

2.3.2 Logic as the basis for formal methods

Formal methods encompass a wide range of techniques and languages used in the development of software. Wing (1990) defined a method as formal "if

it has a sound mathematical basis, typically given by a formal specification language" (p. 8). Cooke (1992) explained:

> The term 'Formal Methods' alludes to the facility to be able to reason formally (in a mathematically precise, logical way) about the properties of programs and systems. It covers not only programming languages and the common data types, their operators and their properties, but also logic, particularly the notion of deduction — 'if something is true then something else is true'. (p. 420)

Goldson, Reeves, and Bornat (1993) projected the "hope that the use of formal methods will make programs conform to specification and make them more reliable but such 'methods' are really nothing more than a collection of techniques imported from discrete mathematics, logic and set theory" (p. 373).

In the article "Logic as a formal method", Galton (1992) outlined a representative selection of the ways in which formal logic has been used in computer science. As applications of classical first-order predicate logic, he included program specification, program verification, program synthesis, and logic programming. Beyond classical logic, Galton sketched applications based on intuitionistic logic, temporal logic, modal logic, and logics for non-monotonic reasoning. He admitted that, even in his extensive survey, he had neglected many areas of computing in which logic is important; this serves to punctuate the breadth of the role of logic in computer science.

This subsection focuses on a specialized area within the range of formal methods that evolved from work in the formal development of algorithms. Researchers needed to manipulate formulae of the predicate calculus on a regular basis yet found that conventional logics, such as natural deduction, required a great deal of formal detail while providing little or no insight into the

development process (Gries & Schneider, 1994). The need for a better model gave rise to the development of a collection of formal methods referred to as *formal verification of program correctness* (where correctness of a program is established a posteriori) and *formal derivation of correct programs* (where the proof of correctness is developed as the program is developed).

Floyd (1967) was the first to suggest that the specification of proof techniques could provide an adequate formal definition of a programming language; he also analyzed the potential benefits of using an axiomatic approach for program proving and for formal language definition. In his seminal paper "An axiomatic basis for computer programming", Hoare (1969) acknowledged Floyd's suggestion that axioms could provide a simple solution to the problems that arise when aspects of programming languages are left undefined; whereas earlier language definitions had been primarily syntactic and in terms of implementation, axioms made it possible to give a non-operational language definition (i.e., independent of implementation). Hoare presented an axiom system in which programs are expressed as formulae. Such formulae are predicates given in terms of triples {Q}S{R},[2] where S is a statement from the programming language and Q and R are predicates on the variables used in the statement. Referred to as the *precondition* and the *postcondition*, Q and R describe the initial and final program states of the statement S. By using axioms and rules of inference that specified

---

[2] This discussion ignores the steps that led from consideration of only "partial correctness" to "total correctness". Partial correctness does not address the issue of program termination. Minor notational changes that were a part of this evolution are ignored in favor of the later notation.

the meaning of statements in a simple programming language, it was possible to verify the correctness of a given program written in that language.

As an alternative to proving the correctness of *given* programs, Dijkstra (1968) proposed controlling the process of program *generation*. In the early 1970s, Dijkstra introduced the notion of predicate transformers as a systematic way to derive rather than verify programs (Dijkstra, personal communication, April 14, 1994). The process of program derivation with predicate transformers was described in Dijkstra's classic book *A Discipline of Programming* (1976) and expanded in the textbook *A Science of Programming* by Gries (1981). Predicate transformers extended the Hoare axiom system and provided a means for defining programming language semantics in a way that would directly support the systematic derivation of programs from their formal specifications (Dijkstra & Scholten, 1990). In the Hoare-triple {Q}S{R}, the predicate transformer takes as arguments statement S and postcondition R; it returns the predicate Q. Q, the precondition, is a predicate that is satisfied by all of the program states in which execution of statement S is guaranteed to terminate with predicate R true (Gries, 1981). Stated more succinctly, a predicate transformer is a function of two arguments, a statement and a predicate, that returns a predicate as its result. Predicate transformers provide a basis both for deriving correct programs and for verifying the correctness of existing programs.

Over time, the activities of program verification and program derivation have become more formal. As limitations in the use of predicate transformers were encountered, many researchers undertook the task of more formally defining the theory. Dijkstra and Scholten (1990) explain this process as follows:

> Probably conditioned by years of formal program derivation, we approached the task of designing the theory we needed as an exercise in formal mathematics, little suspecting that we were heading for a few of the most pleasant surprises in our professional lives. After a few notational adaptations of the predicate calculus —so as to make it more geared to our manipulative needs— and the adoption of a carefully designed, strict format for our proofs, we found ourselves in possession of a tool that surpassed our wildest expectations. … In the course of the process we profoundly changed our ways of doing mathematics, of teaching it, and of teaching how to do it. Consequently, this booklet is probably as much about our new appreciation of the mathematical activity as it is about programming language semantics. … As time went on, … we were forced to conclude that the formal techniques we were trying out had never been given a fair chance, the evidence being the repeated observations that most mathematicians lack the tools needed for the skillful manipulation of logical formulae. We gave [the tools for manipulating logical formulae] a fair chance; the reader is invited to share our delight. (p. vi)

Dijkstra and Scholten's efforts resulted in, among other things, an equational logic. This approach was used informally in the late 1970s and was refined through the 1980s. In the early 1990s equational logic has come into wider use, for example in discrete mathematics courses (this will be discussed in the next section). The key difference between equational logic and other forms of logic is the extensive use of value-preserving manipulations in the former rather than proofs composed exclusively of chains of implications.

2.3.3 Logic in discrete mathematics

In his article "Mathematics of computing", Saiedian (1992) traced the use of mathematics in computer science through the curricular recommendations listed in sections 2.2.1 and 2.2.2. He observed that Curriculum '68 explicitly recommended a computer science course in discrete mathematics, "Introduction to Discrete Mathematics". Curriculum '78, which listed the discrete mathematics

course “Discrete Structures” as part of the general mathematical requirements, “did not define explicitly the details of topics to be covered in the course” (p. 208) and suggested the course as an applied mathematics component that mathematics departments could provide for computing students. With respect to Computing Curricula 1991 and the evolving role of the discrete mathematics course, Saiedian explained:

> The recommendations very explicitly recommend a course in discrete mathematics with a list of all topics to be covered and further emphasizes that courses in both advanced discrete mathematics and mathematical logic (covering propositional and functional calculi, completeness, proofs, etc.) be considered. As pointed out by a colleague, the Curriculum 1991 view on discrete mathematics and mathematical logic represents an “inverted bell-shaped curve” with respect to Curriculum '68 and Curriculum '78. In 1968, discrete mathematics was considered very important when students going into computer science came mostly from engineering [so that background in] mathematics was no problem. When computing became an area of study for a wider range of students during the 1970s, the emphasis, as reflected in the Curriculum '78, was decreased. However, as the computing discipline matured, it became evident that its foundations are strongly mathematical as shown in the Curriculum 1991. (p. 209)

A wide variety of discrete mathematics textbooks are currently available for discrete mathematics: In an appendix to the MAA Report on Discrete Mathematics in the First Two Years, Siegel (1989a) lists 37 textbooks in “A bibliography of discrete mathematics books intended for lower division courses”. Siegel admits the list “will surely be out-of-date by the time this book appears” (p. 87); in fact, the list does not include three of the textbooks that were used regularly in discrete mathematics courses at the investigator’s home institution from 1988 to 1992. The diversity and number of discrete mathematics textbooks demonstrates the perceived need for this material.

In a comparative review of ten discrete mathematics textbooks, Spresser and LaTera (1992) used as a benchmark the requirements published by the Mathematical Association of America (Ralston, 1989; see section 2.2.2). Spresser and LaTera rated the coverage of formal logic as "very good" for eight of the ten books. For seven of the ten books, computer science students were listed as part of, if not the primary, intended audience.

Franzblau (1993) conducted an informal survey of approaches to teaching discrete mathematics. She presented three course models that she labeled "new": (i) discrete mathematics courses with a focus on proof and (re-)writing, (ii) discrete mathematics as the first course for computer science majors, and (iii) discrete mathematics courses where logic is used as a tool. For the current study, the third model is of the most interest. As an example of this model, Franzblau introduced the approach of Gries and Schneider (1993a, 1993b, 1994). In Franzblau's words, they propose "a radically different and highly structured approach".

Warford (in press) reviewed Gries and Schneider's book *A Logical Approach to Discrete Math* (1993a) after using it as the text for a discrete mathematics course he taught. Warford explained that he found that the central role of the propositional and predicate calculus allowed a unified treatment of other discrete mathematics topics (e.g., sets, mathematical induction, sequences, relations, functions, combinatorial analysis, recurrence relations, algebra, graph theory), as opposed to a "shotgun approach" where the course is made up of several seemingly unrelated topics. In teaching discrete mathematics using Gries and Schneider's approach, Warford found that the character of the course began

to resemble that of the traditional calculus: in traditional calculus, students progress through a series of skill-based exercises such as differentiation, the chain rule, and integration; in the Gries and Schneider approach to discrete mathematics, the skill-based exercises include textual substitution, Leibniz's rule, boolean expressions, and quantification. In addition, both the traditional calculus and the logic-based discrete mathematics course have as an underlying theme the development of students' skills with proofs and reasoning.

2.4 THE CONNECTION BETWEEN LOGIC AND REASONING

Logic has often been called the science of reasoning. Copi (1971) pointed out: "As thinking … reasoning is not the special province of logic, but part of the psychologist's subject matter as well" (p. 1). For psychologists, reasoning is interesting from the point of view of *process*, as a model for human thought; for logicians, the correctness of the *completed* reasoning process is of the most interest. In the field of computing, the use of logic as a tool for expressing and proving theorems is a key focus.

This section explores connections between mathematical logic and reasoning. The section begins by exploring the importance of reasoning in academic success. The role of logic in psychology is discussed, with particular attention to Piaget's theory of developmental stages due to both its use of propositional logic and its major impact on research in this area. Finally, a self-contained instrument designed to measure skill with propositional logic is discussed briefly.

2.4.1 Reasoning skills needed in computer science

In 1986, Powers and Enright (1987) conducted a survey to determine the perceptions of a sample of college faculty members about the importance of analytical reasoning skills for graduate study. Powers and Enright noted, "Despite the perceived importance of reasoning, there seems to be no consensus regarding the impact of formal education on the development of reasoning abilities" (p. 659).

The sample included 255 graduate faculty in six fields of study: chemistry (N = 37), computer science (N = 43), education (N = 42), engineering (N = 43), English (N = 44), and psychology (N = 46). Each participant completed a questionnaire that included items about the importance of various reasoning skills, including the extent to which each skill seemed to differentiate between marginal and successful students. The questionnaire also included items about the importance and frequency of commonly observed errors in reasoning.

Some reasoning skills were consistently rated as very important across the six disciplines. In decreasing order of rated importance, these skills were:

- reasoning or problem solving in situations in which all the needed information is *not* known
- detecting fallacies and logical contradictions in arguments
- deducing new information from a set of relationships
- recognizing structural similarities between one type of problem or theory and another

- taking well known principles and ideas from one area and applying them to a different specialty
- monitoring one's own progress in solving problems
- deriving from the study of single cases structural features or principles that can be applied to other cases
- making explicit all relevant components in a chain of logical reasoning
- testing the validity of an argument by searching for counterexamples

Other skills were rated as important within one discipline but not in others. For example, "knowing the rules of formal logic" was rated as one of the most important skills in computer science but was rated as quite unimportant in the other disciplines. The two reasoning skills that were rated as most important or critical by the computer science educators were "breaking down complex problems or situations into simpler ones" and "reasoning or problem solving in situations in which all facts underlying a situation are known".

While the underlying reasoning processes are important for academic success in many disciplines and logic and reasoning are intimately related, learning about logic is not synonymous with learning to reason. Specifically, logic is a *topic* in computing, with underlying skills as important for the student as the skills of arithmetic. This distinction is important when considering the role of logic in psychological studies: there is a tendency to blend the two, despite the differences. As Copi (1979) has stated: "…paradoxically enough, logic is not concerned with developing our powers of thought but with developing techniques that enable us to get along without thinking!" (p. 246).

2.4.2 Logic and reasoning in psychological theories

Rothaug (1984) surveyed several views of the relationship between formal logic and the natural psychological logic of thought. The descriptive or rationalist view held that there is a close relation between logic and reasoning. Another view held that the laws of logic are only normative and are not useful as a descriptive model for thinking and reasoning. Piaget and other researchers have postulated that higher-level psychological processes reflect logical principles, so that formal logic provides a useful way to characterize a significant component of thinking and reasoning. Anderson (1980), a cognitive psychologist, viewed the use of logic as a useful heuristic method for learning about a subject's behavior when the subject solves problems (rather than as a model for thinking). Anderson maintained "Reasoning is fundamentally a matter of problem solving, not a logical activity … [and] deductive reasoning is a special case of problem solving rather than some special faculty of the mind" (p. 326). Evans (1980) criticized research on reasoning on the grounds that previous experiments in this area involved artificial situations with little relationship to real life situations.

Other views on the role of logic in psychology have been based on the assumption that human intelligence is not a single trait or process but is instead a collection of separate abilities. In these models, the ability to do logic is simply one of many abilities. For example, Guilford (1967) considered a three-dimensional model of intellect, organized around three main aspects of human functioning: operations, products, and content. In Guilford's theory, specific abilities involve a combination of each of these dimensions. Gardner (1985)

argued for the existence of several human potentials, each of which is relatively autonomous. Gardner hypothesized seven intelligences: logical-mathematical, linguistic, musical, spatial, bodily-kinesthetic, interpersonal, and intrapersonal; hence, his theory has been called Multiple Intelligences theory. Gardner and Hatch (1989) explain that there is not necessarily a correlation between any two intelligences and that each may entail distinct forms of perception, memory, and other psychological processes. They characterize the key components of the logical-mathematical intelligence as "sensitivity to, and capacity to discern, logical or numerical patterns; ability to handle long chains of reasoning" (p. 6).

2.4.3 Logic in Piagetian theory

Piagetian theory is complex and broad in scope, so it will only be covered at a very high level in this discussion. Piaget considered intelligence to be the process of adapting through the cooperating and invariant *functions* of assimilation (response based on pre-existing information) and accommodation (response based on new information). Human development comes about not because of changes in function but rather due to changes in behavior over time. *Structure* describes the properties of intellect that govern behavior, with change occurring in response to demands of the environment. Schemata are the structures that allow the mental representation of knowledge; during early life, all of these schemata are based on physical experience. As humans age, structure is defined in terms of less overt behavior, characterized by internal activities.[3]

---

[3] The information in this paragraph and the next was synthesized from discussions by Furth (1969), Ginsburg and Opper (1979), Inhelder and Piaget (1958), Lefrancois (1988), and Stofflett & Baker (1992).

With time, thought becomes subject to certain rules of logic, called *operations*. In Piaget's theory, at the pre-operational stage (ages 2–7), thinking is limited because of the child's reliance on perception and intuition as well as egocentric tendencies. At about age seven, the child enters the concrete operational stage. At this point, the child is able to apply operations to real objects and events. At the formal operational stage, which Piaget's findings showed to begin at about age 12, the child is able to deal with hypothetical situations and can apply a formal set of logic rules or operations. The child thus can go beyond empirical reality (the first order operations) to "formal thought"; the child can now apply second order operations (which use the products of first order operations). Inherent in formal thought is the ability to perform the 16 operations of propositional logic, outlined in Table 2.2. Inferences are drawn through applying logical operations to propositions; underlying this is the system of all possible relations, described by Piaget as the combinatorial system.

Piaget's theory was for childhood and adolescence, with the formal operational stage beginning about age 12. However, Petrushka (1984) cites numerous studies that have shown that a majority of adults, including college students and professionals, fail at many formal tasks.

Ginsberg and Opper (1979) clarified that Piaget uses logic not to describe explicit knowledge but to depict the structure of thought, that is: how does logical thought mediate problem solving? The logical models are not descriptions of actual performance but are instead abstractions intended to capture the essence of thought and to allow psychologists to explain and predict behavior. Parsons (1958) explained that Inhelder and Piaget (1958) used logic as a theoretical tool in

describing the mental structures that govern ordinary reasoning and thought. She pointed out that, while logic is concerned with formalizing internally consistent systems, psychology deals with mental structures independent of any formal training or use of symbols and "regardless of consistency or inconsistency, truth or falsehood" (p. viii). As Furth (1969) pointed out, however, Piaget's models were only intended to reflect developing intelligence, which diffuses the criticisms of the logicians (that the model is not sufficiently sophisticated) and the psychologist (that the model is too far removed from real thinking).

2.4.4 An instrument for measuring ability in logic and reasoning

The Propositional Logic Test (PLT), developed and used over a number of years by science education faculty and students at Rutgers University, assesses a subject's ability to process propositional statements. In taking the PLT, the subject is allowed 15 minutes to interpret truth-functional operators by identifying instances that are consistent or inconsistent with a stated rule (see Figure 2.3 for an example item). The PLT, which consists of 16 items, can be broken into four 4-item subtests. Each subtest addresses one of the Piagetian operations conjunction, disjunction, implication, and biconditional. On the PLT, error patterns are apparent because there are exactly 16 ways that any question could be answered (i.e., the truth tables of the 16 binary operations of logic as shown in Table 2.2).

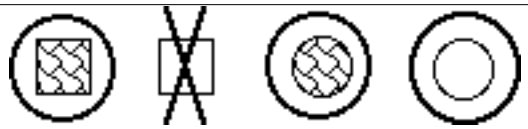
Piburn (1989) has reported that the PLT as a whole has high reliability and that subtest reliability is best for the biconditional subtest and decreases over the

Table 2.2 Piaget's System of 16 Binary Operations

Piaget's Notation	Operation Name	Disjunctive Normal Form			
		$p \quad q$	$p \quad \bar{q}$	$\bar{p} \quad q$	$\bar{p} \quad \bar{q}$
o	1. PLT: negation G&O: *same*	F	F	F	F
$p \cdot q$	2. PLT: conjunction G&O: *same*	T	F	F	F
$p \cdot \bar{q}$	3. PLT: non-implication G&O: inverse of implication	F	T	F	F
$\bar{p} \cdot q$	4. PLT: non-converse implication G&O: inverse of converse implication	F	F	T	F
$\bar{p} \cdot \bar{q}$	5. PLT: conjunctive negation G&O: *same*	F	F	F	T
p [q]	6. PLT: affirmation of p G&O: independence of p to q	T	T	F	F
q [p]	7. PLT: affirmation of q G&O: independence of q to p	T	F	T	F
$p \equiv q$	8. PLT: material equivalence G&O: reciprocal implication	T	F	F	T
$p \bar{\vee} \bar{\wedge} q$	9. PLT: exclusive disjunction G&O: reciprocal exclusion	F	T	T	F
$\bar{q}$ [p]	10. PLT: negation of q G&O: inverse of #7	F	T	F	T
$\bar{p}$ [q]	11. PLT: negation of p G&O: inverse of #6	F	F	T	T
$p \vee q$	12. PLT: inclusive disjunction G&O: disjunction	T	T	T	F
$q \quad p$	13. PLT: reciprocal implication G&O: converse implication	T	T	F	T
$p \quad q$	14. PLT: material implication G&O: implication	T	F	T	T
$p \mid q$	15. PLT: incompatibility G&O: *same*	F	T	T	T
$p * q$	16. PLT: tautology G&O: *same*	T	T	T	T

Note: PLT is the operation name as defined in the key for the Propositional Logic Test, 1990; G&O is the operation name as defined by Ginsburg & Opper, 1979, p. 191.

If it is white then it is round.



If it is white then it must be round, but if it is tiled then it doesn't matter if it is round or not. So the white circle fits but the white square does not. The tiled figures all fit because the statement only tells us about white figures.

Figure 2.3 Sample Item from the Propositional Logic Test (PLT)

implication, conjunction, and disjunction subtests. The PLT has been shown to correlate highly with grades in natural science courses as well as with the Test of Logical Thinking (TOLT), another instrument based on Piagetian theory (Tobin & Capie, 1981). In contrast to other measures of ability with propositional logic, error patterns on the PLT have revealed systematic relationships between age and ability that appear to reflect underlying reasoning processes (Piburn, 1989).

2.5 LOGIC AS A TOOL FOR PREDICTING SUCCESS IN SCIENCE

For science educators, the relationship between logic and science is of special interest because of important parallels between the two: both are systems that seek truth and the systematic procedures employed by each resemble one another (Rothaug, 1984). In addition, Rothaug pointed out that, where logic investigates truth relations between sentences, science seeks to establish in a systematic way the truth of sentences based on the truth of other sentences . Science educators have shown that, given their definitions of *reasoning* and *success*, the ability to reason is strongly related to success in science. Stofflett and Baker (1992) point to results that indicate that students who reason well score

higher on content examinations, have stronger process skills, and have more interest in science.

Various studies with students in college physics courses have shown correlations of .30 to .75 between a variety of measures of logic and achievement in the course (e.g. Baker & VanHarlingen, 1979; Enyeart, VanHarlingen & Baker, 1980; Lockwood, Pallrand & VanHarlingen, 1980; Pallrand & VanHarlingen, 1980; Piburn & Baker, 1988; Seeber, Pallrand, VandenBerg & VanHarlingen, 1979). Similar results were obtained in studies of high school physics students (Lockwood, Pallrand & VanHarlingen, 1982) and college chemistry students (Rothaug & Pallrand, 1982; Rothaug, Pallrand & VanHarlingen, 1981).

The Propositional Logic Test (PLT), described in an earlier subsection, has been used to measure ability in logic in a number of studies. For example, Piburn (1990) considered several questions in a study with Australian high school science students: (1) Is achievement in science positively correlated with ability to reason about logical propositions? (2) Are some logical operators more strongly related to achievement in science than others? (3) Is reasoning about logical propositions related to sex or ability level? (4) Are patterns of error on a test of the ability to reason about logical propositions related to ability?[4] Piburn found that the coefficient of correlation between final grade in science and success on the PLT for the entire sample was .57. He also considered subtest correlations for conjunction, disjunction, implication, and biconditional. He found that advanced students received the highest scores and basic students the

---

[4] In the Australian school system, students are tested and, based on their scores, classified into ability groups as advanced (top 25%), basic (bottom 25%) or intermediate (the middle 50%) (Piburn, 1990).

lowest, with the greatest difference showing up between advanced and intermediate students. Piburn discovered that the pattern of errors on subtests across ability groups in the Australian sample was the same as that across grade levels in cross-sectional studies of American students from grades 7 to the first year in college. Piburn found that correlations between the score on the PLT and achievement in science were significant and relatively high for both the Australian and American samples.

Stager-Snow (1985) designed a study in which the subjects were students in an introductory computer science course for non-computer science majors. Her results indicated that for the females in the sample the PLT was a weak predictor variable; for the males, the PLT had no predictive power. In addition, Stager-Snow found that knowledge of the `if-then` statement contributed more to the variance in explaining computer knowledge for the females than for the males.

2.6 CONTENT ANALYSIS AS A RESEARCH METHODOLOGY

In its simplest form, content analysis is a technique for making replicable and valid inferences from textual data to their context. It is frequently used in the social sciences, especially for the purpose of analyzing communications such as newspaper articles or textbooks. As an example, suppose that a researcher was interested in studying the use of symbols and language in messages posted on electronic bulletin boards for the strategic use of humor. Using content analysis, the investigator could evaluate occurrences of textual figures, icons (e.g., the smiley face `:-)`), and phrases (e.g., parenthetical remarks such as `(wink, wink)`) for intent to express humor.

Content analysis is exploratory, fundamentally empirical in orientation, and predictive in intent (Krippendorff, 1980). As a methodology, content analysis enables the researcher to plan, to communicate, and to critically evaluate the research design independently of the results. Content analysis provides a basis for making inferences through the systematic and objective classification of specified characteristics within a text (Stone, Dunphy, Smith, & Ogilvie, 1966). In the current study, the characteristics of interest were the concepts of logic; the text under consideration was the examination(s) to be analyzed; and the inferences were made by individuals serving as judges, who used a four-category classification system to rate examination items for their relationship to logic.

When designing a content analysis study, the researcher must identify both the phenomena to be studied and potential sources of data. Any content analysis includes two kinds of reality, "the reality of the data and the reality of what the researcher wants to know about" (Krippendorff, 1980, p. 170). Since the two realities seldom map directly onto one another, the researcher must discover ways to analyze the available data so they are indicative of the phenomena of interest. Given a universe of appropriate data, the researcher must define:

- a sampling plan (e.g., every third book from a randomly ordered list of books),
- a method for breaking each sample into units (e.g., words, sentences, paragraphs, or chapters), and
- a coding or classification system that will be used to record information about the units.

While content analysis *can* be conducted by a single individual, usually it involves two or more *coders* or *judges*. As a means for anticipating and solving problems, testing and training are vital aspects of the planning process. Because insight gained during the design and training process may point out problems or better approaches, the design process will be iterative. Problems such as inconsistencies or missing records can emerge even in carefully planned studies. The content analysis proper begins only after the procedure is stable.

