Regular Algebraic Curve Segments (III) –Applications in Interactive Design and Data Fitting

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Abstract

In this paper (part three of the trilogy) we use low degree G^1 and G^2 continuous regular algebraic spline curves defined within parallelograms, to interpolate an ordered set of data points in the plane. We explicitly characterize curve families whose members have the required interpolating properties and possess a minimal number of inflection points. The regular algebraic spline curves considered here have many attractive features: They are easy to construct. There exist convenient geometric control handles to locally modify the shape of the curve. The error of the approximation is controllable. Since the spline curve is always inside the parallelogram, the error of the fit is bounded by the size of the parallelogram. The spline curve can be rapidly displayed, even though the algebraic curve segments are implicitly defined.

Key words: Algebraic curve; tensor product; polygonal chain, parallelogram.

1 Introduction

We use segments of low degree algebraic curves $G_{mn}(u,v) = 0$ in tensor product Bernstein-Bézier(BB) form defined within a parallelogram or rectangle to construct G^1 and G^2 splines. A tensor product BB-form polynomial $G_{mn}(u,v) = \sum_{i=0}^{m}\sum_{n=0}^{n} b_{ij}B_i^m(u)B_j^n(v)$ of bi-degree (m,n) has total degree $m+n$, however,

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the class of $G_{mn}(u,v)$ is a subset of polynomials of total degree $m+n$. G^1 (resp G^2) continuity implies curve segments share the same tangent(curvature) at join points(knots). In each of the G^1 and G^2 constructions, we develop a spline curve family whose member satisfies given interpolation conditions. Each family depends on one free parameter that is related linearly to coefficients of $G_{mn}(u,v)$. Prior work on using algebraic curve spline in data interpolation and fitting focus on using bivariate barycentric BB–form polynomials defined on plane triangles(see [10, 9] for references). Compared with A-spline segments defined in triangular (barycentric) BB-form [1], these algebraic curve segments in tensor product form have the following distinct features: (a) They are easy to construct. The coefficients of the bivariate polynomial that define the curve are explicitly given. (b) There exist convenient geometric control handles to locally modify the shape of the curve, essential for interactive curve design. (c) The spline curves, for the rectangle scheme, are ϵ-error controllable where ϵ is the pre-specified width of the rectangle. This feature is especially important for fitting to "noisy" data with uncertainty or decimated data. (d) These splines curves have a minimal number of inflection points. Each curve segment of the spline curve has either no inflection points if the corresponding edge is convex, or one inflection point otherwise. (e). Since the required bi-degree (m,n) for G^1 and G^2 is low(in this paper, $\min\{m,n\} \leq 2$), the curve can be evaluated and displayed extremely fast. We explore both display via parameterization as well as recursive subdivision techniques(see [6]). (f) In the six spline families we discuss in sections 3 and 4, there are four cases with $\min\{m,n\} = 1$. In these cases, rational parametric expressions are easily derived. Hence, for these cases, we have both the implicit form and the parametric form. Such dual form curves prove useful in several geometric design and computer graphics applications. (g) In treating a non-convex edge in the triangular scheme(see [1]), we need to break the edge into two parts by inserting an artificial inflection point. In the present parallelogram or rectangle scheme, we need not divide the edge, and the inflection point occurs only when necessitated by the end point interpolating conditions. These features make these error-bounded regular algebraic spline curves promising in applications such as interactive font design, image contouring etc.

In the first two parts [10, 9] of this trilogy of papers, we have introduced the concept of a discriminating family of curves by which regular algebraic curve segments are isolated. Using different discriminating families, several characterizations of the Bernstein-Bézier (BB) form of the implicitly defined real bivariate polynomials over the plane triangle and the parallelogram are given, so that the zero contours of the polynomials define smooth and single sheeted real algebraic (called regular) curve segments. In this part three of the trilogy of papers, we characterize the lowest bi-degree of tensor BB-form polynomial to achieve G^1 and G^2 continuous regular algebraic spline curves. Using the lowest bi-degree, we construct explicit spline curve families whose members satisfy given G^1 and G^2 interpolation conditions. We also derive a geometric interpretation of each spline curve family, so that the shape of the individual curves can be intuitively controlled.

The rest of the paper is as follows. In section 2 we show how a number of data