Set Identities

Double Negation	
$p \equiv \neg(\neg p)$	$A = \sim(\sim A)$
Equivalence	
$(p \equiv q) \equiv (p \to q) \wedge (q \to p)$	$(A = B) \equiv (A \subseteq B) \wedge (B \subseteq A)$
Idempotence	
$(p \wedge p) \equiv p$	$(A \cap A) = A$
$(p \vee p) \equiv p$	$(A \cup A) = A$
De Morgan	
$(\neg(p \wedge q)) \equiv (\neg p \vee \neg q)$	$\sim(A \cap B) = \sim A \cup \sim B$
$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$	$\sim(A \cup B) = \sim A \cap \sim B$
Commutativity of *or*	**Commutativity of Union**
$(p \vee q) \equiv (q \vee p)$	$(A \cup B) = (B \cup A)$
Commutativity of *and*:	**Commutativity of Intersection**
$(p \wedge q) \equiv (q \wedge p)$	$(A \cap B) = (B \cap A)$
Associativity of *or*:	**Associativity of Union**
$(p \vee (q \vee r)) \equiv ((p \vee q) \vee r)$	$A \cup (B \cup C) = (A \cup B) \cup C$
Associativity of *and*:	**Associativity of Intersection**
$(p \wedge (q \wedge r)) \equiv ((p \wedge q) \wedge r)$	$A \cap (B \cap C) = (A \cap B) \cap C$
Distributivity of *and* over *or*:	**Distributivity of Intersection over Union**
$(p \wedge (q \vee r)) \equiv ((p \wedge q) \vee (p \wedge r))$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Distributivity of *or* over *and*:	**Distributivity of Union over Intersection**
$(p \vee (q \wedge r)) \equiv ((p \vee q) \wedge (p \vee r))$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Conditional Disjunction:	
$(p \to q) \equiv (\neg p \vee q)$	$(A \subseteq B) \equiv ((\sim A \cup B) = U)$
Contrapositive:	
$(p \to q) \equiv (\neg q \to \neg p)$	$(A \subseteq B) \equiv (\sim B \subseteq \sim A)$
Law of the Excluded Middle:	
$p \vee \neg p$	$A \cup \sim A = U$

Set Inference Rules

Modus Ponens:		
From p and $p \to q$, infer q	From $(x \in A) \wedge (A \subseteq B)$,	infer $x \in B$
Modus Tollens:		
From $p \to q$ and $\neg q$, infer $\neg p$	From $(A \subseteq B) \wedge (x \notin B)$,	infer $x \notin A$
Disjunctive Syllogism:		
From $p \vee q$ and $\neg q$, infer p	From $(x \in A \cup B) \wedge (x \notin B)$,	infer $x \in A$
Simplification:		
From $p \wedge q$, infer p	From $(x \in A \cap B)$,	infer $x \in A$
Addition:		
From p, infer $p \vee q$	From $(x \in A)$,	infer $x \in A \cup B$
Conjunction:		
From p and q, infer $p \wedge q$	From $(x \in A) \wedge (x \in B)$,	infer $x \in A \cap B$
Hypothetical Syllogism:		
From $p \to q$ and $q \to r$, infer $p \to r$	From $(A \subseteq B) \wedge (B \subseteq C)$,	infer $A \subseteq C$
Contradictory Premises:		
From p and $\neg p$, infer q		$A \cap \sim A = \varnothing$
Resolution:		
From $p \vee q$ and $\neg p \vee r$, infer $q \vee r$	From $(x \in A \cup B) \wedge (x \in \sim B \cup C)$, infer $x \in A \cup C$	