During the analysis phase, the researcher must evaluate the coding results for reliability and validity. Reliability techniques in content analysis are targeted toward evaluating agreement among judges and ratings. Several types of reliability can be calculated: overall reliability of the content analysis, item reliability (how consistent were the judges in rating the particular item?), single category reliability (how consistently was the content analysis classification system used?), and judge reliability (how well did each judge agree with the remaining judges?).

In content analysis, the issue of validity is related to external validity; internal validity is simply another term for reliability. External validity concerns two phenomena: 1) how well the findings reflect the true phenomena in the context of the data, and 2) whether there is a correspondence between variations within the analysis process and variations that exist outside of the process.

This section concludes with an informal survey of studies that have used content analysis in their design. The source of this information is the dissertation abstracts reported in the ProQuest Dissertation Abstracts On Disc (January,

1993–February 1994). This survey is essentially a content analysis of the abstracts.

A keyword search using the phrase "content analysis" matched 362 dissertation abstracts in the database. Of these, six were eliminated from consideration because they described research in the natural sciences; in these studies, the content being analyzed was physical phenomena such as hair, water, or digestive tract. Of the remaining 356 abstracts, 111 were for degrees awarded in 1993, 211 were for 1992, 26 were for 1990, four were for 1990, three were for 1989, and one was awarded in 1984.

An average of 2.32 subject categories were listed for each abstract (a maximum of three subject categories could be given for an abstract). Table 2.3 summarizes the subject categories ordered by frequency of citation. Table 2.4 shows the breakdown into component categories of two of the most frequently named categories, Education and Psychology. These categories were chosen as examples for their close connections to the current study. Table 2.5 summarizes the data collection techniques that were used in the abstracts. Figure 2.4 is the synopsis of an example abstract related to the field of computer science (Murfin, 1993).

Table 2.3 Frequency of Subject Categories in Sample of Dissertation Abstracts using Content Analysis

subject category	# abstracts
education	289
sociology	87
psychology	85
health science	70
mass communication	52
political science	43
journalism	29
business administration	21
speech and communication	18
history	15
social work	14
women's studies	13
anthropology	8
law	8
language	7
literature	7
economics	4
geography	4
gerontology	4

subject category	# abstracts
library science	4
agriculture	3
cinema	3
engineering	3
music	3
recreation	3
religion	3
urban and community planning	3
black studies	2
computer science	2
home economics	2
information science	2
theater	2
American studies	1
art history	1
biographical	1
fine arts	1
philosophy	1
transportation	1

Table 2.4 Component Categories for Two Frequently Reported Subject Categories in Sample of Dissertation Abstracts using Content Analysis

subject category	**component categories**	**# abstracts**
Education	curriculum and instruction	43
	administration	37
	higher education	20
	teacher education and training	17
	elementary education	13
	psychology of education	12
	adult and continuing education	11
	general educational issues	11
	intercultural education	11
	guidance and counseling	10
	language and literature	10
	secondary education	10
	sociology of education	8
	special education	8
	early childhood education	7
	religious education	6
	social sciences education	6
	mathematics education	5
	reading education	5
	health education	4
	history of education	4
	educational technology	4
	tests and measurements	3
	art education	2
	business education	2
	community college	2
	industrial education	2
	music education	2
	philosophy of education	2
	physical education	2
	science education	2
	vocational education	2
	agricultural education	1
	finance of education	1
	home economics	1
Psychology	clinical	20
	social	18
	developmental	11
	personality	10
	general	7
	industrial	6
	behavioral	5
	experimental	3
	physiological	3
	psychometrics	2

Table 2.5 Source of Data or Method of Generating Data Reported in Sample of Dissertation Abstracts Using Content Analysis

source of data / method of generating data	# abstracts
official document	125
interview	98
newspaper	36
questionnaire	21
television	16
article	16
textbook	14
survey	13
literature	11
discourse	9
observation	8
advertising (magazine or TV)	6

source of data / method of generating data	# abstracts
electronic communication	5
magazine	4
reference book	3
visual (photographs)	3
arts (e.g. dance)	2
field notes	2
film	2
paper & pencil instrument	2
journal/reflective writing	2
music	2
radio	1
speech	1

Title: An Analysis of Computer-Mediated Communication between Urban Middle School Students and Scientists (Urban Education)

Author: Murfin, Brian Edward

School: The Ohio State University

Degree: Ph.D.

Date: 1993

Source: DAI-A 54/05, p. 1770, Nov 1993.

Subject categories: education, technology
education, sociology of
computer science

Purpose of study: determine the characteristics of effective and ineffective computer-mediated communication between urban middle school students and scientists.

Sample: 20 urban, middle school students and 10 adult scientists and non-scientists.

Units of study: An electronic bulletin board system (BBS) was used to link the scientists and students; 911 messages were posted on the BBS over a 10 week period.

Conclusions: content analysis of all messages revealed, among other things:

(1) the number of positive messages was greater than the number of neutral or negative messages
(2) the students mainly sent messages to only one individual and did not take advantage of the multiloguing [i.e., multiple people in dialogue] capability of computer-mediated communication
(3) non-science messages were more numerous than were science messages.

Figure 2.4 Synopsis of Example Dissertation Abstract in which the Design used Content Analysis

Chapter 3 Research Design

Chapter 3 presents the research design, elaborating, in the process, the research questions posed in Chapter 1. This study was exploratory and descriptive in nature. Preparatory activities included development of a taxonomy of relevant concepts and identification of appropriate computer science materials for analysis. Using a content analysis procedure, expert judges rated multiple-choice items using a four-category classification system. The classification system allowed each judge to indicate the degree of relationship between an item and the subdomain criteria (the taxonomy of concepts). The data that resulted from content analysis were used in two distinct analyses: (a) use of the strength ratings to assign the items to *strongly related* and *not strongly related* partitions and (b) determination of the reliability of the content analysis results. The partitions of items provided the basis for answering the research questions comparing performance data across the partitions.

3.1 IDENTIFYING CONCEPTS IN THE SUBDOMAIN LOGIC

The investigator developed a taxonomy of concepts to provide a concrete definition of concepts in the computer science subdomain of mathematical logic. The taxonomy outlined logic concepts that were particularly relevant to this research, thus focusing the subsequent research effort on a well-defined set of concepts.

The development of the taxonomy was an iterative process influenced by a wide variety of sources, including standard computing curriculum guidelines (e.g., Austing, 1979; Koffman, Miller, & Wardle, 1984; Koffman, Stempel, & Wardle 1985; Tucker, 1990), the way in which the concepts were presented in undergraduate textbooks (e.g., Dale & Walker, in press) and more advanced texts (e.g. Gries, 1981; Sperschneider & Antoniou, 1991), and the draft of an international standard for datatypes (ISO, 1994). During its development, the taxonomy was reviewed by several computer science educators to ensure its content validity.

The resulting taxonomy is a broad outline that provides a breakdown of advanced as well as basic concepts of logic. The taxonomy was labeled as containing concepts in the computer science subdomain "two-valued logic" in order to emphasize the sort of logic that was under consideration in the content analysis procedure.

As a pictorial synopsis of the concepts belonging to the subdomain of interest, the *Quick Reference to the Concepts of "Two-Valued Logic"* was designed to aid the judges in completing the classification task. Figure 3.1 is the quick reference guide that was provided to each judge who participated in the final phase of the content analysis procedure. The full taxonomy is given in Appendix C.

Figure 3.1 Pictorial Representation of the Taxonomy of Concepts in the Computer Science Subdomain of Two-Valued Logic

3.2 IDENTIFYING A SOURCE OF TEST ITEMS FOR ANALYSIS

This research depended on having a source of test items for which:

(a) the content tested by the examination covered beginning computer science concepts, including concepts in the subdomain of logic;

(b) the content of the examination items could be analyzed for strength of relationship to the concepts of logic;

(c) a sufficiently large sample of students had taken the examination(s) from which the items were drawn; and

(d) performance information was available for the sample.

The publicly available Advanced Placement Examinations in Computer Science (APCS examinations) appeared to meet all of these criteria. Because several thousand students take the examination each year, statistics about the examination outcome provide a broad base of information about the performance of novice computer science students.

The College Board's APCS examination is designed to test the material covered during the first two courses of the post-secondary computer science curriculum, generally referred to as "CS1" and "CS2" (Austing, 1979; Koffman, Miller, & Wardle, 1984; Koffman, Stempel, & Wardle 1985). The full topic outline for APCS courses is given in Appendix B. Figure 3.2 summarizes the topics from the APCS outline that most nearly match those in the taxonomy of logic defined for this research. This extract is somewhat generous, in that it

includes several topics that will *sometimes* be related to logic, rather than topics that are *only* related to logic.

Each of the APCS examinations was made up of two parts, a multiple-choice section and a free-response section. Each multiple-choice section included from 35 to 50 items while the free-response section included three or five items. A multiple-choice item comprised a problem statement and five alternative responses, one of which was correct. Each free-response item presented a problem description or specification for which the respondent was to write all or part of an algorithm or computer program. Scoring of the multiple-choice items was done using a mechanical scanning and scoring system. The free-response section was scored manually by teams of computer science educators using scoring rubrics.

Due to the laws in several states, the free-response items must be made publicly available annually. However, ETS is only required to disclose the multiple-choice questions every fourth year. As a result, the full APCS examinations were available for the years 1984, 1988, and 1992. For each of these three years, Educational Testing Services (ETS) published a report that included the full text of all items, the correct answers for the multiple-choice section, grading rubrics for the free-response items, and selected statistical analyses (College Board, 1986, 1989, 1993).

Files obtained from ETS included anonymous individual data for all candidates taking the examinations under consideration. For each respondent, the following information was included: demographic information (gender and

ethnicity), response to each multiple-choice item, the score on each free-response item, and overall APCS grade.

While the sampling represented by the APCS examinations was extensive, it was also self-selective. Only some students took the classes that prepared them to take the examination and, of these students, only some chose to take the APCS examination. Due to the stated purpose of the AP program, a majority of the students who took the examination were college-bound (College Board, 1990).

A. Programming methodology
    1. Specification
        b. Program and subprogram specifications (e.g., pre- and postconditions)
    2. Design
    3. Implementation
        b. Program correctness
            i. Testing and debugging
                A. Reasoning about programs
                B. Assertions
                C. Invariants
            ii. Verification
B. Features of block-structured programming languages
    1. Type and constant declarations
        b. Simple data types
        c. Structured data types
    3. Expressions and evaluation
    4. Assignment statements
    5. Control structures
        b. Conditional execution
        c. Loops
    7. Subprograms
C. Fundamental data structures

Figure 3.2 Topics from the APCS Outline that Correspond to Concepts in Taxonomy of Concepts in the Computer Science Subdomain of Logic

The APCS data used in this study comprised

- four distinct examination packets (1984, 1988, 1992A, 1992AB), and
- five distinct samples (1984, 1988A, 1988AB, 1992A, 1992AB).

The examination packets were used in the content analysis procedure, while the five samples of respondents were used in analyzing performance.

3.3 THE CONTENT ANALYSIS PROCEDURE

This research design adapted methods of content analysis to classify each multiple-choice item from the four APCS examination packets for strength of relationship to logic. The content analysis techniques were based on those described by Krippendorff (1980) in *Content Analysis: An Introduction to Its Methodology*. This book describes the philosophy, sampling techniques, and methods of validation and analysis to be used in designing and carrying out a content analysis procedure.

The most common use of content analysis in the past has been to analyze written text or recorded conversation for the occurrence and meaning of certain words, phrases, and ideas. This study adapted the procedure to allow the judges to rate each APCS multiple-choice item according to how strongly it was related to the subdomain of logic. The content analysis procedure is detailed in the sections that follow.

3.3.1 The taxonomy of concepts as a guideline

The taxonomy of concepts in the subdomain of two-valued logic was initially developed as an outline of topics (see Appendix C). During pre-pilot testing of the content analysis procedure, it was determined that, in outline form,

the taxonomy was too detailed for easy use. The top levels of the taxonomy were extracted and reorganized into a chart that fit on a single page. The taxonomy chart, given earlier in Figure 3.1, was called the *Quick Reference to the Concepts of "Two-Valued Logic"*. The philosophy behind the quick reference guide was to lay out the concepts of logic in an accessible format for easy reference. This provided judges with explicit guidelines of which concepts were considered to be within and outside of the subdomain of interest; it appealed to their understanding of the subdomain and clarified the purposes of the content analysis task.

3.3.2 The units to be classified

Because each free-response item on an APCS examination tends to encompass a wide variety of concepts and skills, isolating the role each component concept plays in the item as a whole is difficult. In contrast, the multiple-choice items are more narrowly focused on a few concepts but are still sufficiently complex to make the performance data significant. As a result, only the multiple-choice items were considered during the content analysis procedure.

The pilot phase used two examination packets, those for 1984 and 1988. The final content analysis phase considered all items from the four examination packets for 1984, 1988, 1992A, and 1992AB.

3.3.3 The content analysis classification system

The classification system in Table 3.1 was used by individual judges to associate each item with one of the categories 'main concept', 'vital subconcept', 'trivial subconcept', and 'not used'. Several changes made to the classification system over the course of its development led to an ordered system with nice

Table 3.1 Classification System for Indicating Strength of Relationship Between APCS Multiple-Choice Items and the Subdomain Under Study

main concept	The item deals directly with the concepts of two-valued logic.
vital subconcept	In this item, one or more concepts of two-valued logic are important subconcepts but not the primary focus.
trivial subconcept	In this item, one or more concepts of two-valued logic appear but have little bearing on the solution.
not used	The concepts in this item bear no meaningful relationship to the subdomain of two-valued logic.

symmetry among categories, since each category focused directly on the notion of strength of relationship to concepts. The range of values was from 'main concept' as the strongest relationship down to 'not used', where no relationship to the subdomain existed. The taxonomy of concepts, described in an earlier subsection and given in Figure 3.1, assisted the judge in determining whether the item included any logic concepts. If not, the item was rated 'not used'; otherwise, the judge had to determine whether to rate the item as having logic as a 'trivial subconcept', a 'vital subconcept', or a 'main concept'.

3.3.4 The judges

The judges for both the pilot and final content analysis phases were selected for their expertise in computer science education. Each judge, also referred to in this dissertation as an expert, was in some way involved with the introductory computing curriculum. Most judges were instructors of beginning computer science at the pre-college or post-secondary level, many recruited from the pool of readers (graders) for the free-response items on the 1993 APCS

examinations. Another group of judges was recruited from among the participants in an NSF-sponsored workshop about using formal methods in the introductory computer science sequence[5]. Several of the judges were authors of introductory computer science textbooks.

3.3.5 Pilot phase of the content analysis procedure

To test the content analysis procedure, several pilot runs were conducted. The pilot runs considered only the multiple-choice items from the 1984 and 1988 examinations because the 1992 APCS examinations were not available at that time. Experience gained during the pilot runs led to creation of the quick reference guide (Figure 3.1), refinements to the classification system, and major simplification of the content analysis procedure. The pilot data allowed refinement of the reliability analysis and other statistical procedures. The results from the pilot phase provided preliminary evidence supporting the hypothesis that novice computing students tend to have greater difficulties with items closely related to the concepts of logic than they generally have with items dealing with other concepts.

3.3.6 Final phase of the content analysis procedure

In the final content analysis procedure, each judge received the following materials: (1) a cover letter with instructions (see Appendix D), (2) the quick reference guide, which provided an overview of the taxonomy of two-valued logic concepts (see Figure 3.1), (3) a table with the correct answers to all items in the

---

5 NSF UCC grant USE9156008, "Program Derivation for First-Year Computing Students", Walter M. Potter (principal investigator), June 3-5, 1993, Southwestern University, Georgetown, TX.

four examination packets, (4) the four examination packets, (5) four copies of the coding form (see Appendix E), and (6) a stamped envelope addressed to the researcher for return of the completed forms.

Judges completed the rating task at their convenience. A key requirement was that they not discuss their rating of the APCS examination items with anyone until after the task was complete. This constraint avoided the situation where two judges, by collaborating, produced a single combined rating rather than two individual ratings. This constraint also reduced the likelihood of inconsistencies that could arise in a judge's ratings because of outside influences.

A total of 150 forms were returned: 38 for the 1984 examination packet, 36 for the 1988 examination packet, and 38 each for the two 1992 examination packets. With few exceptions, the forms were complete and filled in correctly. In the few instances where a judge had inadvertently omitted the rating for an item, the researcher contacted the judge directly for the missing rating.

3.4 THE DATA

The ratings that resulted from the final content analysis procedure, together with the five APCS data sets from ETS, provided the basis for studying student performance on the examinations. The content analysis ratings identified the APCS items most relevant to informing this research, while the ETS data provided information about student performance on all of the multiple-choice items. Table 3.2 summarizes the data and the sources of each. The following subsections clarify the data.

3.4.1 The ETS data sets

Educational Testing Service (ETS) provided files containing anonymous data for every individual who took the five examinations under consideration. The data that was relevant to this research was the response on each multiple-choice item (or an indication that the item was omitted).

In analyzing the response data, several additional variables were developed. For every respondent, the following variables were calculated:

(1) For each multiple-choice item, a dichotomous indicator of whether the item was answered correctly.

(2) For each multiple-choice item, a dichotomous indicator of whether the item was omitted by the student (i.e., no answer was given).

(3) The total number of multiple-choice items answered correctly.

Item (3) differed from the multiple-choice score reported by Educational

Table 3.2 Data Used in Study and Source from which Obtained or Derived

	Educational Testing Services Data Sets	**Content Analysis Procedure**
per test	• number of multiple-choice items • number of respondents in individual data files	• number of judges • classification categories
per multiple-choice item	• correct choice • frequency of each of the five answer choices being selected by a respondent • proportion of respondents answering item correctly • proportion of respondents omitting item	• rating assigned by each judge • summary of ratings, that is, the frequency of each category • indication of partition under each partitioning algorithm
per respondent	• response for each multiple-choice item or indication that item was omitted • number of items answered correctly on the multiple-choice portion • number of items attempted on the multiple-choice portion • proportion of multiple-choice items answered correctly	• number of correctly answered items in each partition under each of the partitioning algorithms • number of items attempted in each partition under each of the partitioning algorithms

Testing Services, which factored in a penalty for incorrect answers. The ETS score was calculated as Number Correct – (1/4 * Number Wrong), where Number Wrong was the number of multiple-choice items that were attempted but incorrectly answered.

3.4.2 The content analysis data

The content analysis data included the number of judges as well as information about the classification system (the number of categories and a code for each). The ratings from the individual judges were accumulated into a ratings summary table showing the frequency of occurrence of each category for each item. The ratings summary table provided the basis for the reliability calculations as well as for the partitioning algorithms described in the following subsection.

3.4.3 Partitioning of multiple-choice items

The partitioning procedure created two sets of multiple-choice items, one for each of the two extremes of relationship to the concepts of logic. Assignment of an item to a partition depended on the cumulative ratings from the content analysis procedure. To be assigned to the *strongly related* partition, an item had to be rated as either 'main concept' or 'vital subconcept' by a majority of the judges. To be put into the *not strongly related* partition, an item had to be rated as either 'trivial subconcept' or 'not used' by a majority of the judges.

Two different partitioning algorithms were used to assign each item to the appropriate partition. As a result, two different pairs of partitions were considered.

(1) In the **liberal** partitioning algorithm, a given item was put in the *strongly related* partition if at least 50% of the judges had rated the item as either 'main concept' or 'vital subconcept'. The item was put in the *not strongly related* partition if fewer than 50% of the judges had rated the item as either 'main concept' or 'vital subconcept' (i.e., at least 50% of the judges had rated the item as either 'trivial subconcept' or 'not used').

(2) In the **conservative** partitioning algorithm, a given item was put in the *strongly related* partition if at least 75% of the judges had rated it as either 'main concept' or 'vital subconcept'. The item was put in the *not strongly related* partition if at most 25% of the judges had rated it as either 'main concept' or 'vital subconcept' (i.e., more than 75% of the judges had rated it as either 'trivial subconcept' or 'not used').

The rationale for using two different partitioning algorithms was as follows: In the liberal partitioning algorithm, all items from all four examinations were assigned to one of the two partitions. The 50% level of agreement meant at least half of the judges concurred about the relatedness of the item. Under the conservative model, the items in the mid-range were considered to be noise: all items for which 25% to 75% of the judges rated the item as 'main concept' or 'vital subconcept' were eliminated from consideration. Eliminating the mid-range items compensated for chance choices by judges. This handled the problem of "borderline" items, those items that could shift between partitions based on a change in rating by only one or two judges. Thus, the partitions in the conservative model included only those items for which the cumulative rating of the item's relationship to logic was especially strong or weak.

After the partitions were established, four additional variables were developed for each respondent. These variables were the number of items answered correctly in each of the pairs of partitions, that is, (1) in the *strongly related* partition under the liberal algorithm, (2) in the *not strongly related* partition under the liberal algorithm, (3) in the *strongly related* partition under the conservation algorithm, and (4) in the *not strongly related* partition under the conservative algorithm. In carrying out the subsequent analyses, two parallel analyses were done in most instances, one for each of the two pairs of partitions.

3.5 ANALYSIS OF THE DATA

3.5.1 Analysis of data in ETS files

Summary data was developed for the data in the files from ETS. For each examination, the composition of the examination was considered and performance statistics were generated for the multiple-choice section. Multiple-choice items common to the A and AB versions of the 1988 and 1992 examinations were compared across versions for differences in profiles and performance.

3.5.2 Reliability of the content analysis results

The reliability of the content analysis results was evaluated using a suite of analysis procedures based on Krippendorff (1980). Because the author found no pre-existing software for calculating the reliability figures in a straightforward and easily interpretable fashion, she designed and implemented her own software package. The statistics from this package included overall reliability, the

reliability of each category in the classification system, and individual judge reliability.

In content analysis, reliability is expressed as the amount of agreement that exists among the ratings given by all of the judges. Agreement is calculated using the formula $\alpha = 1 - D_o / D_e$, where α is the agreement coefficient, D_o is the observed disagreement, and D_e is the expected disagreement.[6] The agreement coefficient is interpreted as the reliability of the rating, where a value of 1.0 means "perfect agreement", 0.0 means "agreement entirely due to chance", and –1.0 means "perfect disagreement".

3.6 RESEARCH QUESTIONS AND METHODS OF ANALYSIS

Up to this point, only the procedure for investigating the first research question posed in Chapter 1 has been presented. This question was:

(a) Can a procedure be developed for reliable and valid classification of content-area test items according to their degree of relationship to a pre-defined set of logic concepts?

Due to the nature of this question, no null hypothesis was formulated for statistical analysis. The positive answer to the research question was based on the proof of concept that came from implementing the content analysis procedure and generating the *strongly related* and *not strongly related* partitions of items.

Given the positive result for research question (a), the remaining research questions could be considered. These questions were the following:

---

[6] Expected disagreement is calculated as the average difference within all possible pairings of ratings, both across judges and across items. It is based on the null hypothesis that any differences in ratings are attributable only to chance. See Krippendorff (1980) for a complete description.

(b) In considering student performance on the test items, was the distribution of performance different for items whose content was strongly related to logic than for items whose content was not strongly related to logic?

(c) Was there a relationship between individual performance on the set of items whose content was strongly related to logic and individual performance on the set of items whose content was not strongly related to logic?

Both of these research questions were informed by development of simple descriptive statistics (mean and standard deviation) describing the performance on each partition. The cumulative score on each partition of items was rescaled to reflect the proportion of correctly answered items in the partition. This allowed comparison of performance under each of the partitioning algorithms, across the examination packets, and within each of the five samples.

Question (b) was addressed in terms of the difficulty distribution of the items in each partition. The operational definition for item difficulty was based on the proportion of students who answered the item correctly. Five mutually exclusive categories of difficulty were defined in terms of the proportion of respondents who answered correctly: 'very difficult' for the interval [.0, .2), 'somewhat difficult' for the interval [.2, .4), 'average' for the interval [.4, .6) 'somewhat easy' for the interval [.6, .8), and 'very easy' for the interval [.8, 1.0]. Every test item was assigned to exactly one of these five categories. The null hypothesis was then stated as:

H_1: The difficulty distributions for the partitions of items *strongly related* and *not strongly related* to logic are the same.

To test this hypothesis, a 5X2 table was created in which the rows reflected the difficulty categories, the columns represented the *strongly related* and *not strongly related* partitions, and the value in each cell was the number of items that fit the corresponding row/category criteria. The difficulty distributions defined by the two sets of items were graphed and tested for differences using a hierarchical log-linear test of homogeneity. This analysis was carried out for the item partitions as defined under both the liberal and the conservative partitioning algorithms.

Research question (c) was addressed by considering the null hypothesis:

H_2: The correlation between individual performance on items in the *strongly related* partition and items in the *not strongly related* partition is zero.

To test this hypothesis, the Pearson product-moment coefficient of correlation was calculated, treating individual cumulative score on items in the *strongly related* partition as the first variable and individual cumulative score on items in the *not strongly related* partition as the second variable. Correlation coefficients were developed for the partitions defined under both the liberal and the conservative partitioning algorithms. To aid in interpreting the correlation coefficients, the coefficient of determinacy was calculated, which allowed consideration of the shared variance between the two variables.

