Comments on Arbeitsblatt 3 from o.Prof.Dr.F.L.Bauer.

by Edsger W.Dijkstra and C.S.Scholten

For the integer function $m(x, y)$ of the integer arguments x and y, given by

$$m(x, y) = (\ x \geq y \rightarrow x+1\ [\!]\ x < y \rightarrow y-1) \qquad (1)$$

it is required to prove that

$$(x = y \rightarrow y+1\ [\!]\ x \neq y \rightarrow m(x, m(x-1, y+1))) = m(x, y) \qquad (2).$$

Proof. From (1) follows that the first alternative in the left-hand side of (2) may be replaced by $x = y \rightarrow m(x, y)$; the proof is complete when we show that the second alternative can be replaced by $x \neq y \rightarrow m(x, y)$. This latter replacement is allowed as we shall prove

$$m(x, m(x-1, y+1)) = m(x, y) \qquad (3)$$

From (1) follows

$$x \geq y \Rightarrow (m(x, x) = m(x, y)) \qquad (4)$$

From (1) it also follows that the only possible values of $m(x-1, y+1)$ are x and y. If $m(x-1, y+1) = y$, (3) holds trivially. Otherwise we have $m(x-1, y+1) \neq y$, which implies --on account of (1)--

$$m(x-1, y+1) = x \text{ \underline{and} } x-1 \geq y+1 \qquad (5)$$

As (5) implies $x \geq y$, we conclude by (4) and (5) that (3) then holds as well. (End of proof.)

The above proof has been given, because 33(!) lines of formal text, as dedicated to a proof of this theorem in said Arbeitsblatt 3, is to our tastes a bit unwieldy. We appreciate that the proof in said Arbeitsblatt 3 has been conducted in terms of a limited repertoire of operations that seem suitable for mechanization. When, however, the length of such proofs and their obvious tedium are presented --as is often the case-- as conclusive evidence for the necessity of such mechanizations, it should be clear that at least the above example has failed to convince us of such necessity.

Plataanstraat 5
NL-4565 NUENEN
The Netherlands

prof.dr.Edsger W.Dijkstra
Burroughs Research Fellow