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An open letter to Ross Honsberger.
University of Waterloo.

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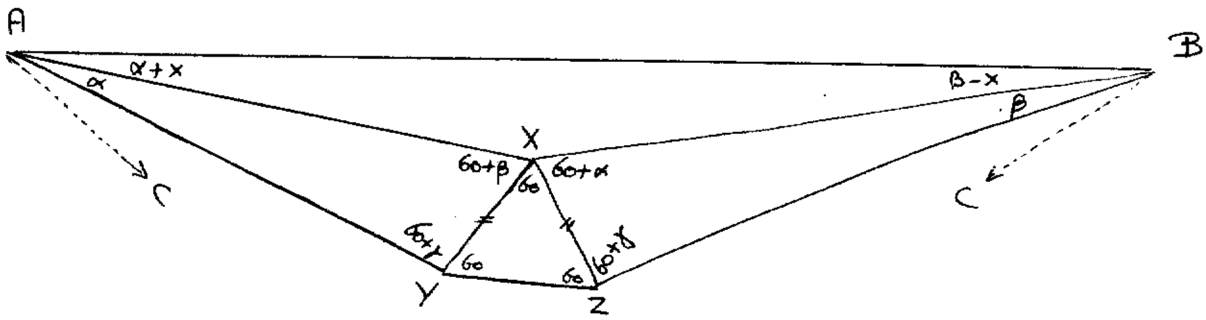
Dear Sir,

the other day I encountered your delightful booklet "Mathematical Gems". On account of Chapter 8, I concluded that you might be interested in the following proof of Morley's Theorem "The adjacent pairs of the trisectors of a triangle always meet at the vertices of an equilateral triangle."



Choose $\alpha, \beta \,\&\, \gamma > 0$ such that $\alpha+\beta+\gamma = 60°$. Draw an equilateral triangle XYZ and construct the triangles AXY and BXZ with the angles as indicated. Because $\angle AXB = 180° - (\alpha+\beta)$, it follows that, if $\angle BAX = \alpha + x$, $\angle ABX = \beta - x$. Using the rule of sines three times (in $\triangle AXB$, $\triangle AXY$, and $\triangle BXZ$), we deduce

$$\frac{\sin(\alpha+x)}{\sin(\beta-x)} = \frac{BX}{AX} = \frac{XZ.\ \sin(60+\gamma)/\sin(\beta)}{XY.\ \sin(60+\gamma)/\sin(\alpha)} = \frac{\sin(\alpha)}{\sin(\beta)}$$

Because in the range considered, this equation has a left-hand-side which is a monotonically increasing function of x (on account of the monotonicity of $\sin(\varphi)$ in the first quadrant) we conclude $x = 0$. Thus Morley's Theorem is proved without any additional lines. I found this proof in the early sixties, but am afraid that I did not publish it. Yours ever,

Edsger W. Dijkstra