Chapter 4 Findings

Chapter 4 begins with a description of the source from which the test items were taken, the Advanced Placement Examinations in Computer Science (APCS examinations). Brief background on the composition of each examination and the associated performance statistics is provided. The outcome of the content analysis procedure is considered from several points of view: the instruments under study (i.e., the APCS examinations), the judges, the partitioning of the multiple-choice items, and the reliability of the content analysis results. The next section presents the results of hypothesis testing for the null hypotheses defined in Chapter 3. Chapter 4 closes with a summary of the research questions and findings.

4.1 SOURCE OF TEST ITEMS: THE APCS EXAMINATIONS

4.1.1 Composition of the APCS Examinations

The 1984 APCS examination was the first offering of an Advanced Placement examination for the subject area computer science; it included 44 multiple-choice items and five free-response items. The 1988 and 1992 APCS examinations were each administered in two different versions, A and AB. The A version for both years covered the concepts of the first computer science course (CS1 in Austing, 1979 and Koffman, Miller, & Wardle, 1984) while the AB version also covered the concepts of the second course (CS2 in Austing, 1979 and Koffman, Stempel, & Wardle, 1985). Version A of the 1988 APCS examination was a simple subset of the AB version: it encompassed the first 35 multiple-

choice items from the AB examination and the first three free-response items from the AB examination. Version AB of the 1988 examination included a total of 50 multiple-choice items and five free-response items. In 1992, the A and AB versions of the APCS examination were distinct (although not disjoint), once again with the A version at the CS1 level and the B version at the CS2 level. The two versions of the 1992 examination shared 15 multiple-choice items and two free-response items, with a total of 40 multiple-choice items and five free-response items on each version.

4.1.2 Performance Statistics

The data reported here are based on files received from Educational Testing Service (ETS). These files included anonymous individual performance and demographic data for all candidates taking the five examinations under consideration. In the official ETS reports (College Board, 1986, 1989, 1993), the statistics for the 1984, 1988, and 1992 examinations were based on only respondents for whom the examination was fully graded before pre-publication analyses were carried out. A small number of respondents were added to the data set after the official ETS reports were published. As a result, in some cases the total number of respondents reported in this research exceeds the number of respondents reported in the official ETS reports (by 22 and one respectively on the A and AB versions of the 1988 examination and by two on the A version of the 1992 examination).

Based on the information in the individual data files from ETS, Tables 4.1 and 4.2 summarize the descriptive statistics for each of the five samples of students who took the APCS examinations. Table 4.1 shows the number of multiple-choice items answered correctly, while Table 4.2 reports the number of multiple-choice items attempted. The descriptive statistics include the number of items under consideration on each examination, the mean value of the variable

Table 4.1 Summary Statistics for Number of Multiple-Choice Items Answered Correctly on Each APCS Examination

examination	*# of multiple-choice items*	*minimum # correct*	*maximum # correct*	*mean*	*standard deviation*	*N*
1984	44	3	44	25.59	8.23	4227
1988A	35	1	34	12.96	5.25	3369
1988AB	50	1	49	26.20	8.75	7375
1992A	40	2	40	19.73	7.26	5231
1992AB	40	0	40	21.74	7.65	4658

Table 4.2 Descriptive Statistics for Number of Multiple-Choice Items Attempted on Each APCS Examination

examination	*# of multiple-choice items*	*minimum # attempted*	*maximum # attempted*	*mean*	*standard deviation*	*N*
1984	44	8	44	41.28	3.73	4227
1988A	35	5	35	27.06	5.44	3369
1988AB	50	5	50	42.24	6.37	7375
1992A	40	6	40	35.04	4.97	5231
1992AB	40	0	40	34.41	5.40	4658

Table 4.3 Rescaled Descriptive Statistics for Number of Multiple-Choice Items Answered Correctly and Number of Multiple-Choice Items Attempted on Each APCS Examination

APCS exam	*mean proportion of items answered correctly*	*standard deviation of proportion of items answered correctly*	*mean proportion of items attempted*	*standard deviation of proportion of items attempted*
1984	0.58	0.19	0.94	0.08
1988A	0.37	0.15	0.77	0.16
1988AB	0.52	0.18	0.84	0.13
1992A	0.49	0.18	0.88	0.12
1992AB	0.54	0.19	0.86	0.14

(i.e., either number correct or number attempted) across all respondents, the standard deviation, the range (minimum and maximum values), and the number of respondents in the sample. Table 4.3 presents the means and standard deviations from Tables 4.1 and 4.2 rescaled to the range [0.0,1.0].

Two different views of performance can be developed from this information: (1) the proportion of respondents from the total sample who answered each item correctly and (2) the proportion of respondents who answered each item correctly among those who attempted the item. Each view has certain advantages; the research reported here used the first figure. This choice was made for two reasons: (a) analyses for a particular examination were based on a constant sample size across all items from that examination and (b) the fact that an item was not attempted provided information about the perceived difficulty of the item. In contrast, ETS used the second view of performance, the proportion of respondents who answered the item correctly among those who attempted the item. Because ETS assesses a "guessing penalty" on the multiple-choice section, many students undoubtedly skipped items about which they were unsure in order to avoid losing points. In addition, since each section of the examination was given during a fixed period of time, some students may have run out of time before completing all items on a section. Thus, students who worked more slowly may not have had sufficient time to answer some items even though in other circumstances they could have answered correctly. The second view takes this time constraint into consideration.

4.2 CONTENT ANALYSIS PROCEDURE RESULTS

The outcome of the final phase of the content analysis procedure is reported in this section. A pilot phase was also run; these results will be discussed as appropriate.

4.2.1 The instruments under study

The final content analysis procedure considered four distinct examination packets: 1984, 1988, 1992A, and 1992B. Although the 1988 examination was administered in two versions to two different samples of students, the multiple-choice section of version A of the 1988 examination was a subset of the items from version AB (i.e., 35 of 50 items appeared on both the A and AB versions). As a result, the content analysis procedure considered only the AB examination packet for 1988. On the 1992 examination, 15 items on the A and AB versions were identical; the packets were not modified to remove this redundancy.

The four examination packets included a total of 174 multiple-choice items. Eliminating the second occurrence of the 15 duplicate items from the 1992 examination (i.e., counting each one only once) resulted in a total of 159 distinct multiple-choice items that were considered in the final phase. The pilot phase of the content analysis procedure used only the 1984 and 1988 APCS examinations, for a total of 94 multiple-choice items.

4.2.2 The judges

The final content analysis procedure was completed by 38 computer science educators during the period from June through November, 1993. Two of the judges did not rate the items from the 1988 examination, so the number of

judges for the 1988 APCS examination was 36. Of these judges at the time of the content analysis:

- 10 were high school teachers
- 25 were instructors at a college or university
- 6 had completed the rating task during both the pilot and final phases
- 21 had served as readers in grading the 1992 APCS free-response items
- 11 had participated in an NSF-sponsored workshop on the use of formal methods in the undergraduate curriculum during summer 1993
- several had written introductory textbooks on various subjects in the field of computer science
- two had been recognized as "Outstanding Educator of the Year" by the Association for Computing Machinery's Special Interest Group on Computer Science Education (ACM SIGCSE)
- one had received the Turing Award, the highest honor available in the computing field (considered analogous in prestige to the Nobel Awards)

4.2.3 Partitioning the items during the final phase

A key goal of this study was to contrast performance on items judged to have a strong relationship to the subdomain of logic with performance on items that were judged to have little or no relationship to logic. During the final phase of the procedure, each judge rated each item with one of four categories that indicated the perceived strength-of-relationship between the concepts covered by the item and the concepts of logic. The categories were 'main concept', 'vital subconcept', 'trivial subconcept', and 'not used'. A rating of 'main concept' or

'vital subconcept' for an item indicated that the judge perceived logic to be an important aspect of that item; a high occurrence of these two categories caused the item to be assigned to the *strongly related* partition. Similarly, 'trivial subconcept' and 'not used' indicated a perceived unimportance of the concepts of logic in the context of the item; a high occurrence of these two categories caused the item to be assigned to the *not strongly related* partition.

Two different algorithms were used in creating the partitions of items. The *liberal partitioning algorithm* classified an item as *strongly related* if at least 50% of the judges had rated the item 'main concept' or 'vital subconcept' and as *not strongly related* otherwise. Under the liberal partitioning algorithm, all items fell into one of the two partitions. The *conservative partitioning algorithm* used stricter criteria for assigning items to partitions, with the result that a number of items were excluded from consideration during further analysis. In the conservative algorithm, an item was classified as *strongly related* if 75% or more of the judges had rated the item 'main concept' or 'vital subconcept'. In order to be put in the *not strongly related* partition under the conservative algorithm, at least 75% of the judges must have rated the item as 'trivial subconcept' or 'not used'. Table 4.4 summarizes the number of items in each partition under the two partitioning algorithms for each of the five examination versions. Appendix F presents the detailed results from the content analysis judging, including the number of judges who chose specific categories for each item and the assignment of individual items to partitions under both the liberal and conservative partitioning algorithms.

Sample APCS examination items are given in Figures 4.1 through 4.4. Figures 4.1 and 4.2 show items that 37 out of 38 judges (97%) rated as either 'main concept' or 'vital subconcept'; these items were assigned to the *strongly related* partition under both the liberal and the conservative partitioning algorithms. Figure 4.3 shows an item that 74% of the judges rated as either 'main concept' or 'vital subconcept'; this item was assigned to the *strongly related* partition under the liberal partitioning algorithm but was eliminated from consideration under the conservative partitioning algorithm. Figure 4.4 shows an item that only one out of 38 of the judges (3%) rated as either 'main concept' or 'vital subconcept'; this item was assigned to the *not strongly related* partition under both the liberal and the conservative partitioning algorithms. Appendix G presents the 22 multiple-choice items that were classified as *strongly related* under the conservative partitioning algorithm.

Table 4.4 Summary of Number of Items in Each Partition under Each Partitioning Algorithm for Each APCS Examination

APCS examination and version	liberal partitioning algorithm			conservative partitioning algorithm		
	strongly related	*not strongly related*	total # of items	*strongly related*	*not strongly related*	total # of items
1984	10	34	*44*	4	30	*34*
1988A	11	24	*35*	4	17	*21*
1988AB	14	36	*50*	5	29	*34*
1992A	16	24	*40*	8	13	*21*
1992AB	14	26	*40*	9	18	*27*

Note: The A version of the 1988 examination consists of the first 35 multiple-choice items from the AB version of the 1988 examination.

Consider the following program fragment:

```
i := 1 ;
while (i <= Max) and (String[i] <> Symbol) do i := i + 1
```

Which of the following is a loop invariant for the `while` loop above; i.e., which is *true* each time the `while`-condition is tested?

(A) `i = Max`

(B) `i = i + 1`

(C) `String[j] = Symbol` for all `j` such that `i < j`

(D) `String[j] ≠ Symbol` for all `j` such that `i ≤ j`

(E) `String[j] ≠ Symbol` for all `j` such that `1 ≤ j < i`

Note: Appeared as item 42 on the 1984 APCS examination; Correct response is (E); From *The Entire 1984 AP Computer Science Examination and Key*, College Entrance Examination Board, 1986, p. 27. Adapted by permission.

Figure 4.1 Sample Multiple-Choice Item, Rated *Strongly Related* by 37 of 38 Judges (97%)

Evaluation of the *Boolean* expression

```
((i <= n) and (a[i] = 0)) or ((i >= n) and (a[i - 1] = 0))
```

is *guaranteed* to cause a run-time error under which of the following conditions?

(A) `i < 0`

(B) Neither `a[i]` nor `a[i - 1]` has the value zero.

(C) Array `a` is of size `n`.

(D) Array `a` is of size 2.

(E) None of the above

Note: Appeared as item 27 on version A and as item 17 on version AB of the 1992 APCS examination; Correct response is (E); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, pp. 27 & 63. Adapted by permission.

Figure 4.2 Sample Multiple-Choice Item, Rated *Strongly Related* by 37 of 38 Judges (97%)

The purpose of a subprogram's precondition is to

(A) initialize the local variables of the subprogram

(B) describe the conditions under which the compiler is to abort compilation

(C) describe the conditions under which the subprogram may be called so that it satisfies its postcondition

(D) describe the algorithm used by the subprogram

(E) describe the effect(s) of the subprogram on its postcondition

Note: Appeared as item 4 on version AB of the 1992 APCS examination; Correct response is (E); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, p. 54. Adapted by permission.

Figure 4.3 Sample Multiple-Choice Item, Rated *Strongly Related* by 26 of 38 Judges (74%)

A standard Pascal compiler runs on several different types of computers, ranging from microcomputers to mainframes. For this compiler, which of the following might be different on the different machines?

I. The value of *maxint*

II. The number of reserved words

III. The maximum size of a set

(A) I only

(B) III only

(C) I and II

(D) I and III

(E) II and III

Note: Appeared as item 7 on both version A and AB of the 1992 APCS examination; Correct response is (D); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, pp. 10 & 55. Adapted by permission.

Figure 4.4 Sample Multiple-Choice Item, Rated *Strongly Related* by 1 of 38 Judges (3%)

4.2.4 Content analysis reliability results

The content analysis reliability procedures were run for each of the four examination packets. For each packet, these calculations resulted in an overall reliability value, a single category reliability value for each of the four classification categories, and individual judge reliability.

4.2.4.1 Overall reliability

In producing the overall reliability figures two different models were considered, the four-category model and the two-category model. In the four-category model, the distinct classification categories were maintained, giving the categories 'main concept', 'vital subconcept', 'trivial subconcept', and 'not used'. The two-category model was based on two collapsed categories: the classification categories 'main concept' and 'vital subconcept' became the single collapsed category *strongly related*, while the classification categories 'trivial subconcept and 'not used' became the single collapsed category *not strongly related.*

The agreement coefficients for the overall reliability of the content

Table 4.5 Agreement Coefficients Showing Overall Reliability of the Content Analysis of the APCS Examinations

APCS examination	liberal partitioning algorithm		conservative partitioning algorithm	
	with all four categories	*with collapsed categories*	*with all four categories*	*with collapsed categories*
1984	0.325	0.458	0.290	0.473
1988 A & AB	0.310	0.434	0.303	0.496
1992A	0.317	0.405	0.458	0.704
1992AB	0.340	0.494	0.391	0.694

analysis results are given in Table 4.5. Comparing the four-category and two-category models, the two-category interpretation consistently resulted in much higher agreement. The conservative partitioning algorithm also tended to result in higher agreement than did the liberal partitioning algorithm. The higher agreement in the conservative partitioning algorithm can be explained by the fact that "noisy" items were eliminated from consideration. The best reliability was for both versions of the 1992 examination using the two-category model and the conservative partitioning algorithm; here the agreement coefficient was .70, while for the 1984 and 1988 examinations the reliability under this same combination was less than .50.

4.2.4.2 Single category reliability

Single category reliability indicates the extent to which a category tends to distinguish itself from the other categories. A low agreement coefficient for a particular category means that judges tended to confuse that category with the remaining categories.

Table 4.6 gives the agreement coefficients for the four classification categories under both partitioning algorithms. The categories 'vital subconcept'

Table 4.6 Agreement Coefficients for Single Category Reliability of the Final Phase of the Content Analysis of the APCS Examinations

	liberal algorithm				conservative algorithm			
APCS examination	*main concept*	*vital subconcept*	*trivial subconcept*	*not used*	*main concept*	*vital subconcept*	*trivial subconcept*	*not used*
1984	0.331	0.199	0.119	0.527	0.415	0.156	0.130	0.419
1988	0.292	0.201	0.105	0.518	0.421	0.198	0.116	0.434
1992A	0.457	0.132	0.148	0.533	0.645	0.161	0.164	0.602
1992AB	0.630	0.145	0.160	0.451	0.704	0.101	0.147	0.452

and ‘trivial subconcept’ tended to be highly unreliable, with agreement coefficients that were at most .20. The ‘not used’ category had the highest reliability, with agreement coefficients ranging from .45 to .53 when considering the partitions defined by the liberal algorithm (which included all items) and from .42 to .60 when considering the partitions defined by the conservative algorithm (which included a reduced set of items). The ‘main concept’ category was relatively reliable, with a range of .29 to .63 for the agreement coefficient when considering the partitions defined by the liberal algorithm and from .41 to .70 when considering the partitions defined by the conservative algorithm.

During the pilot phase, the classification system was composed of four categories that were unordered: ‘direct relationship’, ‘vital subconcept’, ‘parallel concept’, and ‘not related’. After the classification system was refined for use in the final phase, the new set of categories were ordered along a scale that ranged from ‘main concept’ as the most strongly related category, moving down the scale to ‘vital subconcept’, then to ‘trivial subconcept’, and, at the bottom of the scale, ‘not used’. Krippendorff (1980, p. 151) maintained that single-category reliability can only be applied to unordered categories. However, because the single-category reliability results obtained here are fully interpretable, this restriction seems unnecessary.

4.2.4.3 Individual judge reliability

Individual judge reliability indicates the extent to which an individual judge was the source of unreliable data (Krippendorff, 1980). Differences among judges can be explained by work style (e.g., organized, neat, hurried), in

understanding of the instructions, in consistency of understanding of the concepts under consideration, and in the way the items themselves are perceived.

Judge reliability was calculated as the agreement between a particular judge and the pooled set of all other judges. Appendix H presents an overview of individual reliability results for the final phase of the content analysis procedure. Generally, individual judge reliability was much higher under the conservative partitioning algorithm than under liberal partitioning algorithm. This difference was fairly insignificant for the 1984 examination, but was pronounced for the 1988 and 1992 examinations.

An important issue relates to "outliers": should judges whose individual reliability deviated greatly from that of the other judges have been eliminated from further consideration? As Krippendorff (1980) points out, this practice needs to be treated with caution:

> [A] practice, assertedly aimed at bypassing problems of unreliability, is to take averages or majority judgments as true values whenever disagreements among independent observers are encountered. Such data do contain evidence about reproducibility *before* the "undesirable" variance is eliminated. But, since disagreement implies nothing about who is right and who is wrong, neither the mean nor the mode has the wisdom required to improve data reliability by computational means. An even more deceptive practice is to admit only those data to an investigation on which independent coders achieve perfect agreement. The bias here is two-fold. The procedure does not prevent chance agreements from entering the data … and it biases the data toward what is easily codable. (p. 132)

However, Krippendorff does concede that, if a particular judge proves to be very unreliable, the data contributed by that judge "could be removed, checked, or recoded by the other … coders [and] data reliability would improve …" (p. 150). The decision in this study was to retain all judges.

4.2.4.4 Test-retest results for items common to A and AB versions

Several items appeared on both the A and AB versions of the 1988 and 1992 APCS examinations. In 1988, the two versions of the examination had 35 multiple-choice items in common, while in 1992 the two versions had 15 multiple-choice items in common.

In the content analysis procedure, the items for both versions of the 1988 examination were judged in a single packet, where the first 35 multiple-choice items made up the A version and an additional 15 items completed the AB version. As a result, each duplicate item on the 1988 examination was considered only once in judging, as part of the 1988 examination packet. In contrast, the 1992 examination was judged in two separate packets, one for each of the two versions. This meant that, for each of the 15 duplicate items, judges were in essence asked to rate the item twice. This provided an opportunity to consider test-retest stability in the use of the classification system.

When the results of the content analysis are viewed in terms of proportion of judges agreeing on the assignment to partitions, it is possible to associate with each item a value in the range [0.0, 1.0], a scale for the "judged relationship to logic". On this scale, a value of 1.0 would mean that all of the judges agreed that the item belonged in the *strongly related* partition while a value of 0.0 would indicate that all of the judges agreed that the item belonged in the *not strongly related* partition. Table 4.7 shows the duplicate items from the 1992 examination sorted in decreasing order according to judged relationship to logic on the AB version of the examination. For each duplicate item and both versions of the

Table 4.7 Comparison of Content Analysis Ratings on Duplicate Items from A and AB Versions of the 1992 APCS Examination

item	**judged relationship to logic for A-version**	**judged relationship to logic for AB-version**	*item # on A-version of 1992 examination*	*item # on AB-version of 1992 examination*
1	1.00	1.00	1992A-32	1992B-21
2	1.00	1.00	1992A-26	1992B-16
3	0.97	0.97	1992A-27	1992B-17
4	1.00	0.95	1992A-31	1992B-18
5	0.74	0.68	1992A-08	1992B-04
6	0.74	0.68	1992A-20	1992B-06
7	0.42	0.34	1992A-29	1992B-29
8	0.42	0.32	1992A-10	1992B-10
9	0.39	0.32	1992A-09	1992B-09
10	0.29	0.29	1992A-05	1992B-05
11	0.03	0.03	1992A-07	1992B-07
12	0.08	0.11	1992A-02	1992B-02
13	0.08	0.08	1992A-28	1992B-28
14	0.05	0.05	1992A-37	1992B-37
15	0.05	0.05	1992A-38	1992B-38

Note: Items are sorted according to "judged relationship to logic" on AB version; on this scale, 1.0 means all judges agreed the item belonged in the *strongly related* partition; 0.0 means that all judges agreed the item belonged in the *not strongly related* partition

examination, the agreement value and the associated item number is given. Based on the criteria defined by the liberal and conservative partitioning algorithms, each of these 15 items was assigned to the same partition for both versions of the examination.

Differences in rating the same item on the two different versions could indicate any of several phenomena, including problems with the classification system, changes in the judges' perceptions as they proceeded through the rating task, and the influence that the order in which the examinations were considered on classification results. Some judges reported that they recognized the item

repetition and attempted to achieve consistency between versions. For a fully accurate indication of consistency in using the classification system, a judge would have to had to rate each duplicate item on the two versions of the examination without cross-referencing. Because, by their own admission, several judges did do cross-referencing as they noticed duplicate items, the consistency figures are inflated. However, even with cross-checking, none of the 38 judges succeeded in rating all 15 items consistently. This shows that various factors must have influenced the choice of categories from one packet to the other. Appendix I profiles details of the ratings, both in terms of how consistently each judge rated the 15 items and in terms of how consistently each of the 15 items were rated by the 38 judges.

4.3 RESEARCH QUESTIONS AND HYPOTHESIS TESTING

Based on the rating results from the content analysis procedure and the data from ETS, the answers to the research questions posed at the end of Chapter□I can now be considered.

4.3.1 Descriptive statistics for performance differential between partitions

The research question that drove the design of the methodology used in this study was:

(a) Can a procedure be developed for reliable and valid classification of content-area test items according to their degree of relationship to a pre-defined set of logic concepts?

The positive answer to this question was based on the successful implementation of the content analysis procedure and the ability to generate meaningful partitions of *strongly related* and *not strongly related* items.

Tables 4.8 through 4.10 report descriptive statistics for each of the five samples that took the APCS examinations. Table 4.8 summarizes the statistics for the full set of multiple-choice items for each sample, while Table 4.9 describes the partitions defined under the liberal partitioning algorithm and Table 4.10 considers the partitions defined under the conservative partitioning algorithm. In each of the tables and for each partition, the following statistics are given: the number of items, the average proportion of respondents who correctly answered the items in that partition, and the standard deviation of the proportion of respondents who correctly answered the items in that partition. In order to compare performance across partitions, Tables 4.9 and 4.10 also report a "delta" for the mean and a "delta" for the standard deviation of the two partitions for each of the five samples. In each case, the delta value was calculated as the difference between corresponding values for the *strongly related* partition and the *not strongly related* partition. A negative delta value indicates that the value in the

Table 4.8 Number of Multiple-Choice Items plus Mean and Standard Deviation of Proportion Answering Correctly for All Examinations

	number of items	mean proportion answering correctly	standard deviation of proportion answering correctly
1984	44	0.58	0.19
1988 version A	35	0.37	0.15
1988 version AB	50	0.52	0.18
1992 version A	40	0.49	0.18
1992 version AB	40	0.54	0.19

strongly related partition was less than the corresponding value in the *not strongly related* partition.

The general trend that emerged was that the *strongly related* partitions had lower means and smaller standard deviations than the corresponding *not strongly related* partitions had. One exception to this was the 1984 sample, where under both the liberal and the conservative partitioning algorithms, the standard deviation of the *strongly related* partition was larger than the standard deviation of the *not strongly related* partition. The other exception to this trend was for the partitions defined by the liberal partitioning algorithm for both versions of the 1992 examination. On both the A and AB versions in 1992, the mean proportion of respondents answering correctly was higher for the *strongly related* partition than for the *not strongly related* partition. However, the standard deviation of the *strongly related* partition was still smaller for this case.

4.3.2 Differences in difficulty distribution between partitions

Research question (b) asked: In considering student performance on the test items, was the distribution of performance different for items whose content was strongly related to logic than for items whose content was not strongly related to logic? This question was tested using the following null hypothesis:

H_1: The difficulty distributions for the partitions of items *strongly related* and *not strongly related* to logic are the same.

