About 2-coloured 6-graphs.

Let each of the 15 edges of the complete 6-graph be either red or blue. Three of the 6 nodes are said to form a "homogeneous triangle" if the three edges connecting them are of the same colour. Prove the existence of at least two homogeneous triangles.

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We call two edges that meet at a node and are of different colour a "mixed pair" meeting at that node. Because at any node 5 edges meet, at most $2 * 3 = 6$ mixed pairs meet at that node. Because we have 6 nodes, there are at most $6 * 6 = 36$ mixed pairs. Because each mixed pair occurs in one inhomogeneous triangle and each inhomogeneous triangle contains two mixed pairs, we have at most $36/2 = 18$ inhomogeneous triangles. The total number of distinct triangles being $(6*5*4)/(3*2*1) = 20$, we have at least $20 - 18 = 2$ homogeneous triangles.

Plataanstraat 5
5671 AL NUENEN
the Netherlands

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prof. dr. Edsger W. Dijkstra
Burroughs Research Fellow