In testing this hypothesis, the analysis considered one dependent variable and two independent variables. The dependent variable was the number of items in each partition. The first independent variable was the artificial dichotomy that

Table 4.9 Number of Multiple-Choice Items, Mean and Standard Deviation of Proportion Answering Correctly, and Delta for Mean and Standard Deviation under Liberal Partitioning Algorithm

examination	partition	number of items	mean proportion answering correctly	standard dev. of proportion answering correctly	delta for mean	delta for standard dev.
1984	*strongly related*	10	0.56	0.25		
	not strongly related	34	0.59	0.15	–.03	+.10
1988 version A	*strongly related*	11	0.30	0.11		
	not strongly related	24	0.40	0.25	–.10	–.14
1988 version AB	*strongly related*	14	0.46	0.13		
	not strongly related	36	0.55	0.25	–.09	–.12
1992 version A	*strongly related*	16	0.50	0.16		
	not strongly related	24	0.49	0.23	+.01	–.07
1992 version AB	*strongly related*	14	0.56	0.18		
	not strongly related	26	0.54	0.21	+.02	–.03

Table 4.10 Number of Multiple-Choice Items, Mean and Standard Deviation of Proportion Answering Correctly, and Delta for Mean and Standard Deviation under Conservative Partitioning Algorithm

examination	partition	number of items	mean proportion answering correctly	standard dev. of proportion answering correctly	delta for mean	delta for standard dev.
1984	*strongly related*	4	0.42	0.26		
	not strongly related	30	0.60	0.15	–.18	+.11
1988 version A	*strongly related*	4	0.33	0.08		
	not strongly related	17	0.44	0.27	–.11	–.19
1988 version AB	*strongly related*	5	0.48	0.11		
	not strongly related	29	0.57	0.25	–.09	–.14
1992 version A	*strongly related*	8	0.39	0.09		
	not strongly related	13	0.47	0.26	–.08	–.17
1992 version AB	*strongly related*	9	0.48	0.13		
	not strongly related	18	0.53	0.18	–.05	–.05

divided the items into the *strongly related* and *not strongly related* partitions. The second independent variable stratified the items according to difficulty. The operational definition for difficulty of an item depended on the proportion of students who correctly answered the item. The categories of difficulty, which were mutually exclusive, were defined in terms of the proportion of respondents who answered correctly: 'very difficult' for the interval [.0, .2), 'somewhat difficult' for the interval [.2, .4), 'average' for the interval [.4, .6) 'somewhat easy' for the interval [.6, .8), and 'very easy' for the interval [.8, 1.0].

Because the number of items in some categories was so small, the results from the five examinations were pooled for this analysis. Table 4.11 presents the number of items at each level of difficulty in each partition under the liberal partitioning algorithm. Figure 4.5 presents the same data in the form of a graph, where the x-axis is the difficulty category and the y-axis is the proportion of items that fell into a partition; the difficulty distribution for each partition is plotted as a line. Table 4.12 and Figure 4.6 present the same information for the partitions defined by the conservative partitioning algorithm.

To test whether the *strongly related* and *not strongly related* partitions were homogeneous with respect to the five levels of item difficulty, hierarchical log-linear tests of homogeneity were run on the data in Tables 4.11 and 4.12. An important consideration in using this test was the number of cells in the 5X2 table for which there was no item. When this occurs, the empty cells are accidents of the sampling procedure, known as *random zeroes*. When random zeroes are sufficiently prevalent, they can prevent completion of the log-linear test of fit

(Wickens, 1989). In this study, only one random zero occurred and the log-linear tests of fit could be calculated.

The test of fit under the liberal partitioning algorithm resulted in $G^2 = 9.081$, with 4 df; because $p = .06$, the null hypothesis was not rejected. Under the conservative partitioning algorithm, the test of fit resulted in $G^2 = 17.507$, with 4 df; because $p < .002$, the null hypothesis was rejected. The conclusion was that the difficulty distributions of the *strongly related* and *not strongly related* partitions were not homogeneous under the conservative algorithm.

In the definition of the difficulty categories, the choice of range intervals was somewhat arbitrary. During the planning phase, the difficulty categories 'very difficult', 'somewhat difficult', 'average', 'somewhat easy', and 'very easy' were defined to correspond to the intervals [.0, .2), [.2, .4), [.4, .6), [.6, .8), and [.8, 1.0], respectively. As the analysis was being carried out, the investigator noticed that several of the proportions were borderline cases. In order to weigh the effect of the boundary definition, hypothesis H_1 was also tested using the difficulty intervals [.0, .2], (.2, .4], (.4, .6], (.6, .8], and (.8, 1.0]. For these data sets, the difference under the conservative partitioning algorithm was negligible: The test of fit resulted in $G^2 = 17.319$, with 4 df; again this resulted in $p < .002$ and the null hypothesis was rejected. The difference under the liberal partitioning algorithm was more pronounced. The test of fit resulted in $G^2 = 9.86540$, with 4 df; in this case $p < .05$ and the null hypothesis could be rejected. It seems doubtful that this difference is inherent in the definition of the difficulty intervals, but rather that it is a phenomenon associated with this particular data set.

Inspection of both Figure 4.5 and Figure 4.6 reveals that the skew of the difficulty distribution for the *strongly related* items is towards the "very difficult" end of the scale while the difficulty distribution for *not strongly related* distribution has more of a bell shape. Because the distributions under the conservative partitioning algorithm were non-homogeneous, this difference leads to the conclusion that the *strongly related* items were, in general, more difficult than the *not strongly related* items. Based on the alternative definition of difficulty in the preceding paragraph, it can be argued that even under the liberal partitioning algorithm the *strongly related* items were generally more difficult than the *not strongly related* items.

4.3.3 Correlation between number correct in partitions

Research question (c) asked: Was there a relationship between individual performance on the set of items whose content was *strongly related* to logic and the set of items whose content was *not strongly related* to logic? The following null hypothesis was tested:

H_2: The correlation between individual performance on items in the *strongly related* partition and items in the *not strongly related* partition is zero.

In testing this hypothesis, the two partitions of items were treated as distinct subtests. Pearson's product-moment correlation was calculated for the pair of values that were the cumulative scores for the *strongly related* and *not strongly related* partitions. The correlation coefficient was developed under both the liberal and the conservative partitioning algorithms for each of the five samples

Table 4.11 Number and Proportion of Items at Each Difficulty Level in the *Strongly Related* and *Not Strongly Related* Partitions under the Liberal Partitioning Algorithm

	strongly related partition	*not strongly related* partition	totals:
very difficult	3	13	16
somewhat difficult	18	31	49
average difficulty	29	42	71
somewhat easy	10	40	50
very easy	5	18	23
totals:	65	144	209

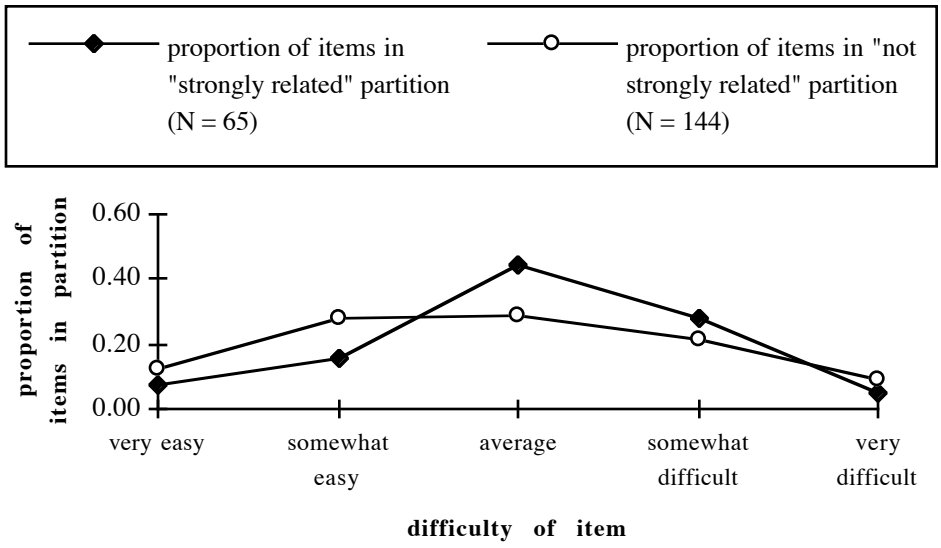


Figure 4.5 Difficulty Distribution of Items in the *Strongly Related* and *Not Strongly Related* Partitions under the Liberal Partitioning Algorithm, With Items Pooled across All Examinations

Table 4.12 Number and Proportion of Items at Each Difficulty Level in the *Strongly Related* and *Not Strongly Related* Partitions under the Conservative Partitioning Algorithm

	strongly related partition	*not strongly related* partition	totals:
very difficult	1	9	10
somewhat difficult	10	20	30
average difficulty	16	33	49
somewhat easy	3	29	32
very easy	0	16	16
totals:	30	107	137

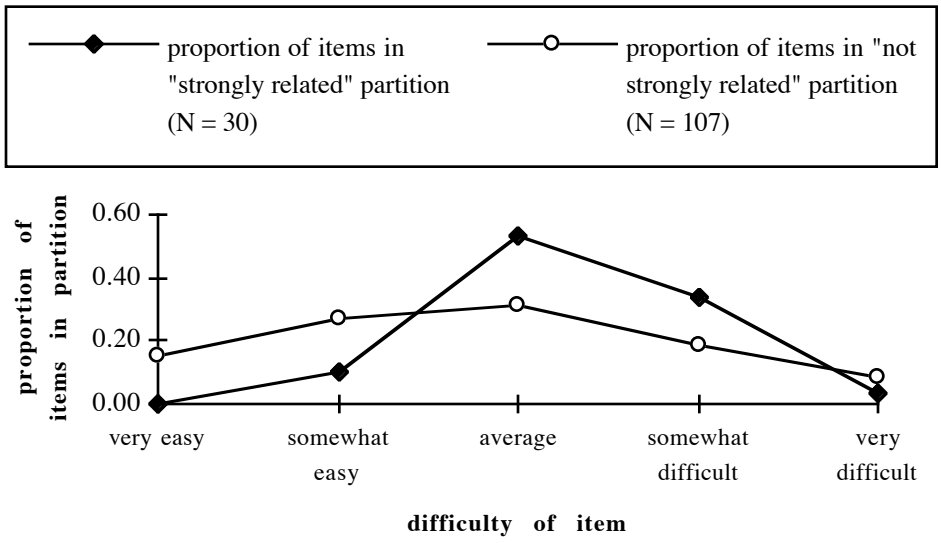


Figure 4.6 Difficulty Distribution of Items in the *Strongly Related* and *Not Strongly Related* Partitions under the Conservative Partitioning Algorithm, With Items Pooled across All Examinations

(1984, 1988 version A, 1988 version AB, 1992 version A, and 1992 version AB). In addition, the coefficient of determination was calculated from each correlation coefficient; this value is the square of the correlation coefficient and is used to describe the shared variance in the two-score distribution.

Table 4.13 reports the correlation coefficients and coefficients of determination for the five samples. Several factors influence the value of the correlation coefficient; these will be considered in the context of this study. First, as the number of items per subtest increases the possible variation of the scores increases and, in turn, the correlations tend to be higher. Here, the subtests defined under the liberal partitioning algorithm had more items than the analogous subtests under the conservative algorithm; the correlations under the liberal partitioning algorithm were consistently larger than the correlations under the conservative partitioning algorithm. Second, as samples become more homogeneous, the possible variation in scores becomes smaller; as a result, correlations tend to be lower for homogeneous samples. In this study, the only instance where a sample was reported to be more homogeneous was for version AB of the 1992 examination. This increased homogeneity was because two distinct examinations were given in 1992 rather than a two-section AB version as in 1988 or a single examination as in 1984 (R. Morgan, personal communication, April 14, 1994). In spite of the increased homogeneity, the correlation coefficients for the sample that took version AB of the 1992 examination were among the highest under both partitioning algorithms.

A factor that could have influenced the value of the correlation coefficient was the effect of removing items from consideration under the conservative partitioning algorithm. In a sense, the eliminated items had something in common with one another, perhaps to the extent that they were measuring some of the same abilities. This reasoning leads to the conclusion that the higher correlations between the partitions defined by the liberal partitioning algorithm could have been due in part to the inclusion of the "mid-range" items (i.e., those items for which only 25% to 75% of the judges agreed about the relationship).

As sample size increases, the correlation coefficient becomes more reliable. In fact, for very large N, very small correlation coefficients can be statistically significant. In this study, the sample sizes were very large, ranging from 3,369 to 7,375. As expected, all correlation coefficients were significant at $p < .001$. These results lead to the conclusion that null hypothesis H_2 should be rejected in all cases. Thus, there is a relationship between performance on items in the *strongly related* partition and performance on items in the *not strongly related* partition under both partitioning algorithms.

The shared variance between the *strongly related* and *not strongly related*

Table 4.13 Correlation between Number Correct in the *Strongly Related* and *Not Strongly Related* Partitions under Both Partitioning Algorithms

APCS examination	liberal partitioning algorithm		conservative partitioning algorithm	
	correlation coefficient	*coefficient of determination*	*correlation coefficient*	*coefficient of determination*
1984	.74	.55	.60	.36
1988A	.60	.36	.42	.18
1988AB	.75	.56	.60	.36
1992A	.74	.55	.56	.31
1992AB	.75	.56	.64	.41

subtests, indicated by the coefficients of determination, was over 50% under the liberal algorithm for the samples taking the 1984 examination, the AB version of the 1988 examination, and both versions of the 1992 examination. Under the liberal partitioning algorithm for the 1988 examination, only 36% of the variance was shared by the two subtests. Under the conservative partitioning algorithm, the shared variance was highest for version AB of the 1992 examination, at 41%. For version A of the 1988 examination, only 18% of the variance was shared. Under the conservative algorithm in all cases, less than 50% of the variability in performance was shared variance. This suggests that to some extent different cognitive abilities are required to answer correctly the items in the two partitions defined under the conservative algorithm. This argues that the model that results from the conservative partitioning algorithm is more descriptive than the model defined under the liberal partitioning algorithm.

4.4 SUMMARY OF FINDINGS

In this section, the research questions are restated and the findings summarized.

4.4.1 Development of the content analysis procedure

The first research question that was posed in Chapter 1 was:

(a) Can a procedure be developed for reliable and valid classification of content-area test items according to their degree of relationship to a pre-defined set of concepts?

This question addressed the feasibility of the methodology proposed for this study. In Chapter 3 it was pointed out that, due to the nature of this question, no

null hypothesis would be formulated. Instead, the answer to the question was based on the results of carrying out the content analysis described in Chapter 3. Because the content analysis procedure was successfully developed and produced acceptable results, the remaining research questions posed in Chapter 1 could be considered.

4.4.2 Comparisons of partitions

In order to consider differences in performance across the partitions of items defined in the content analysis procedure, the following research questions were posed:

(b) In considering student performance on the test items, was the distribution of performance different for items whose content was *strongly related* to logic than for items whose content was *not strongly related* to logic?

(c) Was there a relationship between the pattern of responses on the set of items whose content was *strongly related* to logic and the pattern of responses on the set of items whose content was *not strongly related* to logic?

Both of these research questions were informed by development of simple descriptive statistics (mean and standard deviation) of performance on each set of items. These values showed that, in general, the *strongly related* partitions had lower means and smaller standard deviations than the corresponding *not strongly related* partition had.

Question (b) was tested using the following null hypothesis:

H_1: The difficulty distributions for the partitions of items *strongly related* and *not strongly related* to logic are the same.

The difficulty distributions defined by the two sets of items were graphed and tested for differences using a log-linear test of homogeneity. The null hypothesis was rejected under the conservative partitioning algorithm but not under the liberal partitioning algorithm. The conclusion was that the conservative partitioning algorithm is the superior model for use in future studies.

Research question (c) was addressed by considering the null hypothesis:

H_2: The correlation between individual performance on items in the *strongly related* partition and items in the *not strongly related* partition is zero.

To test this hypothesis, the correlation coefficients for each pairing of item partitions were generated (i.e. under both the liberal and the conservative partitioning algorithms). The null hypothesis was rejected in all cases; however, the practical significance of these results was negligible. The coefficient of determination showed that, under the conservative partitioning algorithm, less than 50% of the variance in performance on the two partitions was shared. Under the liberal partitioning algorithm, this was true in only one case. Hence, the conservative partitioning algorithm appears to provide a more descriptive model.

Chapter 5 Conclusions and Future Research

Chapter 5 discusses the conclusions that can be drawn about the research questions based on the results of this study. The generalizability of the results is considered. Finally, recommendations for future research are presented. The dissertation closes with a brief epilogue that considers the instructional implications of the results.

5.1 Conclusions Regarding Research Questions

This research sought objective evidence as to whether novice computer science students have more difficulty understanding concepts in the computer science subdomain of mathematical logic than they generally have in understanding other novice computer science concepts. This exploratory study produced evidence that supported this conjecture.

The first research question that was posed in Chapter 1, developed in Chapter 3, and explored in Chapter 4 was:

(a) Can a procedure be developed for reliable and valid classification of content-area test items according to their degree of relationship to a pre-defined set of logic concepts?

The content analysis procedure that emerged in the final phase of this study provided a positive answer to this research question. The final procedure was time-effective for the judges to complete, given that the item pool under consideration was rather extensive. Reliability results showed that, in general, the classification that emerged from the content analysis was consistent and reliable.

Content validity was established for the classification results, based on the agreement of nearly 40 experts in computer science. The conclusion was that this procedure resulted in the data needed to allow the study to continue. It is recommended in the Future Research section that the content analysis procedure developed for this study should be refined and applied in other research studies both in the field of computer science and in other fields.

An unanticipated benefit was the insight this research provided into differences in the way the multiple-choice items were perceived by the judges who participated in this study. There was a great deal of variation among the judges in their perceptions of which items included mathematical logic concepts and to what degree. This observation underscores the lack of consensus within the field of computing regarding the relationship between mathematical logic and other computer science concepts. Suggestions for deeper exploration of this phenomenon are discussed in the Future Research section.

While it would have been possible to train the judges to rate items more consistently than they did in this study, such training was deemed undesirable. An explicit decision was made to allow judges to use their expertise, bolstered by the *Quick Reference to Concepts in "Two-Valued Logic"* in Figure 3.1, in doing the classification. This freedom of interpretation and feedback from the judges assisted the researcher in refining the taxonomy of concepts.

Given the results of the classification process during the final content analysis procedure, the test items under consideration were divided into partitions of items that were *strongly related* and *not strongly related* to logic. The remaining research questions posed in Chapter 1 were the following:

(b) In considering student performance on the test items, was the distribution of performance different on items whose content was strongly related to logic than on items whose content was not strongly related to logic?

(c) Was there a relationship between individual performance on the set of items whose content was *strongly related* to logic and the set of items whose content was *not strongly related* to logic?

The answer to research question (b) was explored both through simple descriptive statistics of student performance and by comparing the difficulty distributions of the items in the *strongly related* and *not strongly related* partitions. In general, the items in the *strongly related* partition were more difficult for the five samples of students who took the APCS examinations that were considered in this research. The answer to question (c) was investigated through the development of correlation coefficients comparing individual performance on items in the two partitions. The results of this analysis revealed systematic relationships between the patterns of responses to items in the two partitions, suggesting that the constructs being tested by items in each partition were related to one another. The variability of individual responses to items in the *strongly related* partition explained only a small amount of the variability of individual responses to items in the *not strongly related* partition. Based on the amount of variance shared between the two partitions, the conservative partitioning algorithm was more discriminating than was the liberal partitioning algorithm.

5.2 GENERALIZABILITY OF RESULTS

The experts who participated as judges in the content analysis procedure and who critiqued the taxonomy of concepts were all actively engaged in undergraduate instruction and in research about computer science education. Thus, they represented an informed sample of the total population of computer science instructors. At least 36 judges rated the items in each examination packet, so that their pooled ratings ensured the content validity of the partitioning of the APCS items. As a result, repeating the content analysis procedure on the same item pool with other experts is predicted to produce similar item partitions under both partitioning algorithms.

Because individuals who take the APCS examinations are high school students, the generalizability of these results to novice computer science students at post-secondary institutions comes into question. However, Advanced Placement students tend to be enrolled in advanced or honors high school courses and are generally more successful academically than the average high school student. In addition, secondary students and first-year undergraduates are near one another in age and attitude. The similarities between these two groups argue that conclusions about individual performance for the students taking the APCS examination are generalizable to the results for novice post-secondary students taking the same examinations.

This study focused only on the concepts of logic, grouping all other items together, irrespective of consideration of the relative simplicity or difficulty of the concepts they covered; the difficulty of logic relative to other *specific* concepts

within computer science was not addressed. While the subdomain of logic does cause difficulties for novice computer science students, there may be other subdomains that include concepts that are equally or more difficult for novices to understand.

5.3 SUGGESTIONS FOR FUTURE RESEARCH

Three areas for future research are suggested. Extensions of the current study are proposed as well as ideas for new related research questions. First, ways are outlined in which the content analysis procedure can be developed further and used in other studies. Second, the development of a diagnostic tool based on the findings in this and other research is discussed. Third, studies comparing pedagogical approaches to teaching logic are suggested.

5.3.1 Continued work with the content analysis procedure

The content analysis procedure developed for this study could be refined and extended in four different areas: (1) consideration of additional research questions for the classification results from the current study, (2) development of additional reliability and validity results for the current study, (3) improvement of the content analysis reliability software and procedures, and (4) use of the content analysis procedure in other settings and other disciplines. Each of these areas are developed further below.

(1) Many interesting research questions considered for the current study were not pursued due to time constraints and a need to narrow the scope of the research. Examples of research questions that could be investigated in future studies with the data from the current study are the following:

- Was there a relationship between performance on the free-response section and performance on each partition of multiple-choice items? Because the free-response and multiple-choice sections were completed in different time periods and graded using different techniques (the free-response section is graded manually by groups of educators using grading rubrics, while the multiple-choice section is graded mechanically), the results for each section were connected only by being created by the same team and by being two sections of the same examination. Bennett, Rock, and Wang (1991) found that the free-response and multiple-choice items on the APCS examinations measure approximately the same constructs in different ways, so that relationships in performance could provide insights into differential performance on the *strongly related* and *not strongly related* partitions of items.
- Was there a significant difference in performance by different demographic groups across the partitions of items? For the data sets used in this study, gender and ethnicity were reported. Group performance statistics could be developed for this data set and the results contrasted with other studies that have explored performance differential based on ability in logic (e.g. Stager-Snow, 1985; Stofflett & Baker, 1992).

(2) Additional reliability and validity results could be developed for the current study. Suggestions include the following:

- Consider the reliability of the classification results from the point of view of the groupings of items in each partition. Only the items in a particular partition, for example, in the *strongly related* partition under the

conservative partitioning algorithm, would be included in the calculations. The results could then be compared across partitions and across examination packets.

- Compare the reliability between pairs of judges. These pairings would provide information that could be used to identify cohorts of judges, that is, judges among whom agreement is especially high and consistent. The pairings could also be used to identify judges whose ratings deviate significantly from those given by the other judges. The ratings of the "outlier" judges could be analyzed further to determine their influence on the outcome of the partitioning.
- Conduct clinical interviews with a sampling of the judges to compare backgrounds and the reasoning that went into their ratings. The sampling could be based on information about cohorts of judges and judges whose ratings were significantly different from those of the other judges.
- Develop item reliability, which provides an indication of how easy or difficult it was to rate each item. The utility of this result could be considered in terms of the content analysis results as well as for implications about the way in which the APCS examinations are constructed.
- Evaluate the criterion-related validity of the partitions of items identified in the content analysis procedure. For example, multi-dimensional scaling could be used to form clusters of multiple-choice items based on the patterns of responses. The item partitions from this study would then be compared to the clusters that emerge. This comparison would address the

research question: Do student responses indicate groupings of items that correspond to the partitioning of items that resulted from the content analysis procedure ratings?

- Run the content analysis procedure again with different judges on the same examination packets. Comparing the partitions defined by the new set of ratings with the results obtained in this study would provide further evidence of reliability and content validity.
- Include selected items on in-class examinations for CS1 and CS2 courses. The performance results on those items could be compared with the results from this study to evaluate the generalizability of the results from the samples of students taking the APCS examinations to novice post-secondary computing students.

(3) The software package used for calculating the various types of content analysis reliability could be developed further.

- Generate standard error information for the results produced by the software package.
- Develop an improved software package for calculating the content analysis reliability, especially if the content analysis procedure from this study is to be used by other researchers. The current software package was intended to be a prototype. Time constraints and the acceptable functionality of the prototype rendered a new development effort unnecessary for the purposes of this study. The redesigned software package would concentrate on ease of use, the possibility for interactions

with other statistical analysis tools, and expansion and refinement of the features and reliability procedures.

(4) The content analysis procedure could be applied in other settings and in other disciplines.

- Carry out the procedure for items from other types of examinations for the same concept subdomain, mathematical logic in computer science.
- Use the procedure to investigate differential performance in other subdomains of computer science.
- Apply the procedure in research studies in fields other than computer science.

5.3.2 Development of a diagnostic tool

Based on the results from this study and from studies that showed ability in propositional logic could be used to predict success in science courses (e.g., Piburn, 1990), a diagnostic tool could be developed. Such an instrument could be used in designing experiments to compare approaches to and materials for teaching the concepts of logic and other advanced concepts that depend on an understanding of logic. The instrument would incorporate multiple-choice items identified during this study as well as aspects of other tests that measure understanding of propositional logic (e.g., the Propositional Logic Test as described in Piburn, 1989). The instrument would be simple and brief.

Based on the results of pilot studies, one or more case studies could be developed with individuals whose responses were especially interesting according to predefined criteria. The case study would consist of guided student interviews

and analysis of think-aloud protocols produced as subjects verbally worked through questions similar to those on the instrument.

5.3.3 Approaches to teaching logic to computer science students

In his survey of "Mathematics of computing", Saiedian (1992) claims:

> By integrating courses in discrete mathematics and mathematical logic as part of a pragmatic approach to computer science education, we can increase our students' level of reasoning, prepare them for courses in formal models, improve their theoretical foundation, and prepare them for career growth and/or advanced studies. (p. 220)

While the author of this dissertation supports this statement, she has not discovered objective evidence that supports Saiedian's claim that students' enrollment in discrete mathematics and mathematical logic courses leads to an improvement in their levels of reasoning. Future research must be designed to provide objective evidence of such claims — claims that many computer science educators take as "self-evident".

Curricular guidelines for computer science (e.g., Tucker, 1990) include mathematical logic as a topic in the discrete mathematics course that is recommended for all computer science majors. The current study has shown that, for the samples of novice students who took the Advanced Placement Examinations in Computer Science, items that included logic as a main concept or vital subconcept were generally more difficult. Anecdotal reports suggest that novice computing students at the post-secondary level have similar problems. Warford (in press) has described his experiences teaching a discrete mathematics course that had as its foundation the skillful manipulation of expressions in propositional and predicate calculus. Research should be designed to compare

and contrast the effectiveness of this approach and other approaches to teaching discrete mathematics.

5.4 EPILOGUE

The results of the present study indicate that novice computer science students do experience more difficulty with the concepts of mathematical logic than they do, in general, with other computer science concepts. No attempt was made to establish logic as "the most difficult" subdomain of computer science. However, the evidence from this study does show that the Advanced Placement Examination multiple-choice items judged to be *strongly related* to logic were more difficult than the items that were *not strongly related* to logic.

Two important questions to be considered in continued research are: (1) What can be done to improve beginning students' likelihood of learning the concepts in this content domain well? and (2) How can one provide remediation for novice students who have learned the concepts of logic poorly? The answers to these questions have strong potential for positively influencing the future success of computer science students. In particular, this study has implications for pre-college instruction in mathematical logic and for the college-level discrete mathematics course(s) taken by beginning and novice computing students.

Appendix A Overview of Computing Curricula 1991

The report titled *Computing Curricula 1991* (Tucker, 1990), gives curricular recommendations for a variety of undergraduate programs in the discipline of computing, defined to encompass programs with titles such as "computer science", "computer engineering", and "computer science and engineering".

The guidelines describe a set of nine *subject areas* that comprise the subject matter of the computing discipline. Within the subject area definitions are contained certain fundamental subjects, called "common requirements", that should be covered in all undergraduate programs in computing. The subject areas are given in Table A.1. To reflect the working methodologies that different practitioners apply during the course of their research, development, and applications work, three *processes* were delineated. These processes, *theory*, *abstraction* and *design*, are outlined in Table A.2. Twelve recurring concepts were identified to acknowledge the threads of significant ideas, concerns, and principles that permeate the academic discipline of computing across all subject areas and all processes; these are given in Table A.3.

The subject matter to be covered in all academic computing programs is described in terms of *knowledge units*. The definition of each knowledge unit is based on the subject areas, processes, and recurring concepts discussed in the previous paragraph. The description of each knowledge unit includes:

- a high level description of goals and general subject matter

- the recurring concepts appropriate to the knowledge unit
- suggested lecture topics, including minimum number of lecture hours
- typical laboratory exercises and goals
- related knowledge units
- prerequisite knowledge units
- knowledge units for which the unit is a prerequisite

The report explains that the knowledge units will map into different sets of courses at different institutions and gives several example implementations in an appendix. Table A.1 lists the "common requirements" knowledge units defined in Computing Curricula 1991. For further details, refer to Tucker (1990).

Table A.1 Subject Areas Outlined in Computing Curricula 1991

subject area	*explanation of subject area and major topics*
algorithms and data structures	• specific classes of problems and their efficient solutions • major topics: - performance characteristics of algorithms - the organization of data relative to different access requirements
architecture	• methods of organizing efficient, reliable computing systems • major topics: - implementation of processors, memory, communications, and software interfaces - design and control of large computational systems that are reliable - performance measurement and modeling
artificial intelligence and robotics	• basic models of behavior • major topics: - building of (virtual or actual) machines to simulate animal and human behavior - inference, deduction, pattern recognition, knowledge representation
database and information retrieval	• organization of information and algorithms for the efficient access and update of stored information • major topics: - modeling of data relationships - security and protection of information in a shared environment - characteristics of external storage devices
human-computer communication	• efficient transfer of information between humans and machines • major topics: - graphics - human factors that affect efficient interaction - organization and display of information for effective utilization by humans
numerical and symbolic computation	• general methods for efficiently and accurately using computers to solve equations from mathematical models • major topics: - effectiveness and efficiency of various approaches to the solution of equations - development of high-quality mathematical software packages
operating systems	• control mechanisms that allow multiple resources to be efficiently coordinated during the execution of programs • major topics: - appropriate interfaces for users - effective strategies for resource control - effective organization to support distributed computations

Table A.1 continued on next page

Continuation of Table A.1

subject area	*explanation*
programming languages	• notations for defining virtual machines that execute algorithms • the efficient translation from high-level to machine codes • various extension mechanisms that can be provided in programming languages
software methodology and engineering	• design and production of large software systems that meet specifications • major topics: - principles of programming and software development - verification and validation of software - specification and production of software systems that are safe, secure, reliable, and dependable

Table A.2 Processes as Defined in Computing Curricula 1991

process	*explanation of process*
theory	• akin to processes used in mathematics in the development of coherent theories • used in developing and understanding the underlying mathematical principles that apply to the discipline of computing • major elements: - definitions and axioms - theorems - proofs - interpretation of results
abstraction	• rooted in the experimental sciences • used when modeling potential algorithms, data structures, architectures, etc. • also used when testing hypotheses about models, alternative design decisions, or the underlying theory itself • major elements: - data collection and hypothesis formation - modeling and prediction - design of an experiment - analysis of results
design	• rooted in engineering • used in the development of a system or device to solve a given physical problem • involves conceptualization and realization of systems in the context of real-world constraints • major elements: - requirements - specifications - design and implementation - testing and analysis

Table A.3 Recurring Concepts as Defined in Computing Curricula 1991

recurring concept	*brief characterization*
binding	• processes of making an abstraction more concrete by assigning properties to it
complexity of large problems	• effects of the nonlinear increase in complexity as the size of a problem grows • factor in distinguishing and selecting methods that scale to different data sizes, problem spaces, and program sizes • in large programming projects, factor in determining the organization of an implementation team
conceptual and formal models	• various ways of formalizing, characterizing, visualizing, and thinking about an idea or problem
consistency and completeness	• all concrete realizations of the concepts of consistency and completeness in computing, including related concepts such as correctness, robustness, and reliability
efficiency	• all measures of cost relative to space, time, money and people
evolution	• the fact of change and its implications • impact of change at all levels and the resiliency and adequacy of abstractions • techniques and systems in the face of change
levels of abstraction	• nature and use of abstraction in computing • use of abstraction in managing complexity, structuring systems, hiding details, capturing recurring patterns • ability to represent an entity or system by abstractions having different levels of detail and specificity
ordering in space	• concepts of locality and proximity, including: - distributed systems - networking - software packages
ordering in time	• concept of time, including: - time as a formal parameter in formal models (e.g., in temporal logic) - time as a means of synchronizing processes that are spread out over space - time as an essential element in the execution of algorithms
reuse	• the ability of a particular technique, concept, or system component to be reused in a new context or situation
security	• the ability of software and hardware systems to respond appropriately to and defend themselves against inappropriate and unanticipated requests
tradeoffs and consequences	• the phenomenon of tradeoffs in computing and the consequences of such tradeoffs • technical, economic, cultural, and other effects of selecting one design alternative over another

Table A.4 Knowledge Units Comprising the Common Requirements in Computing Curriculum 1991

"tag"	*knowledge unit name*
AL	algorithms and data structures
AL1	• basic data structures
AL2	• abstract data types
AL3	• recursive algorithms
AL4	• complexity analysis
AL5	• complexity classes
AL6	• sorting and searching
AL7	• computability and undecidability
AL8	• problem-solving strategies
AL9	• parallel and distributed algorithms
AR	architecture
AR1	• digital logic
AR2	• digital systems
AR3	• machine-level representation of data
AR4	• assembly-level machine organization
AR5	• memory system organization and architecture
AR6	• interfacing and communication
AR7	• alternative architectures
AI	artificial intelligence and robotics
AI1	• history and applications of artificial intelligence
AI2	• problems, state spaces, and search strategies
DB	database and information retrieval
DB1	• overview, models, and applications of database systems
DB2	• the relational data model
HU	human-computer communication
HU1	• user interfaces
HU2	• computer graphics
NU	numerical and symbolic computation
NU1	• number representation, errors, and portability
NU2	• iterative approximation methods

"tag"	*knowledge unit name*
OS	operating systems
OS1	• history, evolution, and philosophy
OS2	• tasking and processes
OS3	• process coordination and synchronization
OS4	• scheduling and dispatch
OS5	• physical and virtual memory organization
OS6	• device management
OS7	• file systems and naming
OS8	• security and protection
OS9	• communications and networking
OS10	• distributed and real-time systems
PL	programming languages
PL1	• history and overview of programming languages
PL2	• virtual machines
PL3	• representation of data types
PL4	• sequence control
PL5	• data control, sharing, and type checking
PL6	• run-time storage management
PL7	• finite state automata and regular expressions
PL8	• context-free grammars and pushdown automata
PL9	• language translation systems
PL10	• programming language semantics
PL11	• programming paradigms
PL12	• distributed and parallel programming constructs
SE	software methodology & engineering
SE1	• fundamental problem-solving concepts
SE2	• the software development process
SE3	• software requirements and specifications
SE4	• software design and implementation
SE5	• verification and validation
SP	social, ethical, & professional issues
SP1	• historical and social context of computing
SP2	• responsibilities of the computing professional
SP3	• risks and liabilities
SP4	• intellectual property

Appendix B Topic Outline for the Advanced Placement Examination in Computer Science

Appendix B presents the topic outline for AP Computer Science courses. The outline is extracted from the *Advanced Placement Course Description: Computer Science* (College Board, 1990). The two courses, Computer Science A and Computer Science AB, are described as follows:

> The major emphasis in the Computer Science A course is on programming methodology and procedural abstraction. However, the study of these topics cannot occur in isolation from the study of algorithms, data structures, and data abstraction, so these latter topics are included in the course as needed.
>
> In brief, Computer Science A consists of the study of programming methodology without any discussion of formal correctness proofs or arguments. Algorithms (particularly sorting and searching algorithms) are informally compared and no use is made of the "big-*O*" notation. Data structures and data abstraction are studied in the context of a computer language's built-in types and structures (e.g., arrays and records) and non-linked structures that can be built from these. Recursion is introduced as a control abstraction.
>
> In addition to the topics studied in Computer Science A, Computer Science AB deals more formally with program verification and algorithm analysis. In addition to the study of programming methodology and procedural abstraction that is the core of Computer Science A, there is a major emphasis on the study of data structures and data abstraction. The use of recursive data structures and dynamically allocated data structures is fundamental to Computer Science AB. (p. 8)

The outline that begins on the following page is a reproduction of the topic outline that appears in (College Board, 1990, pp. 10-15). Those topics that are only included in Computer Science AB are indicated by a check mark in the second column. The purpose of the APCS examinations is "…to determine how

well students have mastered the concepts and techniques contained in the respective course outlines" (College Board, 1990, p. 36). All topics in the outline are tested on the AB version of the APCS examination, while the A version covers only those topics that are not checked.

APCS Topic Outline

Area	AB?	Topics
A. Programming Methodology		**1. Specification** a. Problem definition and requirements b. Program and subprogram specifications (e.g., pre- and postconditions, exceptional conditions) c. Abstract data types
		2. Design a. Adaptability i. Simplicity vs. generality ii. Reusable code (software components) b. Subprogram decomposition and data structuring i. Exploring alternatives ii. Information hiding c. Stepwise refinement of subprograms and data structures d. Choice of data structures and algorithms e. User interface (e.g., error checking, help facilities)
	√	**3. Implementation** a. Coding i. Structure ii. Style and clarity of expression b. Program correctness i. Testing and debugging A. Reasoning about programs B. Assertions C. Invariants ii. Verification c. Incremental development i. Top-down ii. Bottom-up iii. Other heuristics, order of implementation
		4. Documentation

APCS Topic Outline continued on following page

Continuation of APCS Topic Outline

Area	AB?	Topics
B. Features of Block-Structured Programming Languages		**1. Type and constant declarations** a. Named constants b. Simple data types (Boolean, character, integer, real, subrange, enumerated) c. Structured data types (e.g., arrays, records, sets, files, strings)
	√	d. Pointer types
		2. Scope
		3. Expressions and evaluation a. Infix notation and operator precedence b. Standard functions
	√	c. Prefix and postfix notation
		4. Assignment statements
		5. Control structures a. Sequential execution b. Conditional execution c. Loops
		6. Input and output a. Terminal b. Text files
	√	c. Files of other types
		7. Subprograms a. Procedures and functions b. Parameters i. Actual and formal ii. Value and reference c. Recursion
		8. Program annotation (comments)
		9. Notation for language definition (syntax diagrams, Backus-Naur form)
C. Fundamental Data Structures		**1. Linear** a. Variations i. Lists
	√	ii. Stacks
	√	iii. Queues b. Representations i. Sequential ii. Random access
	√	iii. Linked (singly and doubly, circular, with and without list heads)
		2. Multidimensional (e.g., matrices, tables)
		3. Records
	√	**4. Tree structures**
		5. Variations (e.g., alternative representation)

APCS Topic Outline continued on following page

Continuation of APCS Topic Outline

Area	AB?	Topics
D. Algorithms		**1. Operations on fundamental data structures**
		a. Insertion
		b. Deletion
		c. Traversals
		2. Searching
		a. Sequential (linear) search
		b. Binary search
	√	c. Hashing
	√	d. Searching an ordered binary tree
		3. Sorting
		a. Quadratic sorts
		b. More efficient sorts
		4. Analysis of algorithms
		a. Informal comparison of speeds
	√	b. The meaning of "big-O" notation
	√	c. Worst-case time
	√	d. Worst-case space
	√	**5. Numerical algorithms**
	√	a. Approximations (e.g., zeros of functions, Monte Carlo method)
	√	b. Numerical accuracy
	√	i. Round-off effects
	√	ii. Precision of approximations
E. Computer Systems		**1. Major hardware components**
		a. Primary and secondary memory
		b. Processors
		c. Peripherals
		2. System software
		a. Language translators
		b. Operating systems
		c. File systems
		3. Types of systems
		a. Single-user systems
		b. Time-sharing and batch-processing systems
		c. Networks
F. Responsible Use of Computer Systems		**1. Privacy**
		2. Reliability of systems
		3. Legal issues and intellectual property
		4. Social ramifications of computer applications

Note: From *Advanced Placement Course Description: Computer Science*, College Entrance Examination Board, 1990 (May 1991 version), pp. 10-14. Adapted by permission.

Appendix C Taxonomy of Concepts in the Computer Science Subdomain Two-Valued Logic

Appendix C presents the full taxonomy of concepts in the computer science subdomain of mathematical logic that was developed for this study. The taxonomy fills in the details for the high-level pictorial outline that was given as the *Quick Reference to the Concepts of "Two-Valued Logic"* in Figure 3.1 of Chapter 3. The qualifying phrase "two-valued" was used to emphasize the sort of mathematical logic that was under consideration in this research, that is, logic restricted to a two-valued domain.

Full Taxonomy of Concepts in the Computer Science Subdomain Two-Valued Logic

1.0 Datatype *boolean*
- 1.1 Set of values {*true*, *false*}
- 1.2 Set of operations {equal(=), not( ), and( ), or( ), implies( ), …}
  - 1.2.1 Truth tables
  - 1.2.2 Precedence rules
- 1.3 Properties
  - 1.3.1 Non-numeric
  - 1.3.2 Unordered
  - 1.3.3 Discrete

2.0 Related simple datatypes
- 2.1 Bit
  - 2.1.1 Set of values {0, 1}
  - 2.1.2 Set of operations {equal, +, *, in-order, …}
  - 2.1.3 Properties
    - 2.1.3.1 Numeric
    - 2.1.3.2 Ordered
    - 2.1.3.3 Discrete
    - 2.1.3.4 Bounded
- 2.2 State, restricted to two values
  - 2.2.1 Set of values: Any set of two distinct literal values (e.g., {on, off}, {left, right})
  - 2.2.2 Set of operations {equal, toggle, …}
  - 2.2.3 Properties
    - 2.2.3.1 Non-numeric
    - 2.2.3.2 Unordered
    - 2.2.3.3 Discrete
- 2.3 Two-valued subrange of integer

3.0 *Boolean*-based calculi
- 3.1 *Boolean* variables or identifiers
- 3.2 States (associating identifiers with values)
- 3.3 Propositions
- 3.4 Tautologies
- 3.5 Quantification
- 3.6 Bound vs. free variables
- 3.7 Predicates
- 3.8 Functions
- 3.9 Laws {equivalence, commutative, associative, distributive, De Morgan's, …}
- 3.10 Axioms, inference rules, theorems

4.0 *Boolean* aspects of Programming Languages
- 4.1 Realization of datatype *boolean*
- 4.2 *Boolean* expressions
  - 4.2.1 Constants and named constants
  - 4.2.2 Relations (functions of other datatypes yielding *boolean* values)
  - 4.2.3 "Predicate calculus expressions" (functions of *boolean* values yielding *boolean* values)
  - 4.2.4 *Boolean*-valued function calls
- 4.3 Assignment with datatype *boolean*
  - 4.3.1 rvalues: [named] constant, variable, …
  - 4.3.2 lvalues: variable, field, attribute, property, …
- 4.4 Conditional control structures in imperative languages
  - 4.4.1 Selection
    - 4.4.1.1 Explicit *boolean* condition (e.g., if)
    - 4.4.1.2 Implicit *boolean* condition (e.g. case in Pascal)
  - 4.4.2 Iteration
    - 4.4.2.1 Explicit *boolean* condition (e.g., while & repeat in Pascal; for in C)
    - 4.4.2.2 implicit *boolean* condition (e.g., for in Pascal)
- 4.5 Parameters of type *boolean*
- 4.6 Other uses in non-imperative languages
  - 4.6.1 Constraints in descriptive or declarative languages
  - 4.6.2 Explicit third value, i.e. {*true*, *false*, *null*/no value}

5.0 Assertions & Formal Verification
- 5.1 Precondition / postcondition
- 5.2 Loop invariant
- 5.3 Calculation of weakest precondition
- 5.4 Data transformations

6.0 Other Topics / Advanced Topics
- 6.1 *Boolean* structures
- 6.2 Deduction systems
- 6.3 Constructivist logic / 3-valued logic

Appendix D Letter Used to Solicit Assistance in Final Phase of Content Analysis Procedure

Dear <name of volunteer>,

Dr. Nell Dale has given me your name as someone who is willing to assist me in my doctoral research. The task I am asking you to complete involves ranking up to 174 multiple-choice items that have appeared on the Advanced Placement examinations in Computer Science. These items are from the years for which the multiple-choice items have been made publicly available: 1984, 1988, and 1992 (which has two separate versions, A and AB).

In this packet, I have provided you with the following:

- A Quick Reference showing the concepts of two-valued logic; these are the concepts that will guide the rating you give to each item.
- The four sets of items to be rated (1984, 1988, 1992A, & 1992B).
- The correct answers to all of the items.
- Four copies of the coding form on which to record your responses.

  The coding form includes the following information:

  – At the upper left, a line where you should enter your name (your specific responses will be anonymous in the analysis).

  – At the upper right, a box where you should circle the year of the test you are rating on this form.

  – In the middle at the top, a box for your reference that gives brief descriptions of the four categories of relationship between a particular item and the concepts of two-valued logic (that is, either the item includes two-valued logic (2vl) as a 'main concept', a 'vital subconcept', or a 'trivial subconcept', or is 'not used').

  – On the remainder of the page, the boxes to be marked with your responses on each item; note that the number of items varies between the tests!! If you change your mind about a rating, either circle your final choice or write the final choice to the side.

In completing this task, please do not spend too much time on any item. This task should take *at most* two hours. It is **very** important that, once you have received the materials, you do not discuss the items or the rating process with anyone else until after you (and they, if they are also completing this for me) have completed *all* of the coding forms. In my experience, your first reaction will usually be the best.

Please return the completed forms to me by mid-September at the latest; for your convenience, I have included a stamped, self-addressed envelope. You are welcome to keep the examination packets and other materials. In the near future I will also have electronic versions of these items available.

Thank you for your interest and for your time. Please feel free to contact me if you have any questions. I will share the results of my research in the future.

Sincerely,

Vicki L. Almstrum

Appendix E Coding Form for Content Analysis Procedure

Appendix E gives the coding form that was used during the final phase of the content analysis procedure. Key features of the coding form are the following:

- The form design allowed one layout to be used for rating any of the examination packets. Judges were instructed to circle the appropriate examination packet year in the upper right-hand corner of the form.
- As an aid, the number of items in each examination packet was given beneath the examination packet years in the upper right-hand corner.
- The judge's name was to be entered at the upper left-hand corner of the form. The researcher assigned a unique identification code to each judge, allowing a particular judge's ratings across examination packets to be traced. All identifying information was kept in confidence so that a particular judge could not be identified based simply on the reported responses.
- Simple clarification of the instructions for completing the classification procedure was given at the top of the form, including the classification categories.

The primary criticism of the form as given was the absence of item numbers along the right side of each column, mirroring those given at the left. It was felt that the extra column would have made it easier for the judge to check the appropriate box. If the form is used again, it should be modified by repeating the first column as the last column.

Appendix F Item Assignment to *Strongly Related* and *Not Strongly Related* Partitions under each Partitioning Algorithm

Appendix F reports the item-by-item assignment to partitions for all of the examination packets. Tables F.1 through F.4 provide detailed information about the 1984, 1988, 1992 version A, and 1992 version AB examination packets, respectively. Each table reports, per item, the number of judges who chose the *strongly related* classifications ('main concept' or 'vital subconcept') and the *not strongly related* classifications ('trivial subconcept' or 'not used'). These two figures were the basis for assigning the item to a partition under each of the two partitioning algorithms; an item's partition assignment under each partitioning algorithm is shown symbolically immediately to the right of the item number in the table.

The partitioning algorithms were as follows:

- Under the *liberal partitioning algorithm*, an item was classified as *strongly related* if 50% of the judges had rated the item as 'main concept' or 'vital subconcept' and *not strongly related* otherwise.
- Under the *conservative partitioning algorithm*, an item was classified as *strongly related* if at least 75% of the judges had rated it as 'main concept' or 'vital subconcept'. If fewer than 25% of the judges rated the item as 'main concept' or 'vital subconcept', it was classified as *not strongly related*. Under the conservative partitioning algorithm, items in the mid-range of agreement were eliminated from further consideration.

Table F.1 Number of Judges Choosing Specific Categories for Each Item and Assignment of Items to Partitions under Each Partitioning Algorithm for 1984 APCS Examination

item number and partition indicators under liberal and conservative algorithms	*# of judges choosing …*	
	main concept or *vital subconcept*	*trivial subconcept* or *not used*
1984-01 * •	0	38
1984-02 † –	24	14
1984-03 * •	2	36
1984-04 * •	4	34
1984-05 * •	1	37
1984-06 * •	5	33
1984-07 * •	0	38
1984-08 † –	25	13
1984-09 * •	8	30
1984-10 * •	2	36
1984-11 * •	2	36
1984-12 † –	27	11
1984-13 * •	6	32
1984-14 * •	4	34
1984-15 * •	0	38
1984-16 † ‡	29	9
1984-17 † ‡	38	0
1984-18 † –	24	14
1984-19 * –	14	24
1984-20 * •	0	38
1984-21 * •	2	36
1984-22 * •	2	36

item number and partition indicators under liberal and conservative algorithms	*# of judges choosing …*	
	main concept or *vital subconcept*	*trivial subconcept* or *not used*
1984-23 * •	0	38
1984-24 * •	0	38
1984-25 † –	28	10
1984-26 * •	1	37
1984-27 * •	0	38
1984-28 * •	0	38
1984-29 * •	0	38
1984-30 † –	23	15
1984-31 * •	2	36
1984-32 * •	1	37
1984-33 * •	9	29
1984-34 * •	8	30
1984-35 * –	10	28
1984-36 * –	14	24
1984-37 * –	17	21
1984-38 * •	9	29
1984-39 * •	1	37
1984-40 * •	4	34
1984-41 † ‡	30	8
1984-42 † ‡	37	1
1984-43 * •	2	36
1984-44 * •	3	35

Note: number of judges = 38

Key to partition indicators:
First column of symbols after the item number: *liberal algorithm*, where:
- † means the item is in *strongly related* partition
- * means the item is in *not strongly related* partition

Second column of symbols after the item number: *conservative algorithm*, where:
- ‡ means the item is in *strongly related* partition
- • means the item is in *not strongly related* partition
- – means the item is excluded from consideration

Table F.2 Number of Judges Choosing Specific Categories for Each Item and Assignment of Items to Partitions under Each Partitioning Algorithm for 1988 APCS Examination

item number and partition indicators under liberal and conservative algorithms	# of judges choosing … *main concept* or *vital subconcept*	# of judges choosing … *trivial subconcept* or *not used*
1988-01 * •	0	36
1988-02 * •	0	36
1988-03 * •	1	35
1988-04 * •	4	32
1988-05 † ‡	28	8
1988-06 † ‡	33	3
1988-07 * •	0	36
1988-08 * •	0	36
1988-09 * •	0	36
1988-10 * •	2	34
1988-11 * –	12	24
1988-12 † –	24	12
1988-13 † –	18	18
1988-14 † ‡	32	4
1988-15 † –	24	12
1988-16 † –	18	18
1988-17 * –	12	24
1988-18 † –	26	10
1988-19 * –	12	24
1988-20 * –	11	25
1988-21 * •	1	35
1988-22 † –	20	16
1988-23 † ‡	35	1
1988-24 * •	0	36
1988-25 * –	17	19

item number and partition indicators under liberal and conservative algorithms	# of judges choosing … *main concept* or *vital subconcept*	# of judges choosing … *trivial subconcept* or *not used*
1988-26 * •	1	35
1988-27 * •	1	35
1988-28 * •	1	35
1988-29 * –	12	24
1988-30 * •	1	35
1988-31 * •	1	35
1988-32 † –	19	17
1988-33 * –	9	27
1988-34 * •	2	34
1988-35 * •	3	33
1988-36 * •	0	36
1988-37 * •	0	36
1988-38 * •	4	32
1988-39 * •	7	29
1988-40 † –	19	17
1988-41 * •	6	30
1988-42 * •	0	36
1988-43 * •	1	35
1988-44 † ‡	30	6
1988-45 * •	0	36
1988-46 * •	8	28
1988-47 * •	3	33
1988-48 * •	1	35
1988-49 * •	3	33
1988-50 † –	23	13

Note: number of judges = 36; version A is items 1-35 and version AB is all 50 items

Key to partition indicators:
First column of symbols after the item number: *liberal algorithm*, where:
- † means the item is in *strongly related* partition
- * means the item is in *not strongly related* partition

Second column of symbols after the item number: *conservative algorithm*, where:
- ‡ means the item is in *strongly related* partition
- • means the item is in *not strongly related* partition

– means the item is excluded from consideration

Table F.3 Number of Judges Choosing Specific Categories for Each Item and Assignment of Items to Partitions under Each Partitioning Algorithm for 1992 APCS Examination, Version A

item number and partition indicators under liberal and conservative algorithms	# of judges choosing … *main concept* or *vital subconcept*	# of judges choosing … *trivial subconcept* or *not used*
1992A-01 * •	4	34
1992A-02 * •	3	35
1992A-03 * •	0	38
1992A-04 * –	14	24
1992A-05 * –	11	27
1992A-06 * •	1	37
1992A-07 * •	1	37
1992A-08 † –	28	10
1992A-09 * –	15	23
1992A-10 * –	16	22
1992A-11 † –	26	12
1992A-12 † ‡	37	1
1992A-13 † –	26	12
1992A-14 * –	17	21
1992A-15 * –	15	23
1992A-16 † –	27	11
1992A-17 * –	16	22
1992A-18 * •	9	29
1992A-19 * –	10	28
1992A-20 † –	28	10

item number and partition indicators under liberal and conservative algorithms	# of judges choosing … *main concept* or *vital subconcept*	# of judges choosing … *trivial subconcept* or *not used*
1992A-21 † –	26	12
1992A-22 † –	19	19
1992A-23 * •	2	36
1992A-24 † ‡	32	6
1992A-25 † –	25	13
1992A-26 † ‡	38	0
1992A-27 † ‡	37	1
1992A-28 * •	3	35
1992A-29 * –	16	22
1992A-30 † ‡	29	9
1992A-31 † ‡	38	0
1992A-32 † ‡	38	0
1992A-33 * –	18	20
1992A-34 * •	5	33
1992A-35 † ‡	31	7
1992A-36 * –	15	23
1992A-37 * •	2	36
1992A-38 * •	2	36
1992A-39 * •	1	37
1992A-40 * •	9	29

Note: number of judges = 38

Key to partition indicators:
First column of symbols after the item number: *liberal algorithm*, where:
- † means the item is in *strongly related* partition
- * means the item is in *not strongly related* partition

Second column of symbols after the item number: *conservative algorithm*, where:
- ‡ means the item is in *strongly related* partition
- • means the item is in *not strongly related* partition
- – means the item is excluded from consideration

Table F.4 Number of Judges Choosing Specific Categories for Each Item and Assignment of Items to Partitions under Each Partitioning Algorithm for 1992 APCS Examination, Version AB

item number and partition indicators under liberal and conservative algorithms	# of judges choosing … *main concept* or *vital subconcept*	# of judges choosing … *trivial subconcept* or *not used*
1992B-01 * •	0	38
1992B-02 * •	4	34
1992B-03 * •	3	35
1992B-04 † –	26	12
1992B-05 * –	11	27
1992B-06 † –	26	12
1992B-07 * •	1	37
1992B-08 * •	5	33
1992B-09 * –	12	26
1992B-10 * –	12	26
1992B-11 † –	24	14
1992B-12 * •	4	34
1992B-13 * •	2	36
1992B-14 † –	28	10
1992B-15 * –	14	24
1992B-16 † ‡	38	0
1992B-17 † ‡	37	1
1992B-18 † ‡	36	2
1992B-19 * •	2	36
1992B-20 * •	2	36

item number and partition indicators under liberal and conservative algorithms	# of judges choosing … *main concept* or *vital subconcept*	# of judges choosing … *trivial subconcept* or *not used*
1992B-21 † ‡	38	0
1992B-22 * •	6	32
1992B-23 * •	4	34
1992B-24 † –	22	16
1992B-25 † ‡	38	0
1992B-26 † ‡	38	0
1992B-27 † ‡	31	7
1992B-28 * •	3	35
1992B-29 * –	13	25
1992B-30 * •	9	29
1992B-31 † ‡	33	5
1992B-32 † ‡	36	2
1992B-33 * –	12	26
1992B-34 * •	6	32
1992B-35 * •	4	34
1992B-36 * •	7	31
1992B-37 * •	2	36
1992B-38 * •	2	36
1992B-39 * –	16	22
1992B-40 * –	11	27

Note: number of judges = 38

Key to partition indicators:
First column of symbols after the item number: *liberal algorithm*, where:
- † means the item is in *strongly related* partition
- * means the item is in *not strongly related* partition

Second column of symbols after the item number: *conservative algorithm*, where:
- ‡ means the item is in *strongly related* partition
- • means the item is in *not strongly related* partition
- – means the item is excluded from consideration

Appendix G Multiple-Choice Items in the *Strongly Related* Partition Defined by the Conservative Partitioning Algorithm

Appendix G presents the full text of several multiple-choice items from the Advanced Placement Examinations in Computer Science for 1984, 1988, and 1992. The items included in this appendix are those that, after the final phase of the content analysis procedure was completed, were classified as *strongly related* under the conservative partitioning algorithm.

Twenty-two multiple-choice items are included in Appendix G. The sources of the items are as follows:

- 4 items from the 1984 examination
  (1984-16, 1984-17, 1984-41, 1984-42)
- 5 items from the 1988 examination
  (1988-5, 1988-6, 1988-14, 1988-23, 1988-44)
- 4 items from the A version of the 1992 examination
  (1992A-12, 1992A-24, 1992A-30, 1992A-35)
- 5 items from the AB version of the 1992 examination
  (1992B-25, 1992B-26, 1992B-27, 1992B-31, 1992B-32)
- 4 items that were common to both versions of the 1992 examination
  (1992A-26 = 1992B-16, 1992A-27 = 1992B-17,
  1992A-31 = 1992B-18, 1992A-32 = 1992B-21)

Because the items are being presented outside of the context of the examination packet, some minor changes have been made to the introductory wording for some items. Such changes were purely cosmetic and had no effect on the content of the multiple-choice item.

1984-16&17

Questions 16-17 are based on the following program segment that searches an array. This array is sorted in increasing order and contains `Num` elements, where *`Num`* is non-negative.

```
First := 1 ;
Last := Num ;
Found := false;
while (First <= Last) and (not Found) do
   begin
      Middle := (First + Last) div 2 ;
      if Item = List[Middle] then
         Found := true
      else
         if Item < List[Middle] then
            Last := Middle - 1
         else
            First := Middle + 1
   end
```

1984-16

How many times will the body of the loop be executed if *`Num`* = `100` and *`Item`* = *`List`*`[1]`?

(A) One

(B) Three

(C) Four

(D) Five

(E) Six

1984-17

Which of the following assertions will be *`true`* every time the program segment completes execution?

(A) `(Item = List[Middle]) or not Found`

(B) `(Item = List[Middle]) and Found`

(C) `First ≤ Middle ≤ Last`

(D) `First < Last`

(E) None of the above

Note: Correct responses to 1984-16 and 1984-17 are (E) and (A), respectively;
From *The Entire 1984 AP Computer Science Examination and Key*, College Entrance Examination Board, 1986, p. 13. Adapted by permission.

1984-41

Let *x* and *y* be variables of type *real* with only positive values. Of the following, which best describes the conditions under which the *boolean* expression, *x + y = x*, can have the value *true*?

(A) Only when *y > x*

(B) Only when *y < 1*

(C) Only when *x* is much greater than y

(D) Only when the computer has 16-bit words

(E) It can never have the value *true*

Note: Correct response to 1984-41 is (C); From *The Entire 1984 AP Computer Science Examination and Key*, College Entrance Examination Board, 1986, p. 27. Adapted by permission.

1984-42

```
i := 1 ;
while (i <= Max) and (String[i] <> Symbol) do i := i + 1
```

Which of the following is a loop invariant for the while loop above; i.e., which is *true* each time the while-condition is tested?

(A) *i = Max*

(B) *i = i + 1*

(C) *String*[*j*] = *Symbol* for all *j* such that *i < j*

(D) *String*[*j*] ≠ *Symbol* for all *j* such that *i ≤ j*

(E) *String*[*j*] ≠ *Symbol* for all *j* such that *1 ≤ j < i*

Note: Correct response to 1984-42 is (E); From *The Entire 1984 AP Computer Science Examination and Key*, College Entrance Examination Board, 1986, p. 27. Adapted by permission.

1988-5

If evaluating `BBB` has no side effects, under what condition(s) can the program segment

```
while BBB do
   Block1
```

be rewritten as

```
repeat
   Block1
until not BBB
```

Without changing the effect of the code?

(A) Under no conditions

(B) If executing *`Block1`* does not affect the value of `BBB`

(C) If the value of `BBB` is *`true`* just before the segment is executed

(D) If the value of `BBB` is *`false`* just before the segment is executed

(E) Under all conditions

Note: Correct response to 1988-5 is (C); From *The 1988 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1989, p. 6. Adapted by permission.

1988-6

Consider the following declarations

```
type
   IntArrayType  =     array[1..Many] of integer ;
   StructureType =     record
                                 IntArray  : IntArrayType ;
                                 Length    : integer
                               end ;
function Search (Structure: StructureType; Key: integer)
              : integer ;
{ Precondition: 0 < Structure.Length < Many                          }
{ Postcondition:}
{     (1)   Returns i such that 0 ≤ i ≤ Structure.Length.            }
{     (2)   If positive i is returned, then                          }
{             Structure.IntArray[i] = Key .                          }
{     (3)   If 0 is returned, then Key ≠ Structure.IntArray[i]       }
{             for all i ≤ Structure.Length .                         }
   var
      Index : integer ;
begin
   Index := 1 ;
   with Structure do
      begin
         while (IntArray[Index] < Key) and (Index < Length) do
            Index := Index + 1 ;
         if IntArray[Index] = Key then
            Search := Index
         else
            Search := 0
      end
end ;
```

Which of the following should be added to the precondition of *Search*?

(A) The value of *Key* appears at least once in *Structure.IntArray* .

(B) The value of *Key* does not appear twice in *Structure.IntArray* .

(C) *Structure.IntArray* is sorted smallest to largest.

(D) *Structure.IntArray* is sorted largest to smallest.

(E) *Structure.IntArray* is unsorted.

Note: Correct response to 1988-6 is (C); From *The 1988 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1989, p. 7. Adapted by permission.

1988-14

Consider the following program segment:

```
const
   Size = 10 ;
type
   GridType = array [1..Size, 1..Size] of char ;
function YesOrNo (    Grid  :  GridType ;
                      Row,
                      Colm  :  integer ;
                      Mark  :  char ) : boolean ;
   var
      i, Count   : integer ;
      OK      : boolean ;
begin { YesOrNo }
   Count := 0 ;
   for i := 1 to Size do
      if Grid[i, Colm] = Mark then
         Count := Count + 1 ;
   OK := (Count = Size) ;
   Count := 0 ;
   for i := 1 to Size do
      if Grid[Row, i] = Mark then
         Count := Count + 1 ;
   YesOrNo := ( OK or (Count = Size) )
end ; { YesOrNo }
```

Which of the following conditions on an array *g* of type *GridType* will by itself guarantee that

```
YesOrNo (g, 1, 1, '*')
```

will have the value *true* when evaluated?

I. The element in the first row and first column is `'*'`.
II. All elements in both diagonals are `'*'`.
III. All elements in the first column are `'*'`.

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Note: Correct response to 1988-14 is (B); From *The 1988 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1989, pp. 13-14. Adapted by permission.

1988-23

If b is a *boolean* variable, then the statement `b := (b = false)` has what effect?

(A) It causes a compile-time error message

(B) It causes a run-time error message

(C) It causes *b* to have value *false* regardless of its value just before the statement was executed

(D) It always changes the value of *b*

(E) It changes the value of *b* if and only if *b* had value *true* just before the statement was executed

Note: Correct response to 1988-23 is (D); From *The 1988 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1989, p. 19. Adapted by permission.

1988-44

Consider the partially completed program below.

```
Root := 0 ;
Lim := n
while BBB do
{ Invariant: (Root)^2 ≤ n < (Lim + 1)^2 }
   begin
      < code to increment Root or decrement Lim,   >
      < leaving Invariant true                     >
   end
```

With which of the following should BBB be replaced in order for the loop above to compute an *integer* approximation of the square root of non-negative *n*?

(A) *Lim <> Root*

(B) *Lim = Root*

(C) *Root * Root <> n*

(D) *Lim * Lim <> n*

(E) *Lim * Lim = Root * Root*

Note: Correct response to 1988-44 is (A); From *The 1988 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1989, p. 32. Adapted by permission.

1992A-12

The code

```
if n = 1 then
   k := k - 1
else
   if n = 2 then
      k := k - 2 ;
```

is rewritten in the form

```
if <condition> then
   <assignment statement> ;
```

where <condition> and <assignment statement> are chosen so that the rewritten code performs the same task as the original code. Assume that both *n* and *k* are *integer* variables.

Which of the following could be used as <condition>?

I. `(n = 1) or (n = 2)`

II. `(n = 1) and (n = 2)`

III. `(n >= 1) and (n <= 2)`

(A) I only

(B) II only

(C) III only

(D) I and III

(E) II and III

Note: Correct response to 1992A-12 is (D); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, p. 14. Adapted by permission.

1992A-24

This item concerns a Pascal function named *Match* that is indicated by the following type declarations and function header

```
type IntArr = array[1..51] of integer ;
function Match(a,b : IntArr) : boolean ;
```

The function compares the first 50 elements of two arrays of integers and returns the value *true* if the elements in corresponding positions are equal, and returns *false* otherwise.

Which of the following is code for the body of the function that fits the specification given above?

(A)
```
i := 1 ;
while (i <= 50) and (a[i] = b[i]) do
   i := i + 1 ;
Match := (i > 50)
```

(B)
```
Match := false ;
for i := 1 to 50 do
   if a[i] = b[i] then
      Match := true
```

(C)
```
for i := 1 to 50 do
   Match := (a[i] = b[i])
```

(D)
```
i := 1;
while (i <= 50) and (a[i] = b[i]) do
   i:= i + 1 ;
Match := (i = 50)
```

(E)
```
i := 1 ;
repeat
   Match := (a[i] = b[i]) ;
   i:= i + 1
until not Match
```

Note: Correct response to 1992A-24 is (A); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, pp. 24-25. Adapted by permission.

1992A-26 same as 1992B-16; appears at end of Appendix G

1992A-27 same as 1992B-17; appears at end of Appendix G

1992A-30

Consider the following code fragments, where all variables are of type *integer*.

Fragment 1

```
x := n ;
y := x ;
while x > 0 do
  begin
    y := y + 1 ;
    x := x div 2 ;
  end ;
```

Fragment 2

```
x := n ;
y := x ;
if x > 0 then
  begin
    repeat
      y := y + 1;
      x := x div 2;
    until x < 0 ;
  end ;
```

Assume that the two fragments start with the same value for variable *n*. For which value(s) of *n* do the two code fragments compute the same value for variable *y*?

I. Any value less than zero

II. The value zero

III. Any value greater than zero

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

Note: Correct response to 1992A-30 is (D); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, p. 30. Adapted by permission.

1992A-31 same as 1992B-21; appears at end of Appendix G

1992A-32 same as 1992B-22; appears at end of Appendix G

1992A-35

Consider the following definitions.

```
const
   Length = <some positive integer> ;
type
   ListType = array[1..Length] of integer ;

function State(List : ListType; Value : integer) : boolean ;
var
   Counter : integer ;
   Flag : boolean ;
begin
   Flag := false ;
   for Counter := 1 to Length do
      begin
         Flag := (List[Counter] = Value) ;
      end ;
   State := Flag ;
end ;
```

Under which of the following conditions must the function above return *true*?

(A) Under all conditions

(B) Under the condition that *Value* = *List*[*Length*]

(C) Under the condition that *Value* = *List*[*i*] for some *i* such that 1 ≤ *i* ≤ *Length*

(D) Under the condition that *Value* ≠ *List*[*i*] for all *i* such that 1 ≤ *i* ≤ *Length*

(E) Under no conditions

Note: Correct response to 1992A-35 is (B); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, p. 34. Adapted by permission.

1992B-16 same as 1992A-26; appears at end of Appendix G

1992B-17 same as 1992A-27; appears at end of Appendix G

1992B-18 same as 1992A-31; appears at end of Appendix G

1992B-21 same as 1992A-32; appears at end of Appendix G

1992B-25&26

These questions concern the definition of two new *boolean* operators, "conditional and" and "conditional or," denoted cand and cor, respectively.

1992B-25

Given *boolean* expressions *First* and *Second*, the cand operator is defined as follows.

```
                    / Second if First = true
First cand Second = <
                    \ false if First = false (and in this case,
                                              Second is not evaluated)
```

In which of the following fragments could the use of cand in place of and prevent run-time errors that might otherwise occur?

I.
```
while (Node <> nil) and (Node^.Datum < NewDatum) do
   begin
      Node := Node^.Next ;
   end ;
```

II.
```
if (List[i] mod 2 = 1) and (List[i] = 5) then
   begin
      writeln('Found it!') ;
   end ;
```

III.
```
repeat
   x := 2 * x ;
until (0 <= x) and (x < 5) ;
```

(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

item 1992B-26 given on next page

Continuation of 1992B-25&26

1992B-26

Boolean operator cor is to be defined so that whenever the expression *First* or *Second* evaluates without error, the expression *First* cor *Second* also evaluates without error, and furthermore, so that *First* or *Second* = *First* cor *Second*. In some cases, evaluating *First* or *Second* will cause a run-time error, while *First* cor *Second* evaluates without error. Of the following, which is the best definition of the cor operator?

(A)
```
                        / First if Second = false
First cor Second = <
                        \ false if Second = true (and in this case,
                                                  First is not evaluated)
```

(B)
```
                        / First if Second = true
First cor Second = <
                        \ true if Second = false (and in this case,
                                                  First is not evaluated)
```

(C)
```
                        / Second if First = false
First cor Second = <
                        \ true if First = true (and in this case,
                                                Second is not evaluated)
```

(D)
```
                        / Second if First = false
First cor Second = <
                        \ false if First = true (and in this case,
                                                 Second is not evaluated)
```

(E)
```
                        / Second if First = true
First cor Second = <
                        \ true if First = false (and in this case,
                                                 Second is not evaluated)
```

Note: Correct responses to 1992B-25 and 1992B-26 are (A) and (C), respectively; From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, p. 68. Adapted by permission.

1992B-27

Assume that the following declarations have been made.

```
const
   MaxNum = <some positive integer> ;
type
   ListType = array[2..MaxNum] of boolean ;
var
   List : ListType ;
```

Consider the following code segment.

```
for i := 2 to MaxNum do
   begin
      List[i] := true ;
   end ;
for i := 2 to MaxNum do
   begin
      for j := 1 to (MaxNum div i) do
         begin
            List[i * j] := not(List[i * j]) ;
         end ;
   end ;
```

For *i* in the range 2..*MaxNum*, which of the following characterizes the entries of *List* that will have value *true* after the segment above has executed?

(A) *List*[*i*] = *true* for no values of *i*.

(B) *List*[*i*] = *true* for all values of *i*.

(C) *List*[*i*] = *true* for all values of *i* that are even.

(D) *List*[*i*] = *true* for all values of *i* that are prime.

(E) *List*[*i*] = *true* for all values of *i* that are perfect squares.

Note: Correct response to 1992B-27 is (E); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, p. 69. Adapted by permission.

1992B-31&32

are based on the following code framework.

```
var
   n,i,v : integer ;
begin
   read(n) ;
   i := 1 ;
   v := 1 ;
   while <condition> do
      begin
         <body> ;
      end ;
   writeln(v) ;
end ;
```

The placeholders <condition> and <body> are to be replaced with code so that whenever the value read into variable *n* is positive, the value output is *n*! (*n* factorial). Further, the expression *v* = *i*! is to be maintained as an invariant of the while loop.

1992B-31

Which of the following choices for <body> maintains *v* = *i*! as the loop invariant?

(A) `i := i + 1 ;` `v := v * i`

(B) `v := v * i ;` `i := i + 1`

(C) `i := i + 1 ;` `v := n * i`

(D) `v := n * i ;` `i := i + 1`

(E) `i := i * (i - 1) ;` `v := v + 1`

1992B-32

Assume that <body> has been replaced with code that maintains *v* = *i*! as the loop invariant. Which of the following choices for <condition> ensures that if the loop terminates, the value *n*! is output?

(A) *i* = *n*

(B) *i* <> *n*

(C) *i* = *v*

(D) *i* <> *v*

(E) *i* = *v* * *n*

Note: Correct responses to 1992B-31 and 1992B-32 are (A) and (B), respectively; From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, p. 73. Adapted by permission.

1992A-26
and
1992B-16

Under which of the following conditions must the following *boolean* expression have the value *true*?

```
((i <= n) and (a[i] = 0)) or ((i >= n) and (a[i - 1] = 0))
```

(A) `(i <= n) or (i >= n)`

(B) `(a[i] = 0) and (a[i - 1] = 0)`

(C) `i = n`

(D) `i < n`

(E) `i > n`

Note: Correct response to 1992A-26/1992B-16 is (B); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, pp. 27 & 63. Adapted by permission.

1992A-27
and
1992B-17

Evaluation of the *boolean* expression

```
((i <= n) and (a[i] = 0)) or ((i >= n) and (a[i - 1] = 0))
```

is *guaranteed* to cause a run-time error under which of the following conditions?

(A) `i < 0`

(B) Neither `a[i]` nor `a[i - 1]` has the value zero.

(C) Array `a` is of size *n*.

(D) Array `a` is of size 2.

(E) None of the above

Note: Correct response to 1992A-27/1992B-17 is (E); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, pp. 27 & 63. Adapted by permission.

1992A-31
and
1992B-18

Consider the following declarations.

```
type
   ListType = record
         Items : array[1..MaxLength] of integer;
         NumItems : integer ;
      end ;

procedure Find(List : ListType ; Num : integer ;
var Found : boolean ; var Loc : integer) ;
{precondition: 0 ≤ List.NumItems ≤ MaxLength}
begin
   Found := false ;
   Loc := 0;
   while not Found and (Loc < List.NumItems) do
      begin
         Loc := Loc + 1 ;
         if List.Items[Loc] = Num then
            Found := true ;
      end ;
end ;
```

Which of the following is a correct postcondition for procedure *Find*?

(A) `Found`

(B) `Found and (Loc >= List.NumItems)`

(C) `(List.Items[Loc] = Num) or (Loc = List.NumItems)`

(D) `Loc = List.NumItems`

(E) `not Found and (Loc < List.NumItems)`

Note: Correct response to 1992A-31/1992B-18 is (C); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, pp. 31 & 64. Adapted by permission.

1992A-32
and
1992B-21

The *boolean* expression

```
(Num > Max) or not(Max < Num)
```

can be simplified to

(A) `Max <> Num`

(B) `Max = Num`

(C) `(Num < Max) and not (Max < Num)`

(D) `false`

(E) `true`

Note: Correct response to 1992A-32/1992B-21 is (E); From *The 1992 Advanced Placement Examinations in Computer Science and their grading*, College Entrance Examination Board, 1993, pp. 32 & 66. Adapted by permission.

Appendix H Reliability of Individual Judges

Appendix H provides an overview of the reliability of individual judges during the final phase of the content analysis procedure. Figures H.1 through H.4 present graphs that show the reliability of each judge on the 1984, 1988, 1992 version A, and 1992 version AB examination packets respectively. Judge numbering across the examinations is consistent, that is, judge number *x* on the 1984 examination is also judge number *x* on the 1988 and 1992 examinations.

Individual judge reliability indicates the extent to which an individual judge was the source of unreliable data (Krippendorff, 1980). Each graph shows the overall reliability values under the liberal and conservative partitioning algorithms (graphed as horizontal lines), individual judge reliability under the liberal algorithm (graphed as a black diamond), and individual judge reliability under the conservative algorithm (graphed as a white circle). In this study, individual judge reliability was calculated based on the test-test condition by which the ratings were generated. In a test-test situation, the reliability of particular individual is calculated by comparing the outcome of that individual's ratings of the items to the pooled ratings by all of the other judges. (This is in contrast to a test-standard condition, where a judge's rating performance during a training session would be compared to a pre-existing standard. The test-standard condition could be used for the purpose of identifying highly capable judges and for eliminating inconsistent judges, important issues when accuracy is a key goal of the content analysis procedure.)

Figure H.1, which reports judge reliability on the 1984 examination, includes two judges, #24 and #26, whose agreement coefficients differed drastically from those of the other judges. Under the conservative partitioning algorithm, these two judges' reliability was –.30 to –.40, indicating a fairly systematic deviation from the classifications assigned by the other 36 judges. In 1988 (see Figure H.2), the reliability of judge #26 was still rather different than the reliability of the remaining judges, particularly under the conservative partitioning algorithm. On the 1992 examinations (see Figures H.3 and H.4), the reliability of judge #26 was comparatively low on the A version but nearer that of other judges on the AB version.

In 1984 as well as in other years, several other judges consistently had lower reliability than the others (although they differed less radically than the judges in the previous paragraph). For example, the reliability of judge #9 hovered at about .25 for the liberal partitioning algorithm on the 1984, 1988, and 1992A examinations and increased to about .40 for the 1992AB examination. Individual reliability for judge #34 was about the same as the individual reliability of the other judges in 1984 and 1988 but, for both versions of the 1992 examination, was lowest under the liberal partitioning algorithm and among the lowest under the conservative partitioning algorithm.

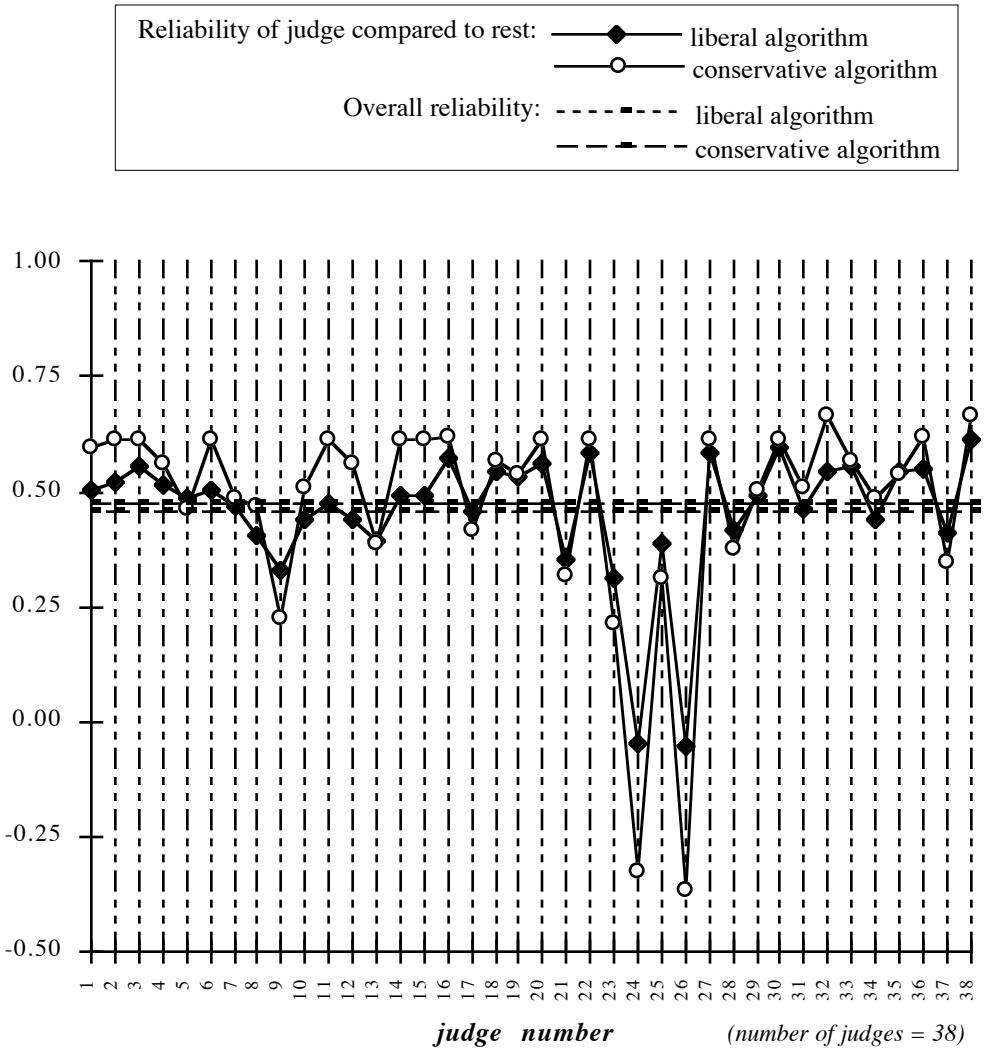
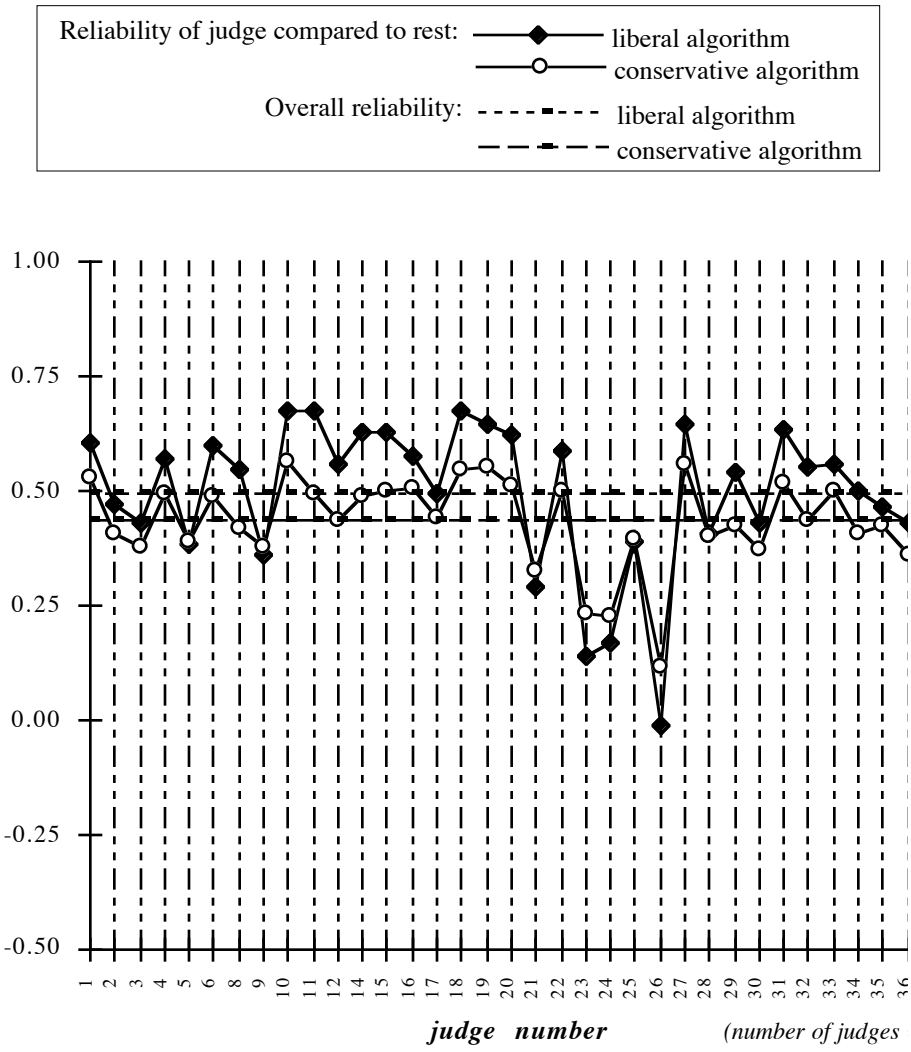


Figure H.1 Comparison of Agreement Coefficients of Individual Judges on Examination Packet for 1984 under Liberal and Conservative Partitioning Algorithms



Note: Judges #7 and #13 did not complete the content analysis procedure for this examination.

Figure H.2 Comparison of Agreement Coefficients of Individual Judges on Examination Packet for 1988 under Liberal and Conservative Partitioning Algorithms

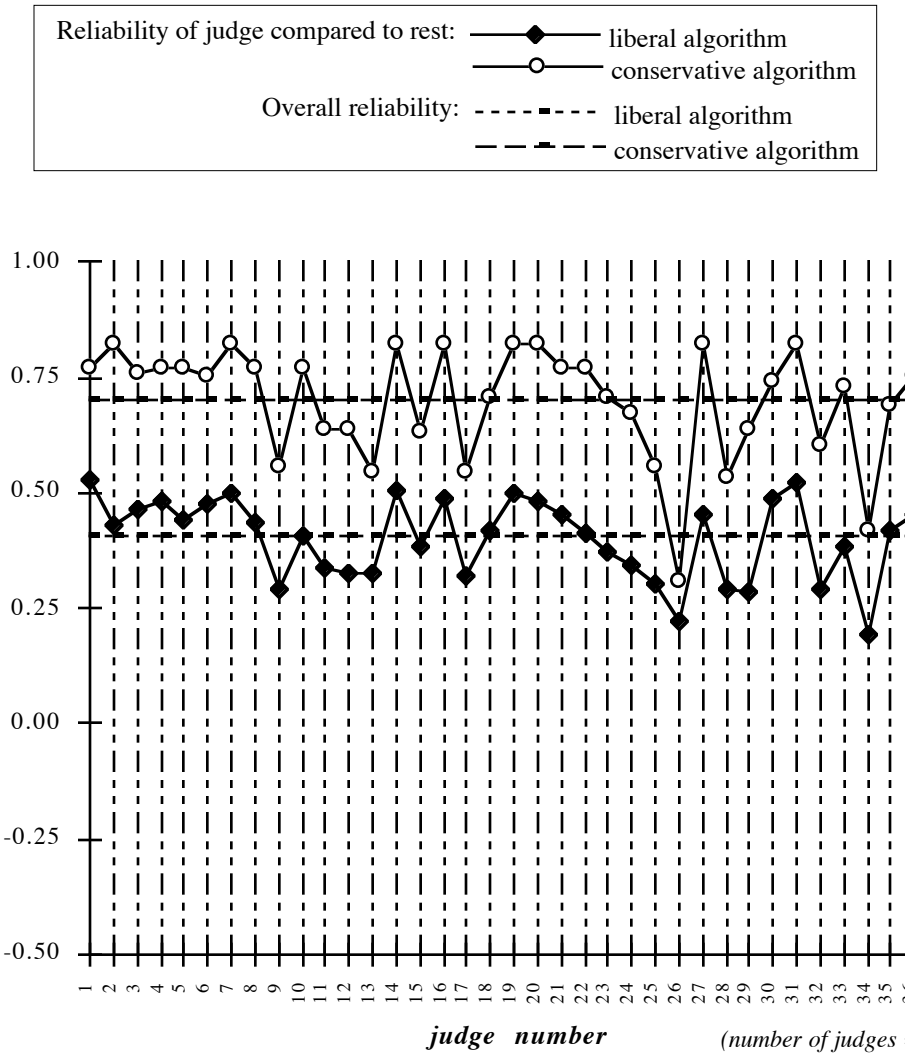


Figure H.3 Comparison of Agreement Coefficients of Individual Judges on Examination Packet for Version A of 1992 Examination under Liberal and Conservative Partitioning Algorithms

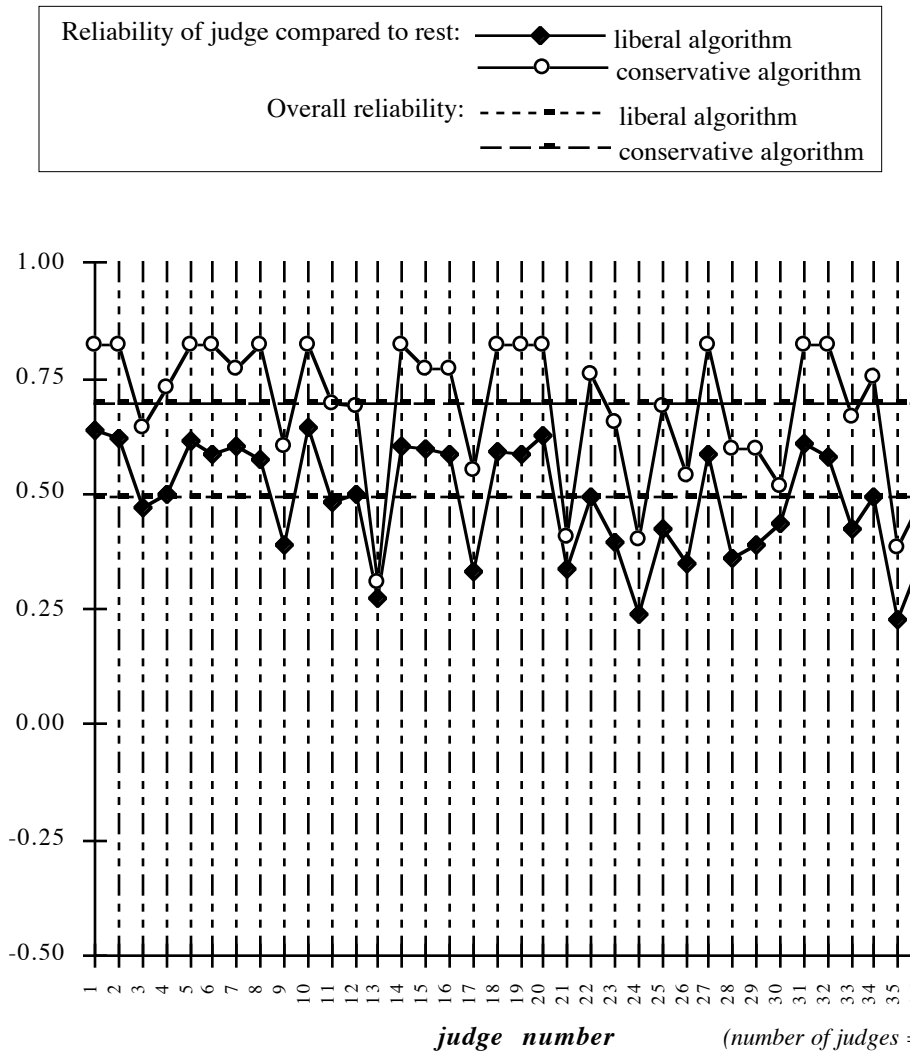


Figure H.4 Comparison of Agreement Coefficients of Individual Judges on Examination Packet for Version AB of 1992 Examination under Liberal and Conservative Partitioning Algorithms

Appendix I Rating Comparison for Duplicate Items on the A and AB Versions of the 1992 APCS Examination

Appendix I compares the judging results for the 15 items that appeared on both the A and AB versions of the 1992 APCS examination. The results for the duplicate items are considered in terms of the paired ratings from each judge for the A and AB versions of the examination packets. A rating pair is expressed as the pair of categories (p_{ij} , q_{ij}), where p_{ij} is the rating given by judge i to duplicate item j on the one version of the examination and q_{ij} is the rating given by judge i to duplicate item j on the other version of the examination. The item numbering corresponds to that given in Table 4.9. In doing the content analysis, the judge was free to complete the examination packets in any order. As a result, each rating pair is unordered with respect to the two versions of the examination. Instead, differences in ratings will be ordered by the categories 'main concept', 'vital subconcept', 'trivial subconcept', and 'not used' without regard to the version for which the rating was given.

For every rating pair, either

$p_{ij} = q_{ij}$ (the judge gave the same rating to the item on both versions of the examination), or

$p_{ij} \neq q_{ij}$ (the judge gave different ratings to the item on the two versions of the examination).

Tables I.1 and I.2 are 15X2 tables, with one row for each duplicate item. Each entry in Table I.1 is the number of judges for whom $p_{ij} = q_{ij}$, while each

entry in Table I.2 is the number of judges for whom $p_{ij} \neq q_{ij}$. (For both Table I.1 and Table I.2, he sum of the row marginals is 15, the number of duplicate items.)

Tables I.3 and I.4 are 38X2 tables, with one row for each individual judge. Each entry in Table I.3 shows the number of items for which $p_{ij} = q_{ij}$, while each entry in Table I.4 shows the numbers of items for which $p_{ij} \neq q_{ij}$. (For both Table I.3 and Table I.4, the sum of the row marginals in is 38, the number of judges who rated the items.)

In all of the tables, the column marginals for the consistently-rated items are much higher than the column marginals for the inconsistently-rated items. (The consistently-rated items are those in Tables I.1 and I.3, where $p_{ij} = q_{ij}$; the inconsistently-rated items are those in Tables I.2 and I.4, where $p_{ij} \neq q_{ij}$.) In considering the all ratings pairs with $p_{ij} \neq q_{ij}$, four classes of mismatches emerge:

- The rating pair was either (‘main concept’, ‘vital subconcept’) or (‘trivial subconcept’, ‘not used’). Since these combinations did not affect the pooling of the categories into the dichotomous scale of *not strongly related* and *strongly related*, they did not affect the outcome.
- The rating pair was (‘vital subconcept’, ‘trivial subconcept’). This combination straddled the dividing line between the dichotomous categories *not strongly related* and *strongly related*, so complicated the process of simplifying the data.
- The rating pair was either (‘vital subconcept’, ‘not used’) or (‘main concept’, ‘trivial subconcept’). This combination was a more extreme version of the second class of mismatch, since these combinations could

be considered to have a greater "distance" between them, thus making the process of simplifying the data more complicated.

- The rating pair was ('main concept', 'not used'). Since this pairing represents a complete change of opinion, this rating mismatch is the most troublesome.

In the final content analysis results, 76% of the rating pairs were consistent. Of the remaining 24% of the ratings pairs, 53% fell into the first class of rating mismatch, 21% fell into the second class of rating mismatch, 22% fell into the third class of rating mismatch, and only 4% fell into the fourth class of rating mismatch. (These figures represent 13%, 5%, 5%, and 1% of the total number of ratings pairs respectively.)

Table I.1 Number of Judges Giving Same Rating to Duplicate Items in the Content Analysis of the A and AB Versions of the 1992 APCS Examination

	rating on A /AB version of 1992 examination				
item numbers	*main / main*	*vs / vs*	*triv / triv*	*nu / nu*	*totals:*
92A-02 / 92B-02	0	3	11	22	*36*
92A-05 / 92B-05	2	6	16	5	*29*
92A-07 / 92B-07	0	1	2	35	*38*
92A-08 / 92B-04	8	11	6	2	*27*
92A-09 / 92B-09	0	10	18	1	*29*
92A-10 / 92B-10	0	12	17	2	*31*
92A-16 / 92B-16	10	0	0	0	*10*
92A-17 / 92B-17	1	1	0	0	*2*
92A-20 / 92B-06	9	12	7	0	*28*
92A-28 / 92B-28	1	1	5	26	*33*
92A-29 / 92B-29	2	10	15	5	*32*
92A-31 / 92B-18	25	7	0	0	*32*
92A-32 / 92B-21	38	0	0	0	*38*
92A-37 / 92B-37	0	1	5	28	*34*
92A-38 / 92B-38	0	1	5	28	*34*
totals:	*96*	*76*	*107*	*154*	*433*

Note: *main* is 'main concept'; *vs* is 'vital subconcept'; *triv* is 'trivial subconcept'; *nu* is 'not used'

Table I.2 Number of Judges Giving Different Ratings to Duplicate Items in the Content Analysis of the A and AB Versions of the 1992 APCS Examination

rating on A /AB version of 1992 examination

item numbers	*main / vs*	*triv / nu*	*vs / triv*	*main / triv*	*vs / nu*	*main / nu*	*totals:*
92A-02 / 92B-02	0	1	1	0	0	0	*2*
92A-05 / 92B-05	2	5	1	0	1	0	*9*
92A-07 / 92B-07	0	0	0	0	0	0	*0*
92A-08 / 92B-04	7	2	2	0	0	0	*11*
92A-09 / 92B-09	1	3	4	0	1	0	*9*
92A-10 / 92B-10	0	3	2	0	2	0	*7*
92A-16 / 92B-16	17	0	0	8	0	3	*28*
92A-17 / 92B-17	14	1	5	14	0	2	*36*
92A-20 / 92B-06	3	1	5	1	0	0	*10*
92A-28 / 92B-28	0	3	2	0	0	0	*5*
92A-29 / 92B-29	0	1	4	0	1	0	*6*
92A-31 / 92B-18	4	0	1	0	0	1	*6*
92A-32 / 92B-21	0	0	0	0	0	0	*0*
92A-37 / 92B-37	0	2	1	0	1	0	*4*
92A-38 / 92B-38	0	2	1	0	1	0	*4*
totals:	*48*	*24*	*29*	*23*	*7*	*6*	*137*

Note: *main* is 'main concept'; *vs* is 'vital subconcept';
triv is 'trivial subconcept'; *nu* is 'not used'

Table I.3 Number of Duplicate Items Given Same Rating in the Content Analysis of the A and AB Versions of the 1992 APCS Examination

	rating on A /AB version of 1992 examination				
	main / main	*vs / vs*	*triv / triv*	*nu / nu*	*totals:*
Judge #1	5	0	4	5	*14*
Judge #2	1	1	2	3	*7*
Judge #3	1	3	0	7	*11*
Judge #4	2	4	2	5	*13*
Judge #5	3	2	0	5	*10*
Judge #6	1	2	7	1	*11*
Judge #7	2	1	0	5	*8*
Judge #8	1	1	6	5	*13*
Judge #9	4	5	0	2	*11*
Judge #10	2	0	3	4	*9*
Judge #11	2	0	0	5	*7*
Judge #12	1	1	3	7	*12*
Judge #13	5	1	3	0	*9*
Judge #14	2	1	3	6	*12*
Judge #15	2	2	4	5	*13*
Judge #16	4	0	3	6	*13*
Judge #17	6	3	0	5	*14*
Judge #18	2	0	3	6	*11*
Judge #19	1	2	3	6	*12*
Judge #20	2	1	4	4	*11*
Judge #21	2	1	4	4	*11*
Judge #22	3	5	1	4	*13*
Judge #23	4	4	3	3	*14*
Judge #24	4	1	3	1	*9*
Judge #25	7	3	3	1	*14*
Judge #26	1	8	3	1	*13*
Judge #27	1	3	0	5	*9*
Judge #28	1	9	0	4	*14*
Judge #29	1	3	2	4	*10*
Judge #30	2	0	6	3	*11*
Judge #31	2	1	7	3	*13*
Judge #32	2	0	6	5	*13*
Judge #33	3	4	2	5	*14*

Table continued on following page

Continuation of Table I.3.

	rating on A /AB version of 1992 examination				
	main / main	*vs / vs*	*triv / triv*	*nu / nu*	*totals:*
Judge #34	3	1	1	2	*7*
Judge #35	4	2	1	5	*12*
Judge #36	2	0	6	2	*10*
Judge #37	3	1	4	4	*12*
Judge #38	2	0	5	6	*13*
totals:	*98*	*76*	*107*	*154*	*433*

Note: *main* is 'main concept'; *vs* is 'vital subconcept'; *triv* is 'trivial subconcept'; *nu* is 'not used'

Table I.4 Number of Duplicate Items Given Different Ratings in the Content Analysis of the A and AB Versions of the 1992 APCS Examination

	rating on A /AB version of 1992 examination						
	main / vs	*triv / nu*	*vs / triv*	*main / triv*	*vs / nu*	*main / nu*	*totals:*
Judge #1	0	0	0	1	0	0	*1*
Judge #2	2	4	1	1	0	0	*8*
Judge #3	1	0	0	0	1	2	*4*
Judge #4	2	0	0	0	0	0	*2*
Judge #5	3	0	2	0	0	0	*5*
Judge #6	1	1	2	0	0	0	*4*
Judge #7	2	2	1	0	1	1	*7*
Judge #8	0	0	1	1	0	0	*2*
Judge #9	2	2	0	0	0	0	*4*
Judge #10	1	2	1	2	0	0	*6*
Judge #11	0	6	1	1	0	0	*8*
Judge #12	1	0	1	1	0	0	*3*
Judge #13	2	0	4	0	0	0	*6*
Judge #14	2	0	0	1	0	0	*3*
Judge #15	0	0	0	2	0	0	*2*
Judge #16	0	0	0	2	0	0	*2*
Judge #17	1	0	0	0	0	0	*1*
Judge #18	2	1	0	1	0	0	*4*
Judge #19	0	0	1	2	0	0	*3*

Table continued on following page

Continuation of Table I.4.

rating on A /AB version of 1992 examination

	main / vs	*triv / nu*	*vs / triv*	*main / triv*	*vs / nu*	*main / nu*	*totals:*
Judge #20	1	1	2	0	0	0	*4*
Judge #21	1	0	0	1	2	0	*4*
Judge #22	2	0	0	0	0	0	*2*
Judge #23	1	0	0	0	0	0	*1*
Judge #24	3	0	2	0	0	1	*6*
Judge #25	1	0	0	0	0	0	*1*
Judge #26	1	0	0	1	0	0	*2*
Judge #27	4	1	1	0	0	0	*6*
Judge #28	1	0	0	0	0	0	*1*
Judge #29	2	1	2	0	0	0	*5*
Judge #30	3	0	1	0	0	0	*4*
Judge #31	1	0	0	1	0	0	*2*
Judge #32	1	0	0	1	0	0	*2*
Judge #33	1	0	0	0	0	0	*1*
Judge #34	2	2	1	0	3	0	*8*
Judge #35	1	0	1	1	0	0	*3*
Judge #36	0	0	3	2	0	0	*5*
Judge #37	0	1	1	0	0	1	*3*
Judge #38	0	0	0	1	0	1	*2*
totals:	*45*	*24*	*29*	*24*	*7*	*6*	*137*

Note: *main* is 'main concept'; *vs* is 'vital subconcept';
triv is 'trivial subconcept'; *nu* is 'not used'

Glossary

abstract datatype: A datatype described only at the logical level, without details of implementation.

ACM: The Association for Computing Machinery, one of the key professional organizations in the field of computer science. Provides a variety of forums for the dissemination of technical information and discussions of issues important to the computing profession.

APCS: Advanced Placement Computer Science, one of the subject-area examinations offered annually by ETS to high school students.

ASL: The Association for Symbolic Logic, a worldwide organization dedicated to the study of logic.

assertion: A predicate within the context of a program. The predicate asserts what must true about the program state at that point.

boolean: An abstract datatype that is based on the set of values {*true*, *false*}[7]. The operations are the logical connectives.

boolean expression: An expression constructed from boolean constants, boolean variables, boolean operations, and operations from other domains whose result type is boolean (e.g., 9□+□3□=□12, 7□<□4, ax□+□by□≠□c, 'xy'□<□'xyz'). [8]

classical logic: A branch of logic that assumes that the values *true* and *false* form a dichotomy, so that anything that is not *true* is *false* and vice versa.

constant: A computational object associated with a particular datatype; has an unchanging and unchangeable value from the datatype domain.

content analysis: A research technique for making replicable and valid inferences from data to their context.

data object: A variable or constant; defined by means of a datatype.

data structure: Representation of a data object within a computer program.

---

[7] Because these values are part of a set, the values of type boolean considered in "pure" form are unordered. However, some programming languages impose an artificial ordering on these values. In Pascal, for example, boolean is implicitly defined by the declaration

```
type boolean = (false, true)
```

which defines `false < true` to be valid.

[8] Note that, while the expression 7 < 4 evaluates to "false", it is still a valid boolean expression.

data type: See datatype.

datatype: The formal description of the characteristics of a group of related data objects; includes a domain (or set of distinct values), a collection of relationships among the values of the domain, and a set of operations on the values.[9]

ETS: Educational Testing Service, the organization that provides the operational services for the College Entrance Examination Board. Responsible for the Advanced Placement Program and the Graduate Record Examination Program, as well as other programs.

identifier: A sequence of one or more digits and letters; the name of a data object, datatype, proposition, or predicate.

IEEE: The Institute of Electronics and Electrical Engineers; has as its purpose to advance the theory and practice of computer science and engineering and to promote the exchange of technical information.

ISO: International Standards Organisation.

logic: A science that deals with the rules and criteria of valid inference and demonstration; the science of the formal principles of reasoning.

logical connectives: Operators defined over values of type boolean; include negation ("not" or ¬), conjunction ("and" or ∧), disjunction ("or" or ∨), implication ("implies" or ⊃), and equality ("equals" or =; also "equavales" or ≡).

LID: Language-Independent Datatype, the topic of an ISO standard (1994).

MAA: Mathematical Association of America.

mathematical logic: A branch of logic that uses a formalized system consisting of primitive symbols, combinations of these symbols, axioms, and rules of inference. Also called symbolic logic, logistic.

---

[9] Whether datatype is one word or two is an unresolved issue. B. Meeks has argued that the one-word spelling conveys that this is a "reserved word" denoting a specific concept, not to be confused with other notions of "types" of data. The translation into French of "data type" was established as "type des donne's", literally the "type of the data", which could be interpreted in many ways, whereas the translation of "datatype" would be "type de donne's" — the "type of data", which implied reference to some specific system of categories. A quick survey of programming language standards revealed that only Fortran refers to "data type" and only Prolog uses "datatype". COBOL refers to "classes of data". The Ada, Pascal, C/C++, and SmallTalk standards refer only to "type". (E. Barkmeyer, personal communication, April 12, 1994)

operator: A mathematical or logical symbol denoting an operation to be performed.

postcondition: The assertion that is the second half of the specification of a sequential program or statement; specifies that which must hold after the program or statement is executed.

precondition: The assertion that is the first half of the specification of a sequential program or statement; specifies that which must hold before the program or statement is executed.

predicate: A formula of the predicate calculus.

predicate calculus: The branch of mathematical logic that uses symbols for quantifiers and for arguments and predicates of propositions as well as for unanalyzed propositions and logical connectives; also called functional calculus.

program space: The set of data objects that are defined at a particular time during execution of a computer program.

program state: The values of the data objects in the program space at a particular time during execution of a computer program.

proposition: An expression in language or signs of something that can be believed, doubted, or denied or is either true or false. A symbolic proposition is formed according to the following rules: (1) *true* and *false* are propositions; (2) an identifier is a proposition; (3) if *b* is a proposition, then so is ¬ b; and (4) if b and c are propositions, then so are (a ∧ b), (a ∨ b), (a ⇒ b), and (a ≡ b).

propositional calculus: The branch of mathematical logic that uses symbols for unanalyzed propositions and logical connectives only; also called sentential calculus.

PLT: Propositional Logic Test.

specification: A formal description of the desired behavior of a computer program or other code fragment. In sequential programs, the specification consists of a pair of assertions, the precondition and the postcondition.

two-valued logic: Propositional logic specifically restricted to a domain of two values.

type: A synonym for “datatype” when used in the context of describing a data object.

variable: A computational object associated with a particular datatype; has a specific value from the datatype domain at any given time and may have different values of the same domain at different times.

Credits:

- The data-related definitions were influenced by Dale & Walker (in press).
- The calculus and boolean definitions have been influenced by Gries (1981), ISO (1994), and Tucker, Bradley, Cupper, & Garnick (1992).
- The logic definitions have been influenced by Cumbee (1993) & *Webster* (1972).
- The definition of content analysis is based on Krippendorff (1980, p. 21).
- The definition of operator is suggested by *Webster* (1972).

Bibliography

Anderson, J. R. (1980). Cognitive Psychology and its implications. San Francisco: W. H. Freeman.

ASL. (in press). Association of Symbolic Logic Guidelines for Logic Education. *Bulletin of Symbolic Logic*.

Atchison, W. F. (chair). (1968). Curriculum '68: Recommendations for academic programs in computer science. *Communications of the ACM*, *11*(3), 151–197.

Austing, R. H. (chair). (1979). Curriculum '78: Recommendation for the undergraduate program in computer science. *Communications of the ACM*, *22*(3), 147–166.

Baker, D., & VanHarlingen, D. (1979, March). The relationship of Piagetian and Piagetian-like tasks to physics achievement. Paper presented at the meeting of the National Association for Research in Science Teaching, Atlanta, GA (abs.).

Belnap, N. D, Jr., & Grover, D. L. (1973). Quantifying in and out of quotes. In H. Leblanc (Ed.), *Truth, syntax and modality: Proceedings of the Temple University Conference on Alternative Semantics* (pp. 17–47). Amsterdam: North-Holland.

Bennett, R. E., Rock, D. A., & Wang, M. (1991). Equivalence of free-response and multiple-choice items. *Journal of Educational Measurement*, *28*, 77–92.

Berztiss, A. (1987). A mathematically focused curriculum for computer science. *Communications of the ACM*, *30*(5), 356–365.

Boute, R. T. (1990). A heretical view on type embedding. *SIGPLAN Notices*, *25*(1), January. 25–28.

Boute, R. T. (1991). Letter to the Editor. *SIGPLAN Notices*, *26*(2), February. 9–10.

Church, A. (1956). *Introduction to Mathematical Logic* (Vol. 1). Princeton, NJ: Princeton University Press.

College Board. (1986). *The Entire 1984 AP Computer Science Examination and Key*. College Entrance Examination Board.

College Board. (1989). *The 1988 Advanced Placement Examinations in Computer Science and their grading*. College Entrance Examination Board.

College Board. (1990). *Advanced Placement Course Description: Computer Science*. College Entrance Examination Board. May 1991 version.

College Board. (1993). *The 1992 Advanced Placement Examinations in Computer Science and their grading*. College Entrance Examination Board.

Cooke, J. (1992). Formal methods — mathematics, theory, recipes, or what? *The Computer Journal*, *35*(5), 419–423.

Copi, I. M. (1979). *Symbolic Logic* (5th ed.). New York: Macmillan.

Cumbee, J. (1993). Mathematical logic. *Academic American Encyclopedia* [electronic data file from database on UTCAT PLUS system]. Danbury, CT: Grolier Electronic Publishing, September update from 1991 version.

Dale, N., & Walker, H. (in press). *Abstract data types: Specification, implementation, and application*. Lexington, MA: D. C. Heath.

Denning, P. J. (chair). (1989). Computing as a discipline. *Communications of the ACM*, *32*(1), 9–23.

Dijkstra, E. W. (1968). A constructive approach to the problem of program correctness. *BIT*, *8*, 174–186.

Dijkstra, E. W. (1976). *A discipline of programming*. Englewood Cliffs, NJ: Prentice-Hall.

Dijkstra, E. W. (1989). On the cruelty of really teaching computing science. *Communications of the ACM*, *32*(12), 1398–1404.

Dijkstra, E. W., & Feijen, W. H. J. (1988). *A method of programming*. Menlo Park, CA: Addison-Wesley.

Dijkstra, E. W., & Scholten, C. S. (1990). *Predicate calculus and program semantics*. New York: Springer-Verlag.

Enyeart, M., VanHarlingen, D., & Baker, D. (1980). The correlation of inductive and deductive reasoning to college physics achievement. *Journal of Research in Science Teaching*, *17*(3), 262–267.

Evans, J. St. B. T. (1980). Current issues in the psychology of reasoning. *British Journal of Psychology*, *71*, 227–239.

Floyd, R. W. (1967). Assigning meaning to programs. *Proceedings of the American Mathematical Society Symposia in Applied Mathematics*, *19*, 19–32.

Franzblau, D. (1993). New models for courses in discrete mathematics. *SIAG DM News*, November 15.

Furth, H. G. (1969). *Piaget and knowledge: Theoretical foundations*. Englewood Cliffs, NJ.

Galton, A. (1992). Logic as a Formal Method. *The Computer Journal*, *35*(5), 431–440.

Gardner, H. (1985). *Frames of mind: The theory of multiple intelligences*. Basic Books.

Gardner, H., & Hatch, T. (1989). Multiple Intelligences go to school: Educational implications of the theory of multiple intelligences. *Educational Researcher*, November, 4–10.

Gibbs, N. E., & Tucker, A. B. (1986). A model curriculum for a liberal arts degree in computer science. *Communications of the ACM*, *29*(3), 202–210.

Ginsberg, H., & Opper, S. (1979). *Piaget's theory of intellectual development* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Goldson, D., Reeves, S., & Bornat, R. (1993). A review of several programs for the teaching of logic. *The Computer Journal*, *36*(4), 373–386.

Gries, D. (1981). *The Science of Programming*. New York: Springer-Verlag.

Gries, D. (1990). Calculation and discrimination: A more effective curriculum. *Communications of the ACM*. *34*(3). 44–55.

Gries, D., & Schneider, F. B. (1993a). *A logical approach to discrete mathematics*. New York: Springer-Verlag.

Gries, D., & Schneider, F. B. (1993b). *Instructor's manual: A logical approach to discrete mathematics*. Ithaca, NY: Computer Science Department, Cornell, University.

Gries, D., & Schneider, F. B. (1994). *A new approach to teaching mathematics*. Unpublished manuscript, Cornell University, Computer Science Department, Ithaca, NY.

Guilford, J. P. (1967). *The nature of human intelligence.* New York: McGraw-Hill.

Hasenjaeger, G. (1972). *Introduction to the basic concepts and problems of modern logic*. Dordrecht, Holland: D. Reidel Publishing.

Hilbert, D., & Ackermann, W. (1950). *Principles of mathematical logic*. New York: Chelsea Publishing.

Hoare, C. A. R. (1969). An axiomatic basis for computer programming. *Communications of the ACM*, *12*, 576–583. (Reprinted in *Communications of the ACM*, 1983, *26*(1), 53–56).

IEEE Computer Society Education Committee. (1976). *A Curriculum in Computer Science and Engineering.* IEEE Computer Society, Model Curriculum Subcommittee.

IEEE Model Program Committee. (1983). *The 1983 IEEE Computer Society Model Program in Computer Science and Engineering*. IEEE Computer Society. Educational Activities Board.

Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence*. New York: Basic Books.

Irons, E. T. (1961). A syntax directed compiler for ALGOL 60. *Communications of the ACM*, *4*(1), 51–55.

ISO. (1994). *Information technology — Language-independent datatypes.* ISO/IEC draft International Standard 11404: 1994, Geneva: International Organization for Standardization.

Jensen, K., & Wirth, N. (1974). *Pascal User's Manual*. New York: Springer-Verlag.

Koffman, E. B., Miller, P. L., & Wardle C. E. (1984). Recommended Curriculum for CS1, 1984. *Communications of the ACM*, *27*(10), 998–1001.

Koffman, E. B., Stempel, D., & Wardle C. E. (1985). Recommended Curriculum for CS2, 1984. *Communications of the ACM*, *28*(8), 815–818.

Krippendorff, K. (1980). *Content Analysis: An Introduction to Its Methodology*. Beverly Hills, CA: Sage Publications.

Lefrancois, G. R. (1988). *Psychology for Teaching* (6th ed.). Belmont, CA: Wadsworth Publishing Company.

Lewis, C. I., & Langford, C. H. (1959). *Symbolic Logic* (2nd ed.). New York: Dover.

Lockwood, W., Pallrand, G. & VanHarlingen, D. (1980, April). The relationship between logical operators and achievement in university physics. Paper presented at the meeting of the National Association for Research in Science Teaching, Boston, MA (abs.).

Lockwood, W., Pallrand, G., & VanHarlingen, D. (1982, April). Logical ability and achievement in high school level physics. Paper presented at the meeting of the National Association for Research in Science Teaching, Lake Geneva, WI (abs.).

Meeks, B. (1990). Two-valued datatypes. *SIGPLAN Notices*, *25*(8) (August) 75–79.

Meeks, B. (1991). Letter to the Editor. *SIGPLAN Notices*, *26*(8), August. 24.

Merritt, S. (chair). (1993). *ACM Model High School Computer Science Curriculum*. New York: Association for Computing Machinery, Task Force of the Pre-College Committee of the ACM Education Board.

Murfin, B. E. (1993). An Analysis of Computer-Mediated Communication between Urban Middle School Students and Scientists (Urban Education) [CD-ROM]. Abstract on ProQuest: Dissertation Abstracts On Disc, January, 1993 – February 1994. Available: UMI, Order No. AAC 9325561. (Doctoral Dissertation, Ohio State University; reference to *Dissertation Abstracts International*, 5405A, p. 1770, Nov. 1993).

Myers, Jr., J. P. (1990). The central role of mathematical logic in computer science. *SIGCSE Bulletin*, *22*(1), 22–26.

Nocolescu, R. (1991). Letter to the Editor. *SIGPLAN Notices*, *26*(2), February. 9–10.

Pallrand, G., & VanHarlingen, D. (1980, April). Cognitive structures and achievement in physics. Paper presented at the meeting of the National Association for Research in Science Teaching, Boston, MA (abs.)

Parsons, A. (1958). Translator's introduction: A guide for psychologists. In B. Inhelder & J. Piaget, *The growth of logical thinking from childhood to adolescence*, New York: Basic Books.

Petrushka, D. (1984). *A study of the effect of content on the ability to do syllogistic reasoning: An investigation of transferability and the effect of practice.* Unpublished doctoral dissertation, Rutgers University, NJ.

Piburn, M. D. (1989). Reliability and validity of the Propositional Logic Test. *Educational and Psychological Measurement*, *49*, 667–672.

Piburn, M. D. (1990). Reasoning about logical propositions and success in science. *Journal of Research in Science Teaching*, *27*(9), 887-900.

Piburn, M. D., & Baker, D. (1988, April). Reasoning about logical propositions and success in science. Paper presented at the meeting of the American Educational Research Association, New Orleans, LA.

PLT Key. (1989). Key to the Propositional Logic Test. Unpublished document. Obtained through M. Piburn, Arizona State University.

Pollack, S. V. (Ed.). (1982a) *Studies in Computer Science*. The Mathematical Association of America.

Pollack, S. V. (1982b). The development of computer science. In S. V. Pollack (ed.), *Studies in Computer Science*, The Mathematical Association of America.

Popkin, R. H. (1993a). Philosophy. *Academic American Encyclopedia* [electronic data file from database on UTCAT PLUS system]. Danbury, CT: Grolier Electronic Publishing, December update; copyright 1991.

Popkin, R. H. (1993b). Traditional logic. *Academic American Encyclopedia* [electronic data file from database on UTCAT PLUS system]. Danbury, CT: Grolier Electronic Publishing, September update; copyright 1991.

Powers, D. E., & Enright, M. K. (1987). Analytic reasoning skills involved in graduate study: Perceptions of faculty in six fields. *Journal of Higher Education*, *58*, pp. 658-682.

ProQuest: Dissertation Abstracts On Disc [CD-ROM]. (January, 1993 – February, 1994). Available: UMI.

Proulx, V. K., & Wolf, C. E. (1993). Appendix F: Breadth approach using applications and programming modules. In Merritt, S. (chair), *ACM Model High School Computer Science Curriculum*, 1993, New York: Association for Computing Machinery, Task Force of the Pre-College Committee of the ACM Education Board.

Ralston, A. (Ed.) (1989). *Discrete Mathematics in the First Two Years.* MAA Notes No. 15. The Mathematical Association of America.

Ralston, A., & Shaw, M. (1980). Curriculum '78 — Is computer science really that unmathematical? *Communications of the ACM*, *23*(2), 67–70.

Romberg, T. A. (1989) *Curriculum and Evaluation Standards for School Mathematics*. National Council of Teachers of Mathematics

Rothaug, W. (1984). *Logical connectives and community college science course achievement*. Unpublished doctoral dissertation, Rutgers University, NJ.

Rothaug, W., & Pallrand, G. (1982, April). *Reasoning skills and science course achievement*. Paper presented at the meeting of the National Association for Research in Science Teaching, Lake Geneva, WI (abs.)

Rothaug, W., Pallrand, G., & VanHarlingen, D. (1981, April). *Logical operations and achievement in community college students.* Paper presented at the meeting of the National Association for Research in Science Teaching, Grossingers, NY (abs.)

Saiedian, H. (1992). Mathematics of computing. *Computer Science Education*, *3*(3), 203-221.

Sakkinen, M. (1990). On embedding Boolean as a subtype of Integer. *SIGPLAN Notices*, *25*(7), 95–96.

Seeber, F., Pallrand, G., VandenBerg, G., & VanHarlingen, D. (1979, April). *Logical ability, formal thought and achievement in physics*. Paper presented at the meeting of the National Association for Research in Science Teaching, Atlanta, GA (abs.)

Shaw, M. (Ed.). (1985). *The Carnegie-Mellon Curriculum for Undergraduate Computer Science*. New York: Springer-Verlag.

Siegel, M. (1989a). Afterthoughts: Discrete Mathematics in the First Two Years. In A. Ralston (ed.), *Discrete Mathematics in the First Two Years*, MAA Notes No. 15, The Mathematical Association of America.

Siegel, M. (1989b). Final report of the MAA committee on discrete mathematics in the first two years. In A. Ralston (ed.), *Discrete Mathematics in the First Two Years*, MAA Notes No. 15, The Mathematical Association of America.

Sperschneider, V., & Antoniou, G. (1991). *Logic: A foundation for computer science*. International Computer Science Series. Reading, MA: Addison-Wesley.

Spresser, D. M., and LePera, T. (1992). Comparative review of 10 discrete mathematics textbooks. *Computing Reviews*, February, 100–108.

Stager-Snow, D. (1985). *Analytical ability, logical reasoning and attitude as predictors of success in an introductory course in computer science for*

non-computer science majors. (Doctoral dissertation, Rutgers University, NJ). *Dissertation Abstracts International*, *45*, 08A, p. 2473, February, 1985.

Stofflett, R. T., & Baker, D. R. (1992). The effects of training in combinatorial reasoning and propositional logic on formal reasoning ability in high school students. *Research in Middle Level Education*, *16*(1), 159–177.

Stolyar, A. A. (1970). *Introduction to elementary mathematical logic*. Cambridge, MA: MIT Press.

Stone, P. J., Dunphy, D., Smith, M. S., & Ogilvie, D. M. (1966). *The General Inquirer: A Computer Approach to Content Analysis*. Cambridge, MA: The M.I.T. Press.

Tobin, K. G., and Capie, W. (1981). The development and validation of a group test of logical thinking. *Educational and Psychological Measurement*, *41*, 413–423.

Tucker, A. B. (Ed.) (1990). *Computing Curricula 1991: Report of the ACM/IEEE-CS Joint Curriculum Task Force.* Final Draft, December 17. ACM Order Number 201910. IEEE Computer Society Press Order Number 2220.

Tucker, A. B., Bradley, W. J., Cupper, R. D., & Garnick, D. K. (1992). *Fundamentals of Computing I.* New York: McGraw-Hill.

Warford, J. S. (in press). Book review of *A Logical Approach to Discrete Math* (1993) by D. Gries and F. B. Schneider. *Computing Reviews*.

Webster's Seventh New Collegiate Dictionary. (1972). Springfield, MA: G. & C. Merriam Co.

Whitehead, A. N. (1911). *An Introduction to Mathematics*. Oxford, England: Oxford University Press. (From I. M. Copi, 1979, *Symbolic Logic*, 5th ed., New York: Macmillan)

Wickens, T. D. (1989). *Multiway contingency tables analysis for the social sciences.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Wing, J. M. (1990). A specifier's introduction to formal methods. *Computer*, *23*(9), 8–24.

Wirth, N. (1976). *Algorithms + Data Structures = Programs*. Englewood Cliffs, NJ: Prentice-Hall.

Vita

Vicki Lynn Almstrum was born February 13, 1955, in Tilden, Nebraska, the daughter of Billy Harrison Almstrum and Lucille Alice Oman Almstrum. She graduated Cum Laude from Arizona State University with a Bachelor of Arts in Education degree, emphasis secondary education, in December, 1977, with a major in mathematics and a minor in German. She continued her studies at Arizona State University, receiving the Master of Arts degree in May, 1980, with a major in mathematics and a minor in computer science. From 1980 to 1984 she worked as a computer scientist at Motorola, Inc. in Tempe, Arizona. In 1984, she moved to Järfälla, Sweden where she worked as a computer scientist for Philips Electronics until 1988. On her return to the United States, she began her graduate studies in the Department of Computer Sciences at the University of Texas at Austin. She received the Master of Science in Computer Sciences degree in December, 1990, and was admitted to candidacy in the doctoral program of Mathematics Education in March, 1991, specializing in computer science education. While pursuing her graduate work, both at Arizona State University and the University of Texas at Austin, she worked as a teaching assistant for undergraduate mathematics and computing science classes. She is a member of the Association for Computing Machinery, the ACM Special Interest Groups on Computer Science Education (SIGCSE), Software Engineering (SIGSoft), and Computer and Human Interaction (SIGCHI), the Institute for Electrical and Electronics Engineers and its Computer Society, Computer Professionals for

Social Responsibility, and Upsilon Pi Epsilon, an honor society in Computer Science. She is married to Torgny Stadler, with whom she enjoys travel, two-stepping, and life's many wonders.

Permanent address:

1412 West 39th 1/2 Street

Austin, Texas 78756

Internet address: almstrum@cs.utexas.edu

This dissertation was prepared by the author using Microsoft® Word version 5.1, Microsoft® Excel version 4.0, Aldus® PageMaker® version 5.0, and SPSS for the Macintosh® version 4.0.